Analytical approaches to wakefield calculations

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Outline

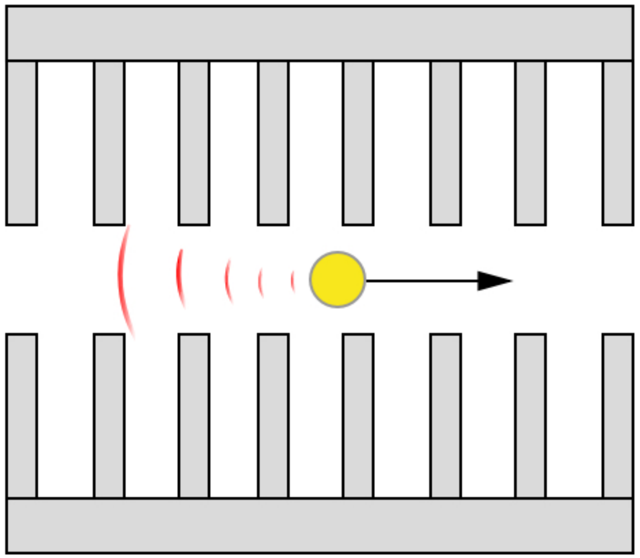
- Introduction to the wakefield concepts
- Simple calculation of the wakefields
- Wakefield based devices

Introduction to the wakefield concepts

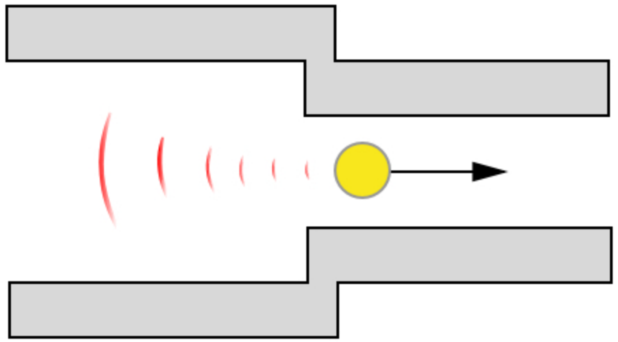
Wakefield. Friend or foe?

- Beam energy spread
- Parasitic transverse momentum induction
- Negative effect on beam dynamics

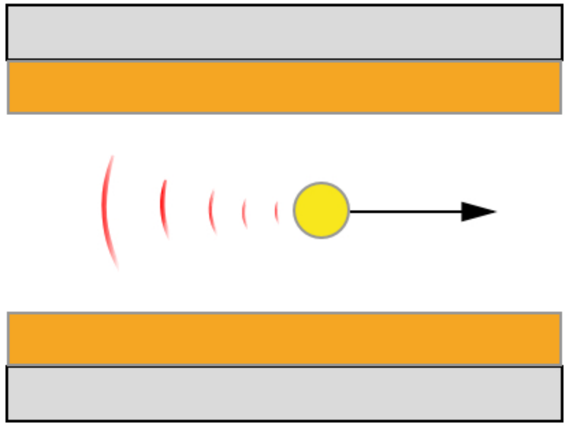
Source of the wakefield



Disk loaded structure

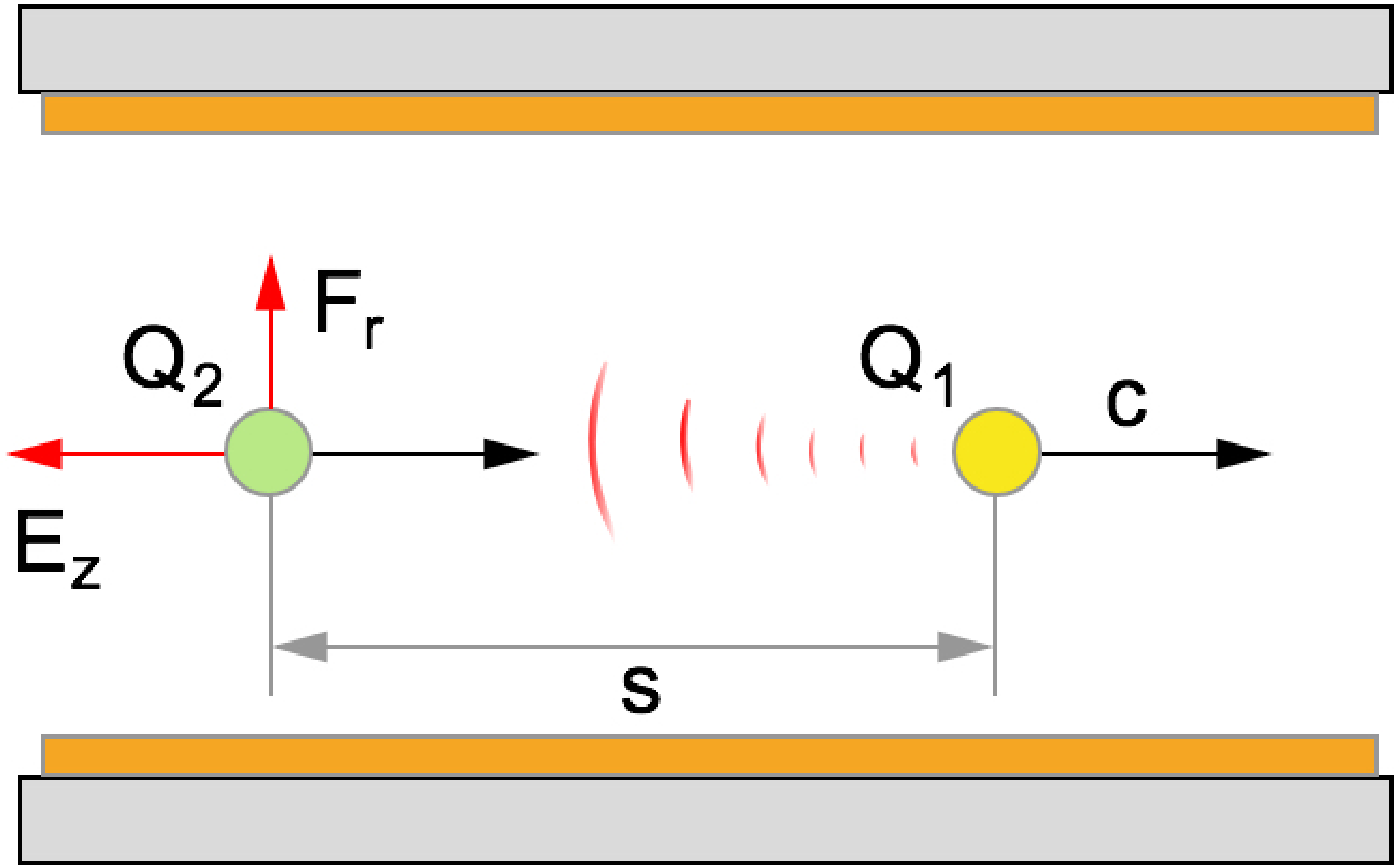


Collimators
and pipe transitions



Structures with
the “retarding” layer

Definition of the wakefield



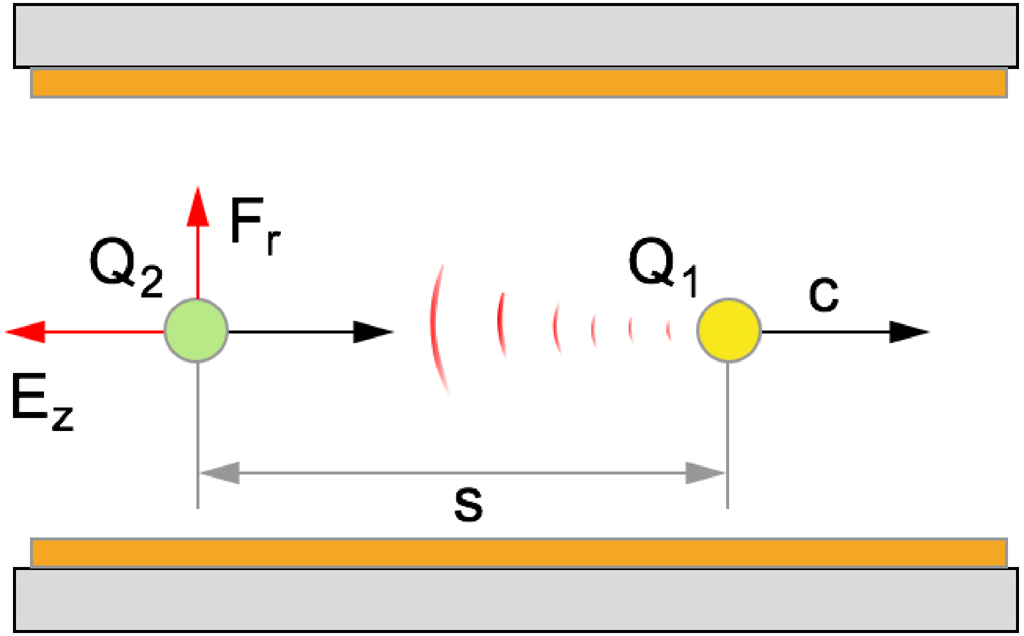
Definition of the wakefield potential

Longitudinal wakefield potential

$$W_z = -\frac{1}{Q_1}\int_{-\infty}^{\infty} E_z\left(z, t = \frac{z+s}{c}\right) dz$$

Transverse wakefield potential

$$\vec{W}_\perp = \frac{1}{Q_1}\int_{-\infty}^{\infty} \vec{F}\left(z, t = \frac{z+s}{c}\right) dz$$



Energy change of the test particle

$$\Delta E = -Q_1 Q_2 W_z(s)$$

Problem:
How to calculate wakefield?

Finite difference methods

Semi-analytical methods

Pen and paper

Calculation of the wakefields

Pen and paper

What are the steps?

The Idea

The Method

The Result

Important theorems

Panofsky-Wenzel theorem

Beam loading theorem

$$\frac{\partial \vec{W}_\perp}{\partial s} = \vec{\nabla}_\perp W_z$$

$$G_z(0^+) = 2G_z(0)$$

A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators (Wiley and Sons, New York, 1993).

One more theorem

Relativistic Gauss Theorem

The Idea

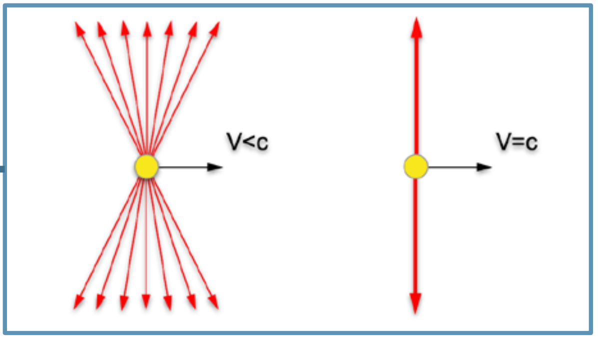
Relativistic Gauss Theorem

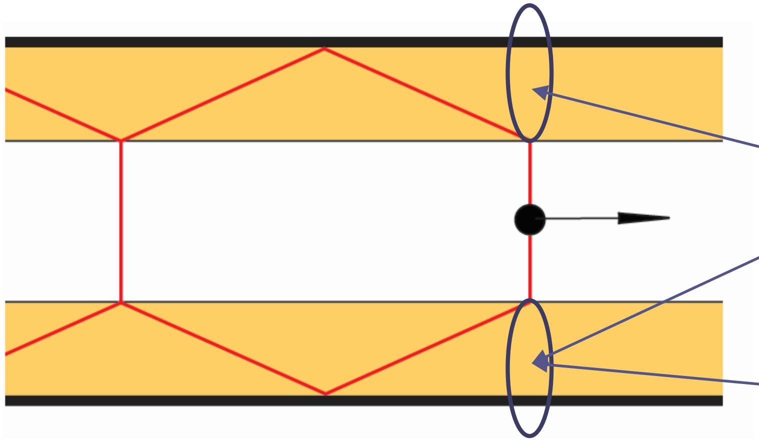
S.S. Baturin, A.D. Kanareykin. PRL 113, 214801, 2014

$$\oint \vec{H} d\vec{l} = \iint_{S_\perp} \left(\frac{\partial D_z}{\partial t} + qnV \right) dS$$

$$n = \delta(x - x_0)\delta(y - y_0)\delta(z - Vt)$$

$$\zeta = Vt - z$$





$$\overset{0}{\cancel{\frac{1}{c}\oint \vec{H} d\vec{l}}} = \iint_{S_\perp} \frac{\partial D_z}{\partial \zeta} dS + q\delta(\zeta)$$

$$\iint_{S_D} E_z^D(0^+) dS = 0$$

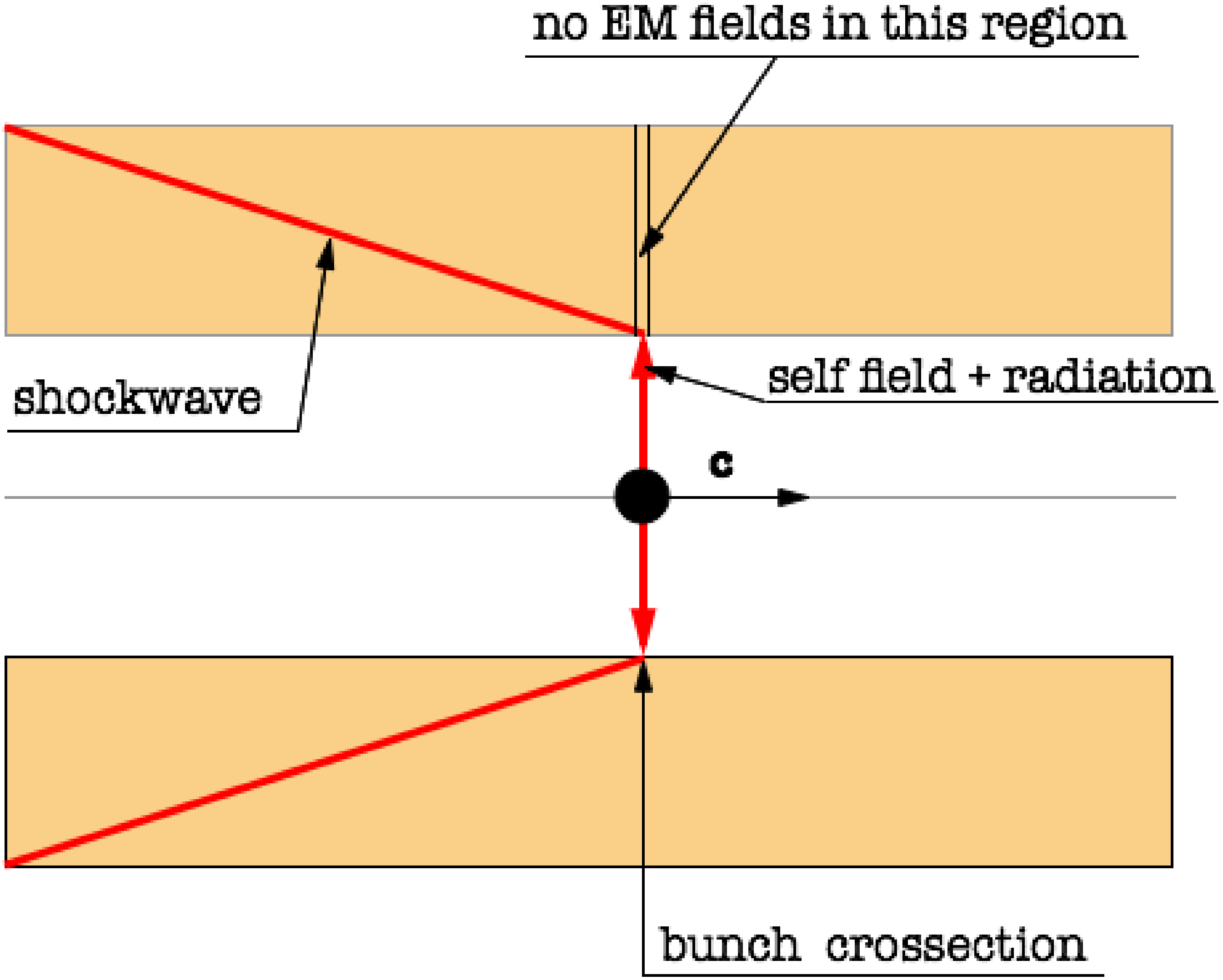
$$\boxed{\iint_{S_V} E_z^V(0^+) dS = -\frac{q}{\varepsilon_0}}$$

$$\boxed{\iint_{S_V} H_z^V(0^+) dS = 0}$$

The Method

Relativistic Gauss Theorem

S.S. Baturin, A.D. Kanareykin. PR-AB 19, 051001 (2016)



Inside vacuum cannel

$$\Delta_\perp E_z = 0$$

$$\Delta_\perp H_z = 0$$

Relativistic Gauss Theorem

Maxwell equations in vacuum channel

$$\nabla_\perp \cdot \mathbf{E}_\perp + i\nabla_\perp \times \mathbf{E}_\perp = 4\pi\rho + \frac{\partial E_z}{\partial \zeta} - i\frac{\partial H_z}{\partial \zeta}$$

$$\nabla_\perp \cdot \mathbf{H}_\perp + i\nabla_\perp \times \mathbf{H}_\perp = i\left(4\pi\rho + \frac{\partial E_z}{\partial \zeta} - i\frac{\partial H_z}{\partial \zeta}\right)$$

$$\zeta = ct - z$$

$$h = H_x + iH_y$$

$$e = E_x + iE_y$$

$$\frac{\partial g_Z}{\partial \chi} = \frac{\partial}{\partial \zeta}(E_z - iH_z)$$

$$\omega = x + iy$$

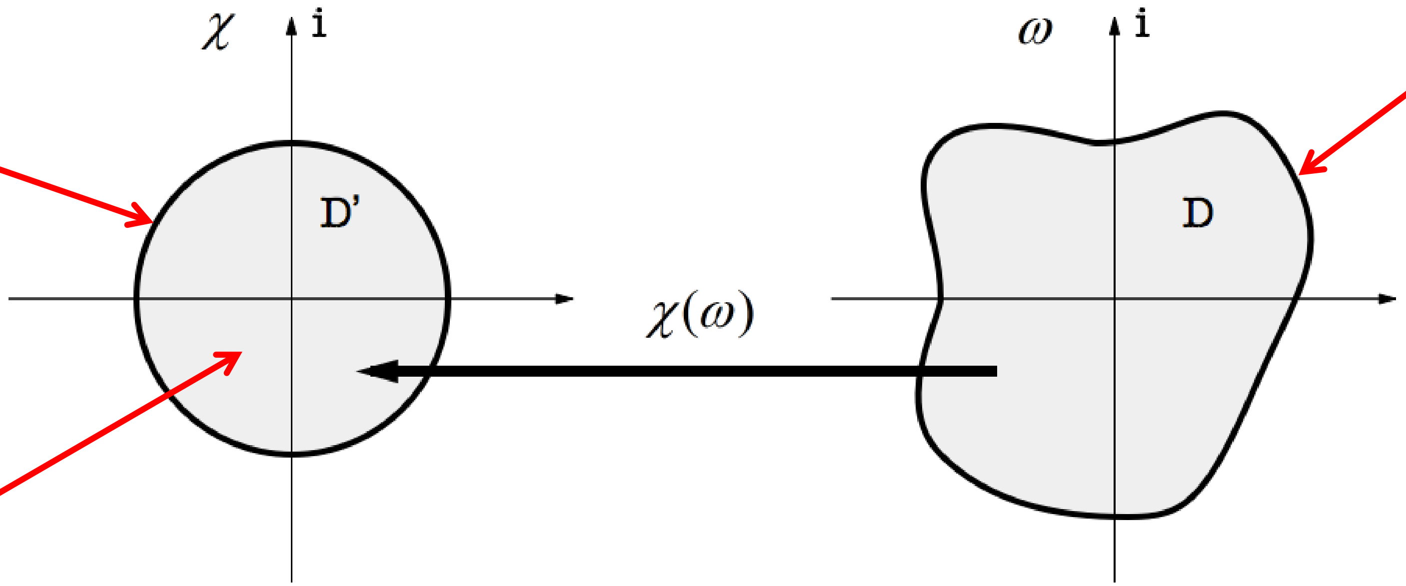
$$\boxed{2\frac{\partial e}{\partial \omega} = 2Q\delta(x')\delta(y') + \frac{\partial \tilde{g}_Z}{\partial \omega}}$$

The Method

Relativistic Gauss Theorem

$$e - g_Z = 0$$

$$E_z = const$$
$$H_z = 0$$



$$e - g_Z = 0$$

$$2\frac{\partial e}{\partial \chi} = 2Q\left(\frac{d\chi}{d\omega}\right)^{*} \delta(x')\delta(y') + \frac{\partial g_Z}{\partial \chi}$$

The Result

Relativistic Gauss Theorem

Longitudinal electromagnetic field

$$E_z(0^+) - iH_z(0^+) = -\frac{4Q}{a^2}\frac{d\chi}{d\omega}\left(\frac{d\chi}{d\omega}\right)^*\Bigg|_{\omega=\omega_0}$$

Transverse Lorentz force

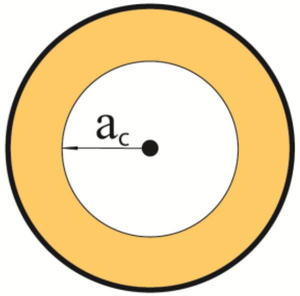
$$F_x(s) + iF_y(s) = s\frac{4eQ}{a^2}\left(\frac{d^2\chi}{d\omega^2}\right)^*\frac{d\chi}{d\omega}\Bigg|_{\omega=\omega_0}$$

$$s \to 0^+$$

The Result

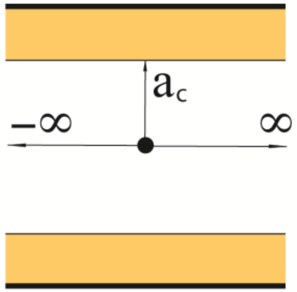
Relativistic Gauss Theorem

S.S. Baturin, A.D. Kanareykin. PR-AB 19, 051001 (2016)



$$\chi(\omega) = \tanh\left[\frac{\pi}{4}\left(\frac{\omega}{a} - i\right)\right]$$

Conformal mapping



General formula

$$E_z(0^+) = -\frac{Q}{\pi\varepsilon_0 a_c^2}\mathrm{Re}\left[-\frac{2Q}{a^2}\frac{d\chi}{d\omega}\left(\frac{d\chi}{d\omega}\right)^*\Bigg|_{\omega=\omega_0}\right]$$

$$E_z^{\omega}(0^+) = |J|E_z^{\chi}(0^+)$$

$$|J| = \left|\frac{d\chi}{d\omega}\right|^2$$

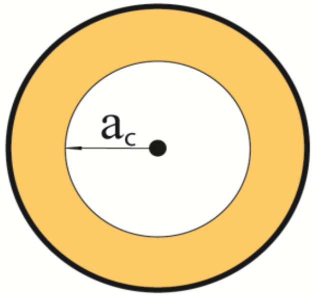
$$E_z^{\chi} = -\frac{q}{\pi\varepsilon_0 a_c^2}$$

$$E_z^{V}(0^+) = -\frac{q}{\pi\varepsilon_0 a_c^2}\frac{\pi^2}{16}$$

The Result

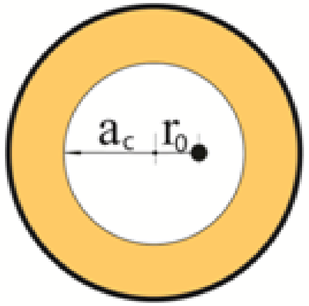
Relativistic Gauss Theorem

S.S. Baturin, A.D. Kanareykin. PRL 113, 214801, 2014



$$\chi(\omega) = a_c \frac{\omega - r_0}{a_c^2 - \omega r_0}$$

Conformal mapping



General formula

$$\frac{F_r(s)}{s} = \left| \frac{eQ}{\pi\varepsilon_0 a_c^2} \left(\frac{d^2\chi}{d\omega^2} \right)^* \frac{d\chi}{d\omega} \Bigg|_{\omega=\omega_0} \right|$$

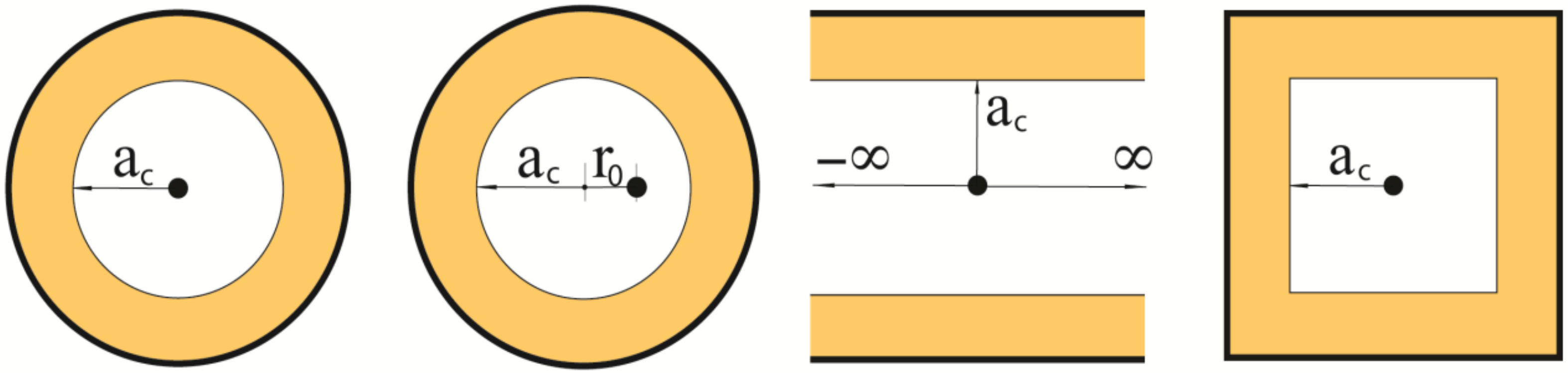
$$\boxed{\frac{F_r(s)}{s} = \frac{eQ}{\pi\varepsilon_0 a_c^4} \frac{2r_0}{\left(1 - \left(r_0 / a_c \right)^2\right)^3}}$$

A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators (Wiley and Sons, New York, 1993).

The Result

Relativistic Gauss Theorem

S.S. Baturin, A.D. Kanareykin. PRL 113, 214801, 2014



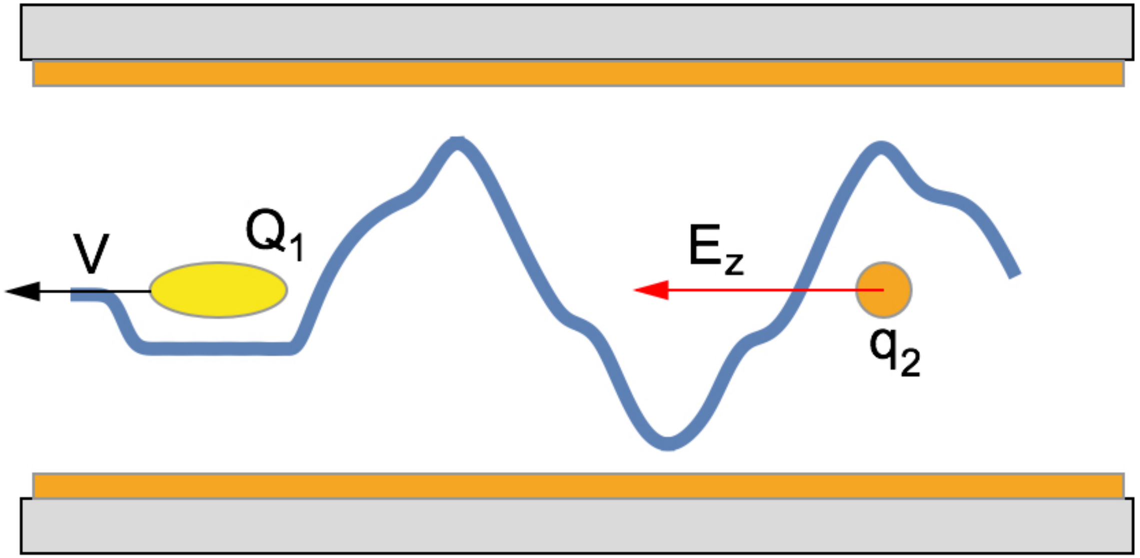
(a) (b) (c) (d)

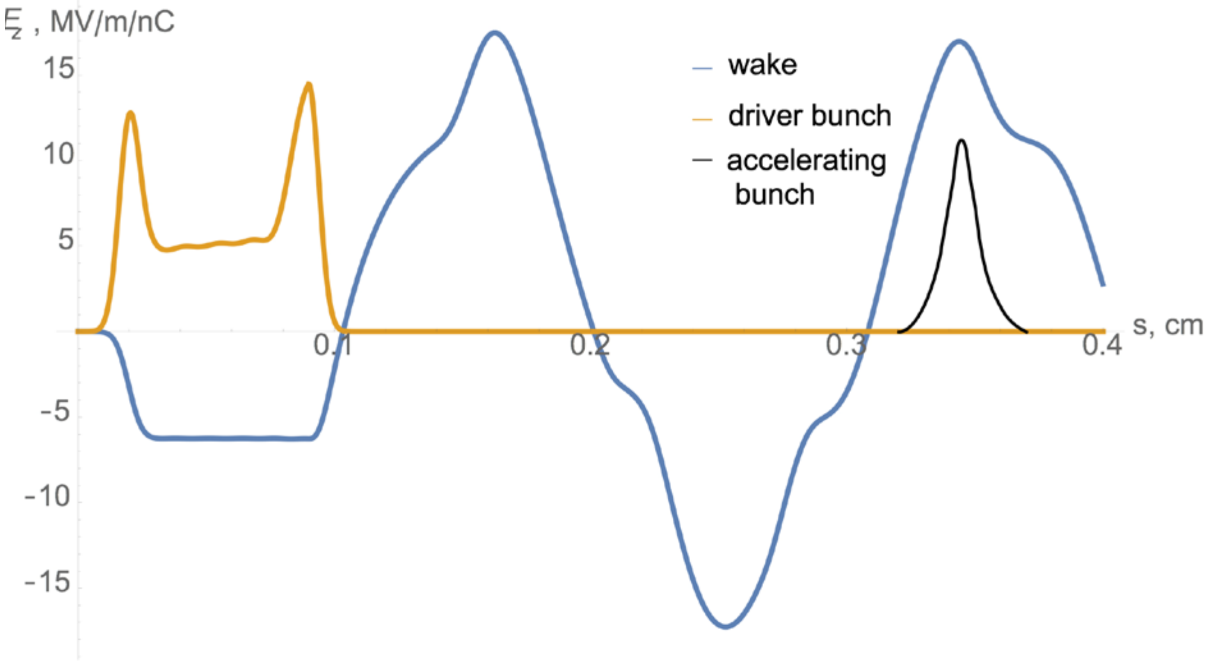
Cylinder	Cylinder displaced	Planar	Rectangular
$f=1$	$f_{dp}=\frac{1}{(1+(r_0/a_c)^2)^2}$	$f_{pl}=\frac{\pi^2}{16}$	$f_{sq}=0.86$

$$E_z(0^+)=-\frac{Q}{\pi\varepsilon_0 a_c^2}\mathrm{Re}\left[-\frac{2Q}{a^2}\frac{d\chi}{d\omega}\left(\frac{d\chi}{d\omega}\right)^*\bigg|_{\omega=\omega_0}\right]$$

Wakefield based devices

Wakefield accelerator

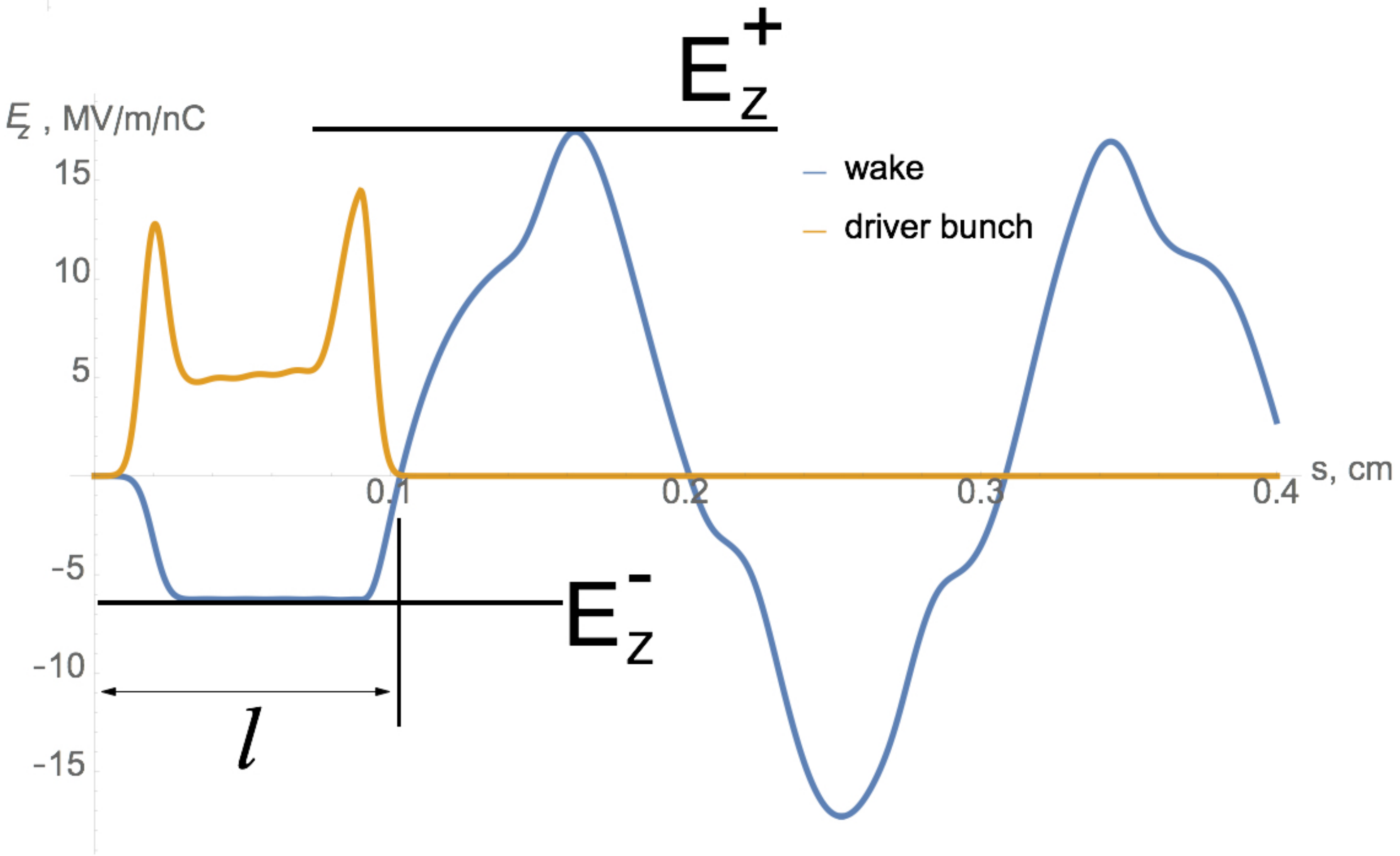




Example of Dielectric Wakefield Accelerator:

- Dielectrics: ε = 4.4 (cordierite-10 (alumina) – low-loss ceramic, ε = 5.7 – diamond, ε = 3.7 - quartz
- Electron beam parameter: σ_z = 20-30 um, Q = 1-3 nC, frequency 0.3-1.2 THz,
- Wakefield maximum 0.1-1.0 GV/m

Transformer ratio



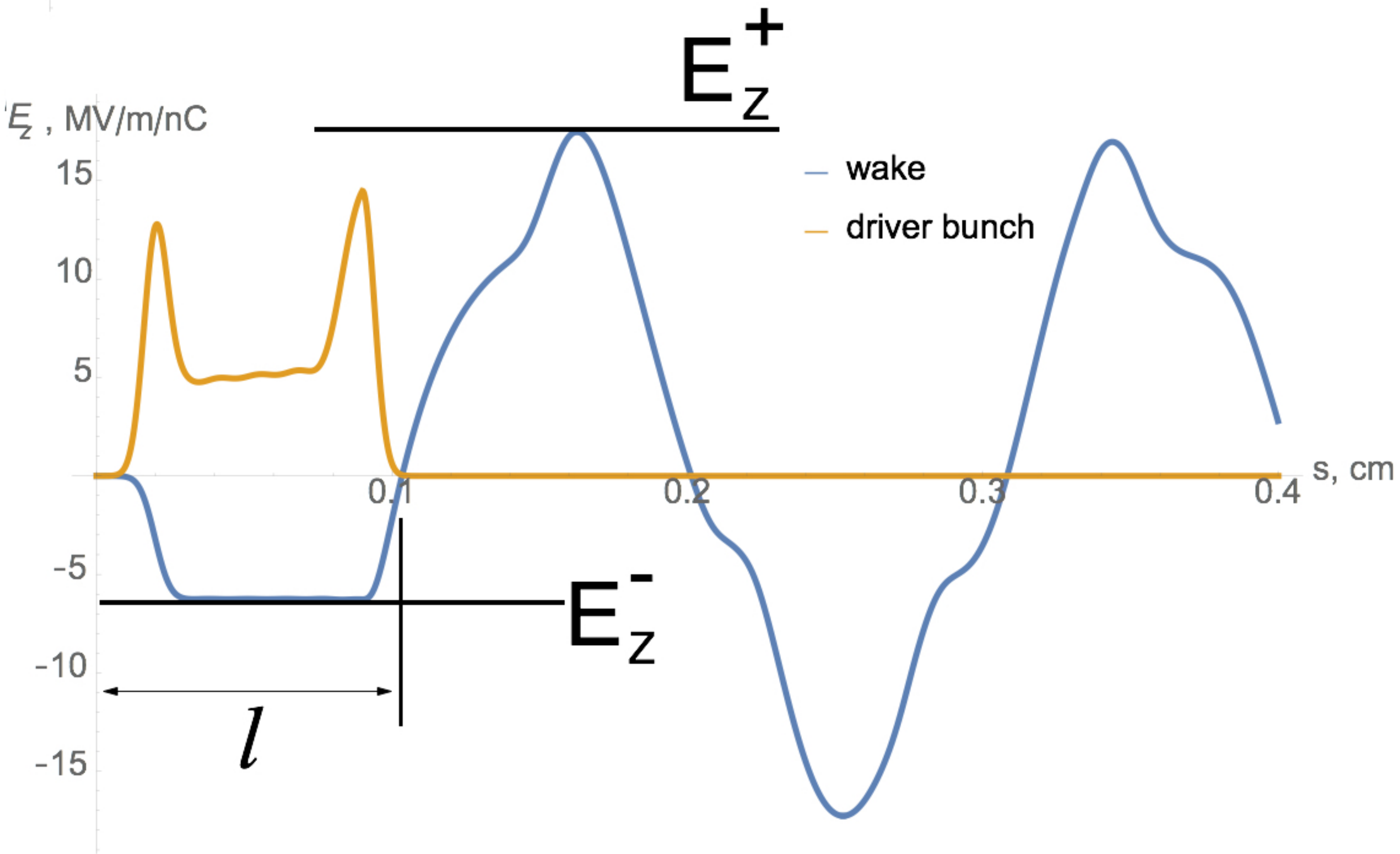
$$R = \frac{\max\left|E_z^+\right|}{\max\left|E_z^-\right|}$$

$$\Delta W^+ = RW^-$$

Wakefield accelerator

Now we define limits on wakefield maximum and transformer ratio.

Limiting Gradient



$$R = \frac{\max\left|E_z^{+}\right|}{\max\left|E_z^{-}\right|}$$

$$E_z^+ \to E_z^+(E^-) \to R$$

$$E_z^+ \to E_z^+(R)$$

The Method

Limiting Gradient

STEP 1

$$E_z^-(s) = A\int_0^s \cos[k(s-s_0)]\rho(s_0)ds_0$$

$$\rho(s) = \frac{E_z^-(s)}{A} + k^2\int_0^s \frac{E_z^-(s)}{A}ds$$

$$E_z^+(s) = A\int_0^l \cos[k(s-s_0)]\rho(s_0)ds_0$$

$$E_z^+ \to E_z^+(E^-)$$

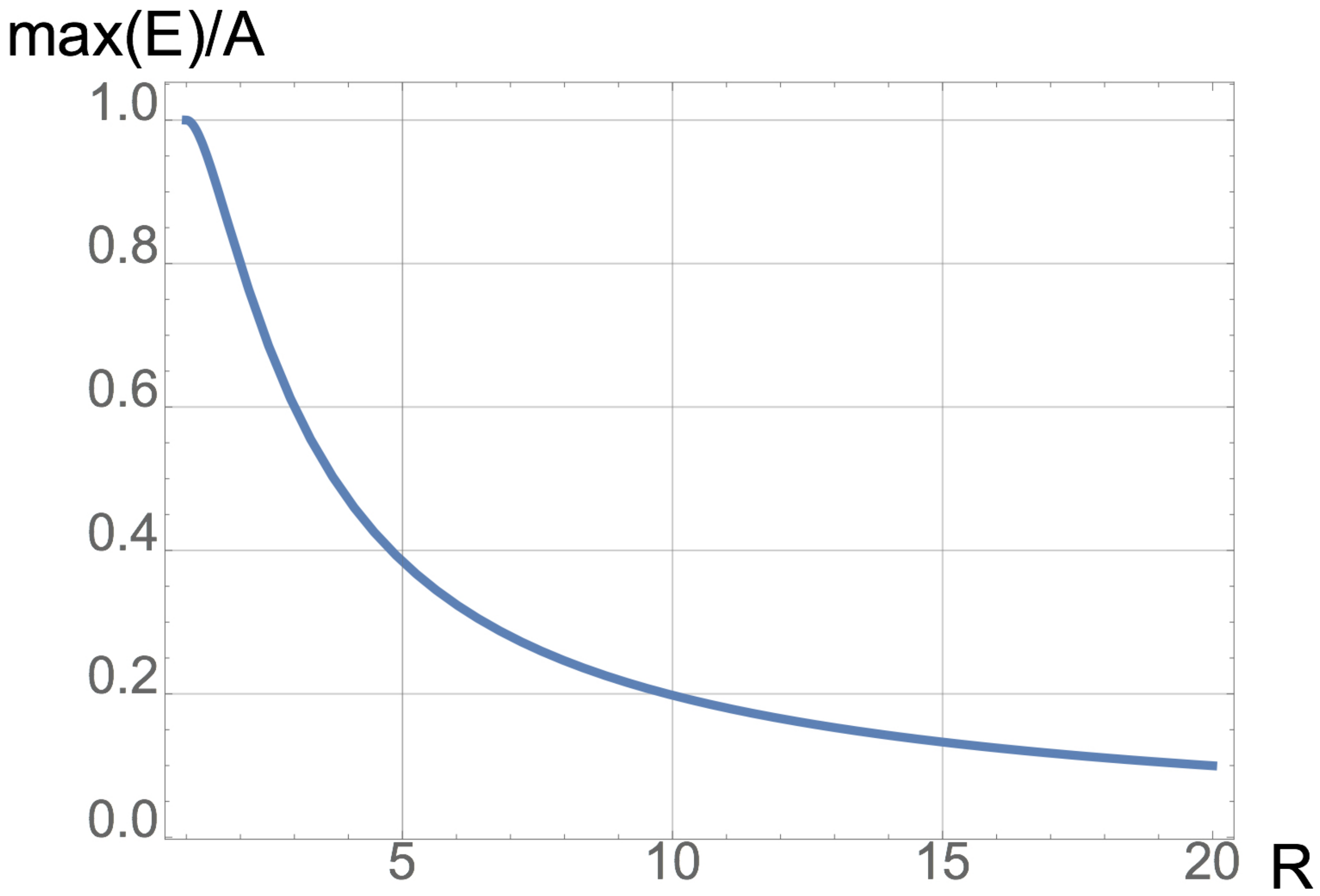
STEP 2

Estimate the integrals in $E_z^+(E^-)$

Limiting Gradient

The Result

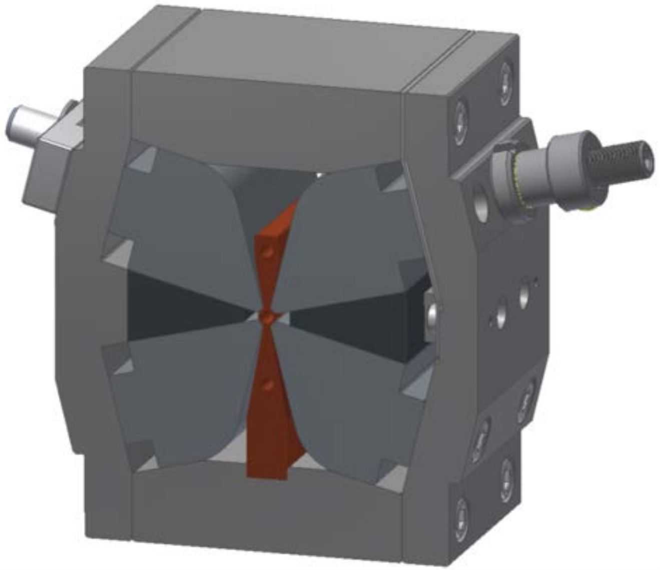
S.S. Baturin, A. Zholents
to be published



$$R \le \sqrt{1+k^2 l^2}$$

$$\max(E_z^+) \le \frac{Qf}{\pi \varepsilon_0 a^2} \frac{2R}{1+R^2}$$

Argonne studies of Wakefield Accelerator*



Driver charge	Aperture	Driver energy	Desired energy gain	Transformer ratio
8nC	2 mm	400 MeV	1.6 Gev	5

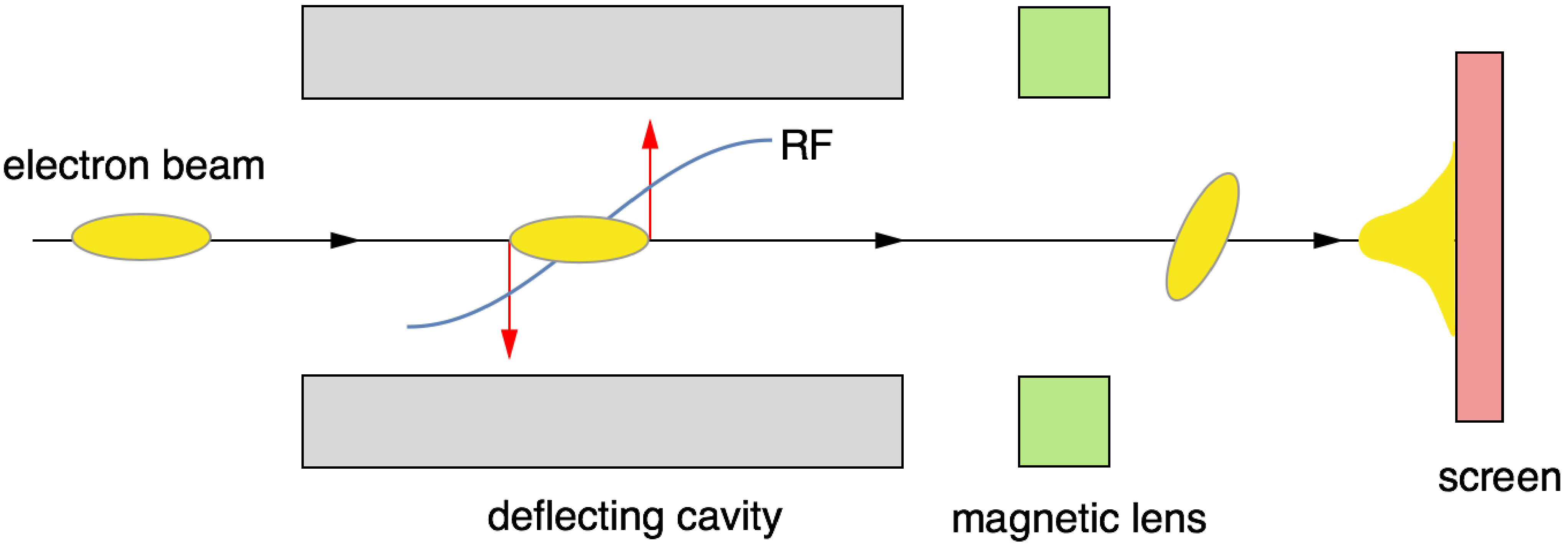
$$\boxed{\max(E_z) = 109\frac{MV}{m}}$$

*A. Zholents et al, Nuclear Instruments and Methods in Physics Research Section A Accelerators Spectrometers Detectors and Associated Equipment 829 February 2016

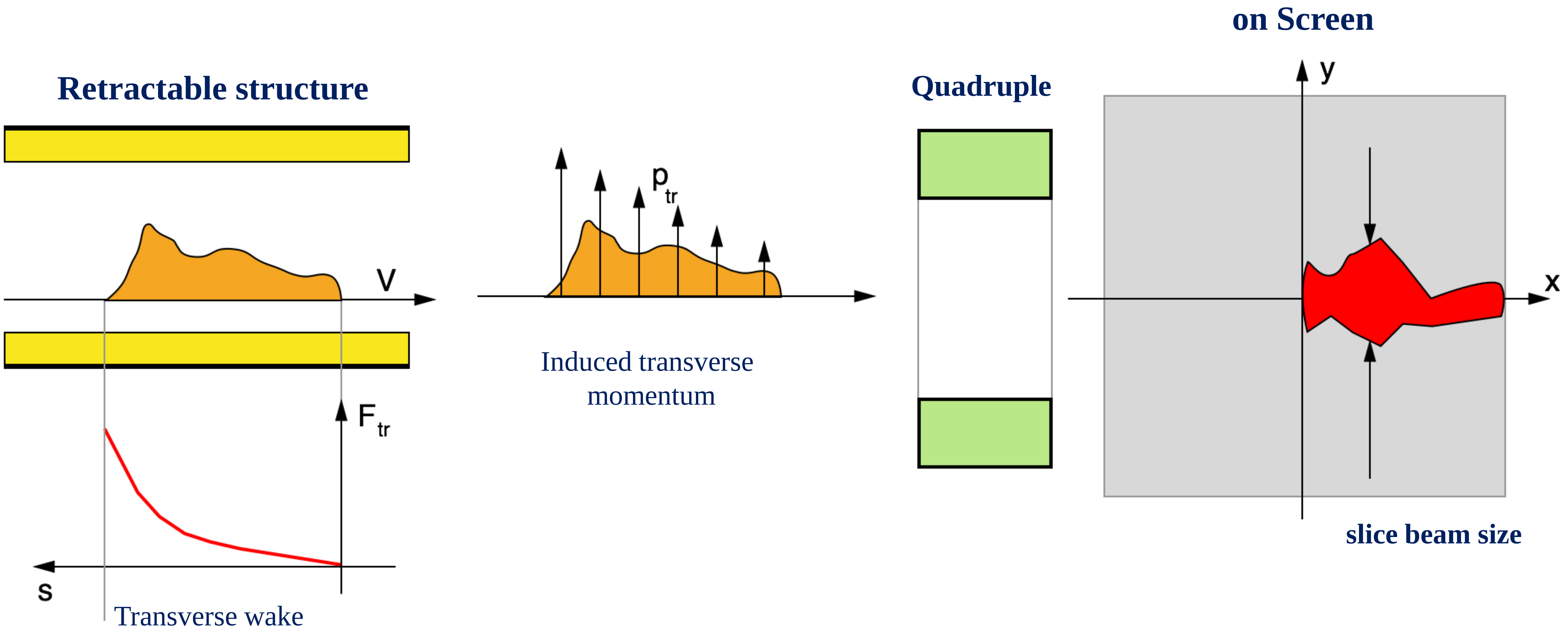
Now we will discuss wakefield application to the bunch diagnostics.

Slice Emittance Measurement

Bunch diagnostics in principle



Slice Emittance Measurement



S. Bettoni, P. Craievich, A. A. Lutman,2 and M. Pedrozzi1

Experimental results PR-AB 19, 021304 (2016)

Slice Emittance Measurement

$$s \approx \frac{\varepsilon_n}{8[mrad]}\sqrt{\frac{I_A}{I_p \gamma}}$$

$$\varepsilon_n = 1 mm \times mrad$$

$$\varepsilon_n = \gamma \sigma_r \sigma_{r'}$$

$$I_A = 17.045 kA$$

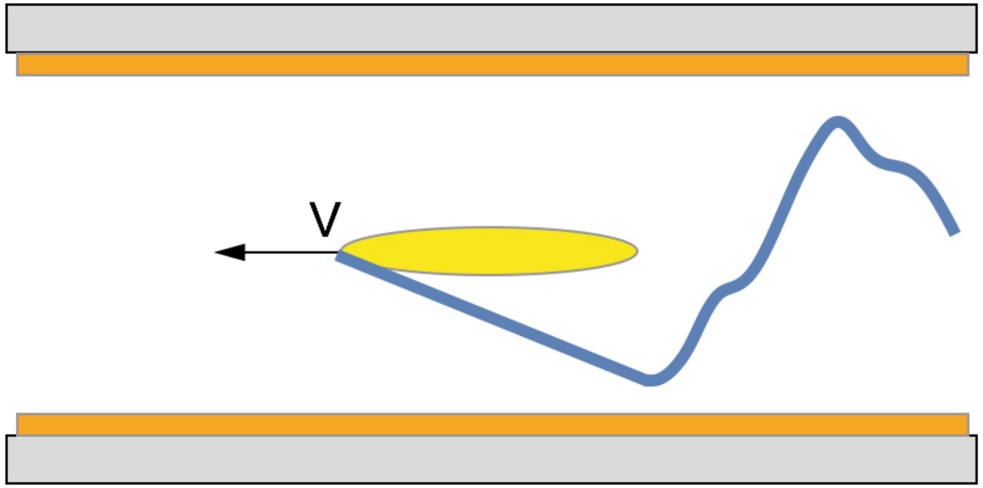
Energy	140 MeV	1.4 GeV	14 GeV
bunch length	1 mm	100um	30 um
current	300 A	1 kA	3.5 kA
resolution	75 um	10 um	1.7 um

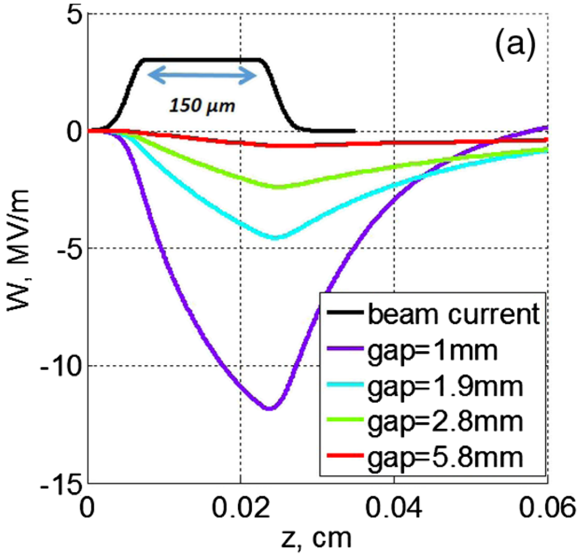
There are more applications that may take advantage of controllable wakefields.

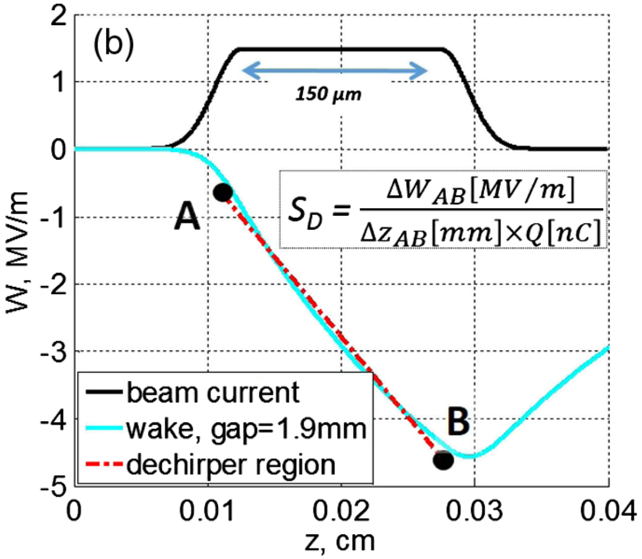
Energy Spread Compensation

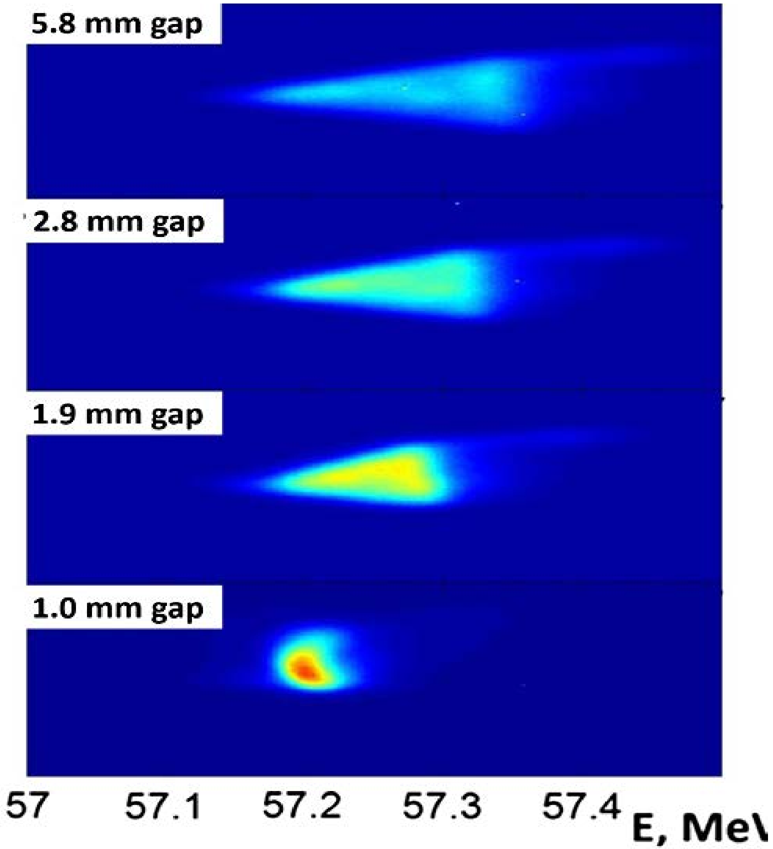
1) S. Antipov, S. Baturin et al. PRL 112, 114801 (2014)

2) P. Emma, M. Venturini, K. L. F. Bane, G. Stupakov et al. PRL 112, 034801



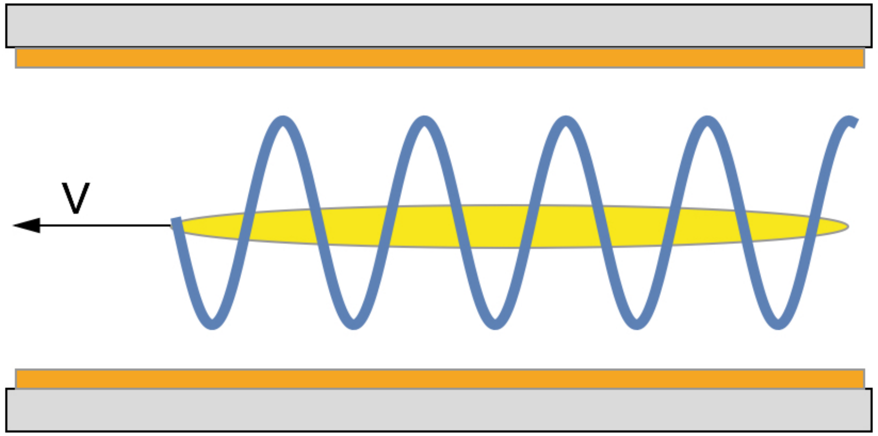


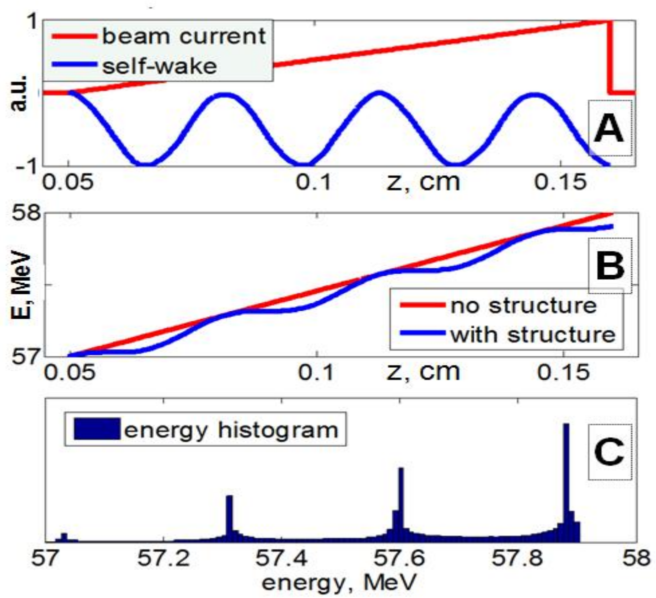


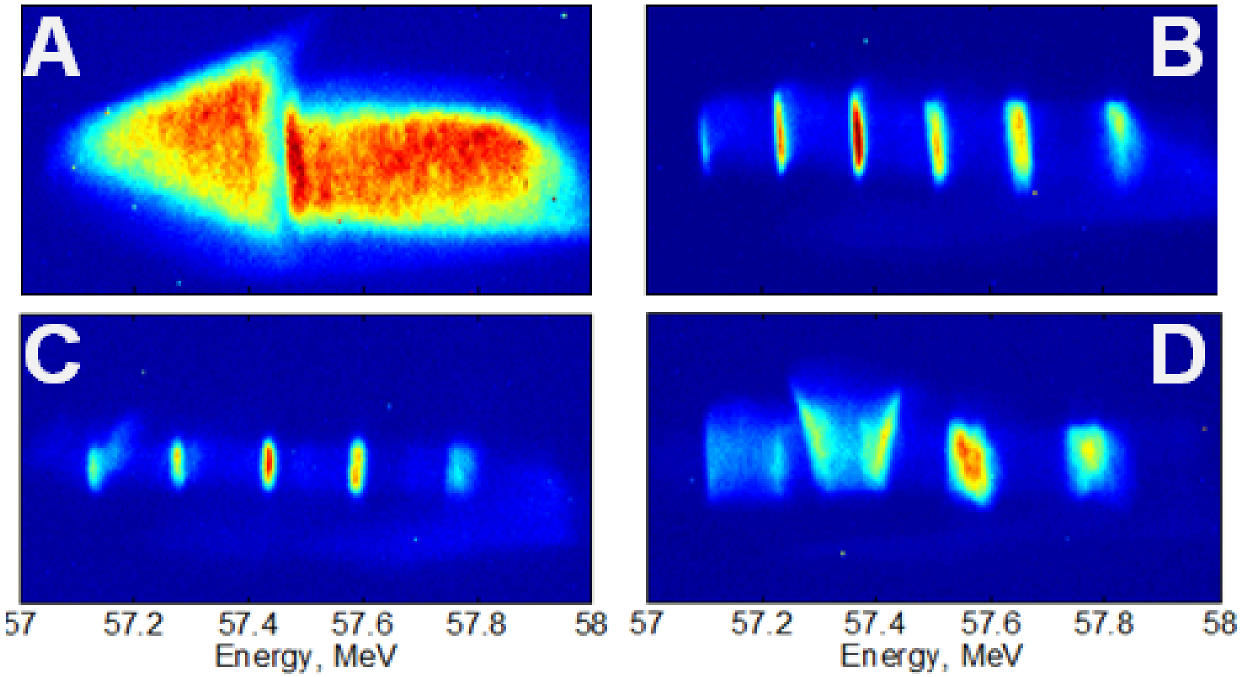


Subpicosecond Bunch Train Production

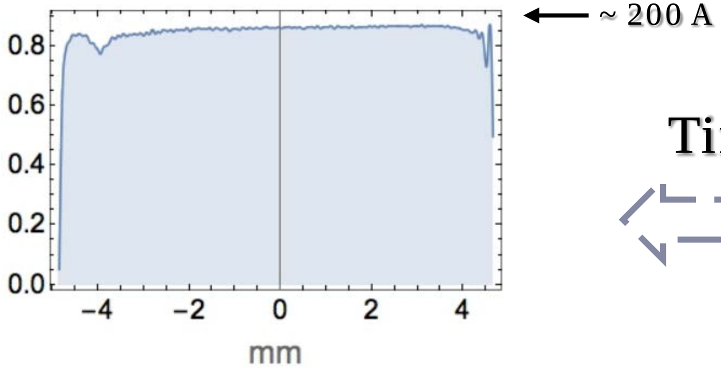
S. Antipov et al. PRL 108, 144801 (2012)



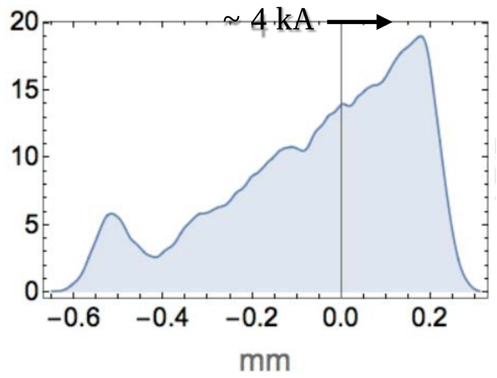


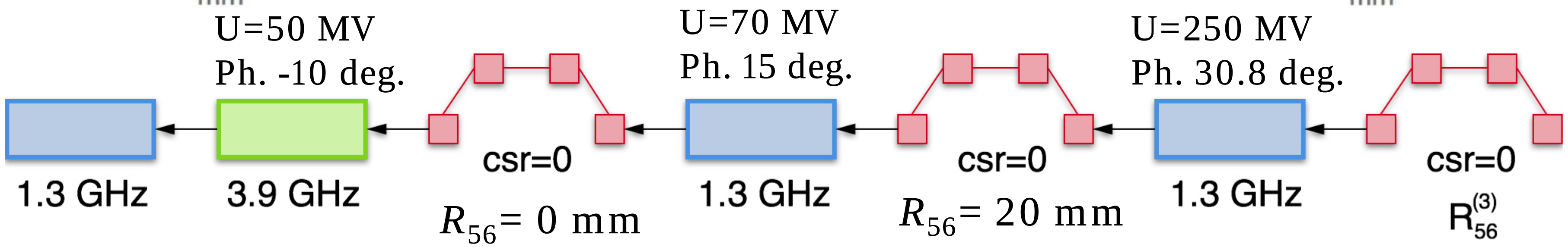


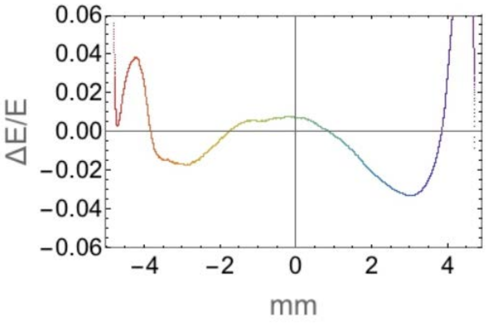
Bunch Shaping



Time reversible tracking





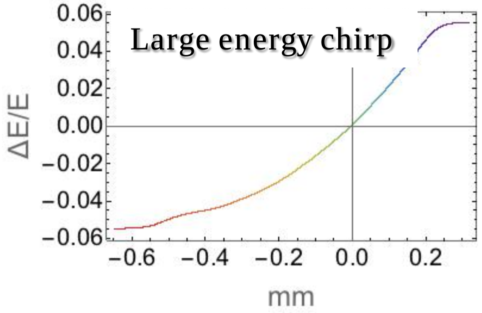


Drive bunch In

Includes:

- Wakefields
- Long. space charge
- T_{566}

CSR is next



Drive bunch Out

Conclusion

- Wakefields are very important part of accelerator science and technology
- Many novel ideas on how we can benefit from wakefields appeared and proved to be working during past years
- New tools for calculation of the wakefields and beam dynamics have been developed and more to come
- Analytical tools for the plasma wakefields

Thank you!