FEL theory: From LEUTL to LCLS

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Coherence in particle and photon beams: Past, Present, and Future Symposium
Free Electron Lasers

- Produced by **resonant interaction** of a relativistic electron beam with EM radiation in an undulator.

\[ \lambda_r = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2}\right) \]

- Radiation intensity \( \propto N^2 \)
- Tunable, Powerful, Coherent radiation sources
Self-Amplified Spontaneous Emission (SASE)

- Initiated by electron shot noise (spontaneous emission) and amplified over a narrow frequency bandwidth \( \sigma_\omega \sim \rho \omega_r \)


\[
\frac{dP}{d\omega} = g_A \left( \frac{dP_0}{d\omega} + g_S \frac{\rho \gamma_0 mc^2}{2\pi} \right) \exp \left( \frac{z}{L_G} - \frac{\Delta \omega^2}{2\sigma_\omega^2} \right)
\]

input power

\( g_A \cdot \text{effective start-up noise power } \approx \text{undulator radiation over } 2L_G \)

- To determine the 3D effects including diffraction and finite beam size, one must solve the initial value problem in terms of a set of guided modes (first introduced by G. Moore)
Vlasov-Maxwell formalism

- The interaction between the electron beam and the FEL radiation can be described in the framework of the Vlasov-Maxwell equations.

- The e-beam is described in terms of a distribution function $F = F(\theta, \eta, x, p; z)$ in 6D-phase space. In view of the importance of stochastic effects such as shot noise, we use the Klimontovich distribution:

$$F(\theta, \eta, x, p; z) = \frac{k_1}{n_e} \sum_{j=1}^{N_e} \delta[\theta - \theta_j(z)] \delta[\eta - \eta_j(z)] \times \delta[x - x_j(z)] \delta[p - p_j(z)],$$

where $n_e$: on-axis electron number density

- The distribution function is governed by the Vlasov equation

$$\frac{\partial F}{\partial z} + \frac{d\theta}{dz} \frac{\partial F}{\partial \theta} + \frac{d\eta}{dz} \frac{\partial F}{\partial \eta} + \frac{dx}{dz} \cdot \frac{\partial F}{\partial x} + \frac{dp}{dz} \cdot \frac{\partial F}{\partial p} = 0,$$

K.-J. Kim, PRL 57, 1871 (1986)
Van Kampen’s normal mode expansion

- After linearizing Vlasov Eq., we seek the self-similar, guided eigenmodes of the FEL. These are solutions of the form:

\[
\Psi = \begin{bmatrix}
a_n(\hat{x}; \hat{z}) \\
f_n(\hat{\eta}, \hat{x}, \hat{p}, \hat{z})
\end{bmatrix} = e^{-i\mu_\ell \hat{z}} \begin{bmatrix}
A_\ell(\hat{x}) \\
F_\ell(\hat{x}, \hat{p}, \hat{\eta})
\end{bmatrix}
\]

- They are characterized by a constant growth rate \(\mu_\ell\) and a \(z\)-independent radiation/density mode profile \(A_\ell/F_\ell\) (Optical guiding)

- Substituting into the Vlasov-Maxwell (FEL) equations, we obtain two coupled relations for the growth rate and the mode amplitudes:

\[
\begin{bmatrix}
\mu_\ell A_\ell + \left(-\frac{\Delta v}{2\rho} + \frac{1}{2} \nabla_\perp^2 \right) A_\ell + i \int d\hat{p} d\hat{\eta} F_\ell \\
\mu_\ell F_\ell + i A_\ell \frac{\partial f_0}{\partial \hat{\eta}} + \left\{ -v \dot{\theta} + i \left( \hat{p} \cdot \frac{\partial}{\partial \hat{x}} - \hat{k}_\beta^2 \hat{x} \cdot \frac{\partial}{\partial \hat{p}} \right) \right\} F_\ell
\end{bmatrix} = 0.
\]
3D solution

- Using Gaussian distributions, we obtain an explicit dispersion relation:

\[
\left( \mu - \frac{\Delta \nu}{2\rho} + \frac{1}{2} \hat{\nu}^2 \right) A(\hat{x}) - \frac{1}{2\pi \hat{k}_\beta^2 \hat{\sigma}_x^2} \int_{-\infty}^{0} d\tau \, \tau e^{-\hat{\sigma}_\eta^2 \tau^2 / 2 - i\mu \tau} \\
\times \int d\hat{p} \, A[\hat{x}_+(\hat{x}, \hat{p}, \tau)] \exp \left[ -\frac{1 + i\tau \hat{k}_\beta^2 \hat{\sigma}_x^2}{2\hat{k}_\beta^2 \hat{\sigma}_x^2} \left( \hat{p}^2 + \hat{k}_\beta^2 \hat{x}^2 \right) \right] = 0.
\]

- There are four dimensionless parameters that affect the growth rate (L.-H. Yu, Krinsky, Gluckstern, Phy. Rev. Lett. 64, 1990)
  - $\hat{\sigma}_x$ is a quantitative measure of the diffraction effect
  - $\hat{\sigma}_x \hat{k}_\beta$ is a measure of the emittance effect
  - $\hat{\sigma}_\eta$ represents the energy spread effect
  - $\Delta \nu / (2\rho)$ is scaled frequency detuning

- Ming Xie obtained a fitting formula that captures all these effects for FEL designs (1995)
Kwang-Je at APS since 1998

• Our work was largely supported by an Argonne LDRD to do “Comprehensive Analysis of SASE”.
• In 1998-2002 we studied short-pulse effects, harmonic generation, 3D SASE start-up, FEL saturation, CSR microbunching instability...
• Together we published >20 journal publications and numerous conference papers, and went to some nice workshops too!

Three-dimensional analysis of harmonic generation in high-gain free-electron lasers
Zhirong Huang and Kwang-Je Kim
Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439

Formulas for coherent synchrotron radiation microbunching in a bunch compressor chicane
Zhirong Huang* and Kwang-Je Kim
Argonne National Laboratory, Argonne, Illinois 60439

Review of x-ray free-electron laser theory
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Argonne National Laboratory, Argonne, Illinois 60439, USA

Sardinia beach (Italy 2002)
LOW-ENERGY UNDULATOR TEST LINE PARAMETERS

PROJECT GOALS

- Perform experiments with the SASE FEL output
- Assess the challenges associated with producing a SASE FEL in preparation for an x-ray regime machine

First SASE saturation (2001)!

Exponential Gain and Saturation of a Self-Amplified Spontaneous Emission Free-Electron Laser

S. V. Milton,1 E. Gluskin,1 N. D. Arnold,1 C. Benson,1 W. Berg,1 S. G. Biedron,1,2 M. Borland,1 Y.-C. Chae,1 R. J. Dejus,1 P. K. Den Hartog,1 B. Deriy,1 M. Erdmann,1 Y. I. Eidelman,1 M. W. Hahne,1 Z. Huang,1 K.-J. Kim,1 J. W. Lewellen,1 Y. Li,1 A. H. Lumpkin,1 O. Makarov,1 E. R. Moog,1 A. Nassiri,1 V. Sajaev,1 R. Soliday,1 B. J. Tieman,1 E. M. Trakhtenberg,1 G. Travish,1 I. B. Vasserman,1 N. A. Vinokurov,1 X. J. Wang, G. Wiemerslage,1 B. X. Yang1

Radiation Pulse Energy [$\text{J}$]

$z$ [meters]
Nonlinear Harmonic Radiation at VISA*

Energy Comparison

<table>
<thead>
<tr>
<th>Mode (n)</th>
<th>Wavelength (nm)</th>
<th>Energy (µJ)</th>
<th>% of E1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>845</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>421</td>
<td>.93</td>
<td>1.8</td>
</tr>
<tr>
<td>3</td>
<td>280</td>
<td>.40</td>
<td>.77</td>
</tr>
</tbody>
</table>

Using the relation of 2nd and 3rd harmonic energies as given by Z. Huang and K.J.Kim:

\[
E_2 = \left( \frac{K}{\gamma k_u \sigma_x} \right)^2 \left( \frac{K_2}{K_3} \right)^2 \left( \frac{b_2}{b_3} \right)^2 E_3
\]

b - bunching parameters
K_n - Coupling coefficients

* A. Tremaine, XJ Wang et al., PRL (2002)
Onto LCLS

• I left Chicago for the Sunny California in late 2002.
• It was realized that undulator wakefield-induced energy loss is an important effect for the LCLS (5 mm gap for >100 m)

![Diagram showing energy deviation vs. time for no wake and with wake.](image)

- Compensate the average energy loss by tapering undulator with XTCAV (Y. Ding)

• Tapered undulator keeps FEL resonance and increase power.
• But, undulator wakefield makes time-dependent energy loss and hence taper only works for the average loss.
• FEL resonance cannot be kept for every slice of the bunch.
• This led to FEL power degradation.
FEL with slowly varying beam and undulator parameters

- E-beam energy $\gamma_c(z)$, undulator parameter $K(z)$

- Initial resonant wavelength $\lambda_0 = \frac{2\pi}{k_0} = \frac{\lambda_u}{2\gamma_c(0)^2} \left[ 1 + \frac{K(0)^2}{2} \right]$

- Resonant energy $\gamma_r(z) = \sqrt{\frac{\lambda_u}{2\lambda_0} \left[ 1 + \frac{K(z)^2}{2} \right]}$

- Longitudinal motion is described by

  $$\theta = (k_0 + k_u)z - ck_0 t^* \quad \text{(ponderomotive phase)}$$

  $$\eta = \frac{\gamma(z) - \gamma_c(z)}{\rho \gamma_c(0)} \quad \text{(normalized energy, change only due to FEL)}$$

\[
\frac{d\theta}{dz} = 2k_u \frac{\gamma(z) - \gamma_r(z)}{\gamma_c(0)} = 2k_u \rho \left[ \eta + \frac{\gamma_c(z) - \gamma_r(z)}{\rho \gamma_c(0)} \right]
\]

\[
\frac{d\eta}{dz} \propto E \cos(\theta + \phi) \quad \text{(E and $\phi$ are radiation field and phase)}
\]

WKB approximation

- Well-known technique in QM for slowly-varying potential

- FEL is characterized by $\rho$: the relative gain bandwidth is a few $\rho$, and radiation field gain length $\sim \frac{\lambda_u}{4\pi\rho}$

- Relative change in beam energy w.r.t resonant energy

$$\delta(z) = \frac{1}{\rho} \frac{\gamma_c(z) - \gamma_r(z)}{\gamma_c(0)}$$

- Apply WKB technique if the relative energy change per field gain length is smaller than $\rho$, i.e.,

$$\left| \frac{d\delta}{d\tau} \right| < 1, \quad \tau = 2\rho k_u z = \frac{z}{\lambda_u/(4\pi\rho)}$$

- We then extend the WKB analysis to 3D via Van Kampen’s method of mode expansion.
Comparison w/ simulations

- Radiation power dependence on $\delta$ is a gaussian

$$P(\delta; z) = P_m(z) \exp \left[ -\frac{1}{2} \left( \frac{\delta(z)-\delta_m(z)}{\sqrt{3}\sigma_\omega/\rho} \right)^2 \right]$$

- GENESIS simulation of LCLS power vs. $\delta$,
  - Power enhancement $\sim 2$ when energy gain $2\rho$ at saturation

- Power vs. $\delta\rho$ has RMS $\sqrt{3}\sigma_\omega$

  FWHM $4\sigma_\omega$ ($\sim 4\rho$ at saturation)
This study led to abandoning 1-nC LCLS.
Teach FEL theory in USPAS

- KJK and I started the USPAS teaching in 2000, later joined force with Ryan. In total we have taught 8 USPAS sessions (+1 this coming summer).

- The lecture notes were steadily improved and became a textbook published by Cambridge Press in 2017.

- The book is translated into Chinese in 2018.

(Kwang-Je Kim -> 金光齐 -> Coherent radiation)
MAY THE COHERENCE BE WITH YOU
The law of Nature tells us to enjoy as we may. Why spoil our joy by sheer vanity of life?

Poem by Fu Du (Tang dynasty, 758)
Calligraphy by T.-D. Lee (Nobel Laureate 1957)