Design considerations for a storage ring resonant cavity beam position monitor / tilt monitor for SPX

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ASD Seminar
Dave Bowman: Open the pod bay doors, HAL.
HAL: I’m sorry, Dave. I’m afraid I can’t do that.
Dave Bowman: What’s the problem?
HAL: I think you know what the problem is just as well as I do.
Dave Bowman: What are you talking about, HAL?
HAL: This mission is too important for me to allow you to jeopardize it.
Rectangular Cavity Modes

\[
f_{mnp} = \frac{c}{2} \left[ \left( \frac{m}{w} \right)^2 + \left( \frac{n}{h} \right)^2 + \left( \frac{p}{d} \right)^2 \right]^{1/2}
\]

Pillbox TM modes \( p = 0 \); independent of \( z \):

\[
f_{mn0} = \frac{c}{2} \left[ \left( \frac{m}{w} \right)^2 + \left( \frac{n}{h} \right)^2 \right]^{1/2}
\]

\[
E_z = E_{z0} \sin(m\pi x/w) \sin(n\pi y/h)
\]

\[
B_x = B_{x0} \sin(m\pi x/w) \cos(n\pi y/h)
\]

\[
B_y = B_{y0} \cos(m\pi x/w) \sin(n\pi y/h)
\]
Second Lowest Mode 120

Vertical BPM / Tilt Monitor

\[ f_{120} = 5.63 \text{ GHz} \]
\[ = 2 \times \text{SPX frequency} \]

\[ E_z = E_0 \sin\left(\frac{\pi x}{w}\right) \sin\left(\frac{2\pi y}{h}\right) \]
\[ B_x = B_{x0} \sin\left(\frac{\pi x}{w}\right) \cos\left(\frac{2\pi y}{h}\right) \]
\[ B_y = B_{y0} \cos\left(\frac{\pi x}{w}\right) \sin\left(\frac{2\pi y}{h}\right) \]
Choice of Transverse Cavity Dimensions

\[ f_{120}[w,h] = 5.63 \text{ GHz} \]

\[(w,h) = (4.39486,6.69288) \text{ cm}\]

\[ f_{210} - f_{120} = f_{120} - f_{110} = 1.55 \text{ GHz}, \]

Maximum mode separation

\[ w = 2.66246 \text{ cm asymptote} \]

\[ h = 5.32491 \text{ cm asymptote} \]

\[ h = w = 5.95343 \text{ cm} \]

\[ f_{120} = f_{210} ; \text{ degeneracy} \]
(w, h) = (4.4, 6.7) cm

\[ f_{210} - f_{120} = f_{120} - f_{110} = 1.55 \text{ GHz}, \]

Maximum mode separation

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>4.08</td>
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<tr>
<td>120</td>
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</tr>
<tr>
<td>210</td>
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<tr>
<td>150</td>
<td>11.71</td>
</tr>
<tr>
<td>330</td>
<td>12.24</td>
</tr>
</tbody>
</table>

BPM / Tilt Mode
Pillbox Mode Symmetries

**Beam-driven modes:**
\[ B_x(h/2 + \delta y) = -B_x(h/2 - \delta y), \]
\[ B_x(y=h/2) = 0 \]

**Vertical BPM-like Modes:**
\[ B_x(h/2 + \delta y) = B_x(h/2 - \delta y). \]
\[ B_x(y=h/2) \text{ non-zero} \]
Output Waveguides
(Cutoff Freq. between modes 110 and 120)
Output Waveguides
(Cutoff Freq. between modes 110 and 120)
Two-sided Waveguide Couplers
Fields

The expressions for the fields shown in figure 1 represent the modes $TM_{mn0}$. In general, the expressions for the fields for modes $TM_{mnp}$ with non-zero $p$ are [4]

\[
E_z = D \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \cos \frac{p\pi z}{d}
\]

\[
E_x = -\frac{D}{k_c^2} \left( \frac{p\pi}{d} \right) \left( \frac{m\pi}{a} \right) \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \sin \frac{p\pi z}{d}
\]

\[
E_y = -\frac{D}{k_c^2} \left( \frac{p\pi}{d} \right) \left( \frac{n\pi}{b} \right) \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \sin \frac{p\pi z}{d}
\]

\[
H_x = \frac{j\omega \epsilon_0 D}{k_c^2} \left( \frac{n\pi}{b} \right) \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \cos \frac{p\pi z}{d}
\]

\[
H_y = -\frac{j\omega \epsilon_0 D}{k_c^2} \left( \frac{m\pi}{a} \right) \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \cos \frac{p\pi z}{d}
\]

(2)

where

\[
k_c^2 = \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2
\]

(3)

and

\[
\frac{\omega^2}{c^2} = \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 + \left( \frac{p\pi}{d} \right)^2
\]

(4)
Voltage, Energy

The voltage gain of a test particle travelling along the z-axis through the field $E_z$ is \[5\]

\[ V = \int_0^d E_z(x, y, z)e^{-j\omega z/c} dz \]  \hspace{1cm} (5)

For the fields of equation 2, this results in

\[ V = \frac{2D\omega}{ck_c^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \exp\frac{-j\omega d}{2c} \begin{cases} 
\sin\frac{\omega d}{2c} & p \text{ even} \\
-j\cos\frac{\omega d}{2c} & p \text{ odd}
\end{cases} \]  \hspace{1cm} (6)

For the case $p = 0$, this reduces to

\[ V = \frac{2D}{\sqrt{(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2}} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \exp\frac{-j\omega d}{2c} \sin\frac{\omega d}{2c} \]  \hspace{1cm} (7)

Total energy stored in cavity with peak longitudinal field $D$: \[ U = \frac{\varepsilon_0 D^2}{8} abd. \]
Bunching Factor
Untilted Beam

Implicit in everything described up until this point is the fact that the fields and voltages have sinusoidal time dependence \( \exp[j\omega t] \). Defining the scalar quantity in equation 7 to be \( V_0 \), a phasor \( \tilde{V} \) can be defined as

\[
\tilde{V} = V_0 \exp j\omega t
\]  

(14)

Equation 3.21 in reference [6] relates the average voltage loss of a longitudinal charge distribution:

\[
\tilde{V}_{\text{avg}} = \frac{1}{q} \int_{-\infty}^{\infty} V_0(x, y) e^{j\omega t} I(t) \, dt
\]  

(15)

where \( I(t) \) is the time-dependent beam current flowing past a fixed point. Now for an untilted Gaussian bunch,

\[
I(t) = \frac{q}{\sqrt{2\pi}\sigma_t} \exp[-(t - t_0)^2 / 2\sigma_t^2]
\]  

(16)

and so

\[
\tilde{V}_{\text{avg}} = V_0(x, y) e^{j\omega t_0} \exp[-\omega^2\sigma_t^2 / 2]
\]  

(17)
Bunching Factor
Tilted Beam

For a vertically tilted bunch, the differential element of beam current can be written

\[ dI(t) = \frac{q}{\sqrt{2\pi}\sigma_t} \exp\left[-(t - t_0)^2/2\sigma_t^2\right] \delta(\delta y - \theta ct) \, d(\delta y) \]  

(18)

where \( \delta(\delta y - \theta ct) \) is the Dirac delta function and \( \theta \) is the angle of tilt in the \( y-z \) plane. (Apologies for the cumbersome notation \( d(\delta y) \), but this is at least consistent - note that \( \delta y \) is not a differential element.)

Then evaluating equation 15 but now integrating over both time \( t \) and vertical displacement \( \delta y \), we have

\[ \tilde{V}_{avg} = \frac{1}{q} \int_{-\infty}^{\infty} d(\delta y) \int_{-\infty}^{\infty} dt \, V_0(x, y) e^{i\omega t} \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left[-(t - t_0)^2/2\sigma_t^2\right] \delta(\delta y - \theta ct) \, d(\delta y) \]  

(19)

\[ = \frac{V_0(x, y)}{\delta y} \frac{q}{\sqrt{2\pi}\sigma_t} \int_{-\infty}^{\infty} dt \, (\theta ct) e^{i\omega t} \exp\left[-(t - t_0)^2/2\sigma_t^2\right] \]  

(20)

\[ = \frac{V_0(x, y)}{\delta y} \theta \left[j \left(\frac{\omega}{c}\right) (c\sigma_t)^2 + ct_0\right] e^{i\omega t_0} \exp[-\omega^2 \sigma_t^2 / 2] \]  

(21)
Loss Factor

The loss factor which relates the amount of energy deposited into the cavity by a relativistic charge $q$ traversing it is given by

$$\kappa = \frac{|V|^2}{4U}$$

$$= \frac{8}{\pi^2 \epsilon_0 d (m^2 b^2 + n^2 a^2)} \frac{ab}{a} \frac{\sin^2 m \pi x}{b} \frac{\sin^2 n \pi y}{\sin^2 \omega d}{2c}$$

The energy deposited into mode TMmn0 by a point charge $q$ is $\kappa q^2$. 
Loss Factor

Peak at \((a, b, d) = (w, h, d) = (5.33, 6.15, 1.98) \text{ cm}\)
Loss Factor, BPM

Finally, note that for $(\omega d)/(2c)$ small compared to one, the $TM_{mn0}$ loss factor for $m$ odd and $n$ even is linear in $d$, taking on the form

$$
\kappa_{mn0} \approx \frac{8}{\pi^2 \epsilon_0} \frac{ab}{(m^2 b^2 + n^2 a^2)} \left( \frac{n\pi \delta y}{b} \right)^2 \frac{1}{d} \sin^2 \frac{\omega d}{2c} \exp \left( -\omega^2 \sigma_t^2 \right)
$$

$$
\approx \frac{2d}{ab \epsilon_0} \left( \frac{n\pi \delta y}{b} \right)^2 \exp \left( -\omega^2 \sigma_t^2 \right)
$$

(13)

Loss Factor, Tilt Monitor

Ultimately, what is desired is the power deposited into the cavity from a pure tilt. The simple replacement

$$
\delta y \Rightarrow \theta \omega c \sigma_t^2 = [\theta \sigma_z] [\omega \sigma_z/c]
$$

(22)

in equation 13 will yield the loss factor for a purely tilted beam:

$$
\kappa_{mn0}^{\text{tilt}} \approx \frac{2d}{ab \epsilon_0} \left( \frac{n\pi}{b} \right)^2 (\theta \sigma_t^2 \omega c)^2 \exp \left( -\omega^2 \sigma_t^2 \right)
$$

(23)
Power deposited into mode TM120 including bunching factor and resonant enhancement:

\[
P_{120} = \frac{\kappa_{120} q^2}{T_{bunch}} \left[ 2F_R(\tau, \delta = 0) \right] \\
= \frac{\kappa_{120} q^2}{T_{bunch}} \left( \frac{1 + e^{-\tau}}{1 - e^{-\tau}} \right) \\
\approx \frac{q^2}{T_{bunch} ab \varepsilon_0} \frac{2d}{b} \left( \frac{2\pi \delta y}{b} \right)^2 \exp \left( -\omega^2 \sigma_t^2 \right) \left( \frac{1 + e^{-\tau}}{1 - e^{-\tau}} \right)
\]

(26)

where \( \tau = (\omega T_{bunch})/(2Q_L) \). The quantity \( \delta \) is a measure of mode’s phase slippage from one bunch to the next, and is zero for this mode. The quantity \( Q_L \) is the loaded \( Q \), \( Q_L = Q_0/(1 + \beta) \), and \( \beta \) is the ratio of power coupled out to wall loss power. In this case the power coupled out is the signal to be sent to the processing electronics, equal to \( \beta \) multiplied by equation 26. \( Q_0 \) is the unloaded \( Q \) resulting from wall losses alone.

Making the substitution equation 22 into equation 26 yields the power deposited into mode \( TM_{120} \) as a consequence of a pure vertical / longitudinal tilt \( \theta \):

\[
P_{120}^{\text{tilt}} \approx \frac{q^2}{T_{bunch} ab \varepsilon_0} \frac{2d}{2\pi b} \left( \frac{2\pi}{b} \right)^2 (\theta \sigma_t^2 \omega c)^2 \exp \left( -\omega^2 \sigma_t^2 \right) \left( \frac{1 + e^{-\tau}}{1 - e^{-\tau}} \right)
\]

(27)
Results

For 102 mA 24-bunch mode, $\sigma_t = 33.5$ ps, $d = 5$ mm, $a = 4.4$ cm, $b = 6.7$ cm, and using a tilt angle equal to 30 $\mu$rad (the resolution requirement for this detector), equation 27 indicates that the total power dumped into mode $TM_{120}$ as a result of this tilt amounts to 172 nanowatts, otherwise known as $-37.6$ dBm. This is assuming no resonant enhancement, $\tau \gg 1$. If for example 10% of this power is coupled to the external electronics, (reducing the $Q$ by about 10%), then $-47.6$ dBm will be seen in the external waveguides. It is important to note, to get a sense of scale, that 30 $\mu$rad of tilt will produce the same amount of power as a parallel displacement of only 0.36 $\mu$m, i.e. 360 nanometers, using equation 22. At the opposite extreme, supposing that the SPX cavities are completely mistuned such that a tilt as large as 100 mrad (6 degrees) occurs. In this case, due to the quadratic dependence on $\theta$, the power dumped into $TM_{120}$ goes up by $+70.5$ dB to $+33$ dBm, i.e. 2 watts. This amount of tilt produces the same amount of power as a parallel displacement of 1.2 mm.
\[ Q_0 = \omega U/W_L: \]

\[ Q_{mn} = \frac{\pi \eta}{2R_s} \frac{[m^2b^2 + n^2a^2]^{3/2}d}{n^2a^3(b + 2d) + m^2b^3(a + 2d)} \] (30)

The quantity \( \pi \eta/2R_s \) is approximately 22673 for copper. Also, the index \( p = 0 \) for this formula, which covers all the modes shown in figure 4. For mode \( TM_{120} \), with \( (a, b, d) = (w, h, d) = (4.4, 6.7, 0.5) \) cm, the unloaded \( Q \) is 3614. Mode \( TM_{110} \) has \( Q = 2563 \).
LOM, HOM Power

Equation 13 relates the energy deposited in modes sensitive to vertical beam offsets $\delta y$. From an impedance standpoint, however, the modes with both $m$ and $n$ odd will be strongly driven and ultimately will require significant damping. For $m$ and $n$ both odd, the loss factor to lowest order is independent of transverse offsets $\delta x$ and $\delta y$, and equation 11 becomes

$$
\kappa_{mn0} \approx \frac{8}{\pi^2 \epsilon_0} \left( \frac{ab}{(m^2 b^2 + n^2 a^2)} \right) \frac{1}{d} \sin^2 \omega d \left[ 1 - \left( \frac{m \pi \delta x}{a} \right)^2 \right] \left[ 1 - \left( \frac{n \pi \delta y}{b} \right)^2 \right] \exp \left( -\omega^2 \sigma_t^2 \right)
$$

$$
\approx \frac{2d}{ab \epsilon_0} \exp \left( -\omega^2 \sigma_t^2 \right)
$$

(24)

To put this in perspective, using $a = 4.4 \text{ cm}$, $b = 6.7 \text{ cm}$, and supposing $d = 5 \text{ mm}$, for 102 mA stored in 24 bunches in the APS storage ring, equation 24 indicates that 300 watts of power will be deposited into the lowest order $TM_{110}$ mode. Clearly this could get out of control pretty quickly if not carefully handled. For completeness, to compute the power deposited starting from the loss factor, the desired formula is for example

$$
P_{110} = \frac{\kappa_{110} q^2}{T_{\text{bunch}}} = \kappa_{110} I_{\text{bunch}}^2 T_{\text{rev}} N_b = \kappa_{110} I_{\text{total}}^2 T_{\text{rev}} / N_b,
$$

(25)
Mafia Model
Mafia Model, Bx at top of Cavity,
\[ \sigma_t = 33 \text{ ps} \]
Mafia Model, $E_y$ in External Waveguide $\sigma_t = 33$ ps

Electric Field $E_y$ at $(w/2+(cd-cx)^3/4, 0, cw/2-d/2)$ for the Beams at Different Positions

- **on-axis**
- **1.25 mm**
- **2.5 mm**

- TM140
- TM120
- TM320?
- TM340?
Odd LOM / HOM Damping Scheme

Broadband Hybrid
3 - 15 Ghz

Top Loop

Bottom Loop

Modes 110, 130, 150, etc.
Terminated
(also 310, 510, etc.)

Modes 120, 140, 160, etc.
Reflected

Input Cables
Phase Matched

Suitable Delay

Short
Waveguide Tee (H-Plane)

Ez ~ Surface Charge Density

http://www.sunist.org/shared%20documents/References/TeeGSG.pdf
$\text{TM}_{120}$
Cavity Mode

$\text{TE}_{10}$
Waveguide Mode

Waveguide Ring Resonator
**TM_{110} Cavity Mode**

**Waveguide Ring Resonator**

**To Waveguide Termination**
Low Order Mode Waveguide Termination Scheme

E-Field Lines

Front View

Side View

Waveguide H-Plane Tee WR-187

Broadband Load

Top View
Summary

- A C-band cavity beam position / tilt monitor design concept has been completed.
- Dimensions are 4.4 cm wide by 6.7 cm high to maximize LOM / HOM mode separation (a bit wider will improve signal strength).
- Operating frequency is 5.63 GHz = 2 * SPX crab cavity frequency.
- Output waveguide prevents strongly beam-driven LOM, HOM modes from getting to processing electronics.
- Sensitivity amounts to -47 dBm = 30 microradians = 360 nanometers (with 24 bunches, 102 mA total stored beam current, 5 mm cavity depth).
- LOM power deposited at 4.08 GHz is 300 watts, scales with cavity depth.
- Mafia simulations support design concept, sensitivity to mode $TM_{140}$ is strongest. Everything coupled to output waveguide scales with vertical position / tilt.
- LOM, HOM mode damping scheme using waveguide appears feasible.