Design considerations for a storage ring resonant cavity beam position monitor / tilt monitor for SPX

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ASD Seminar

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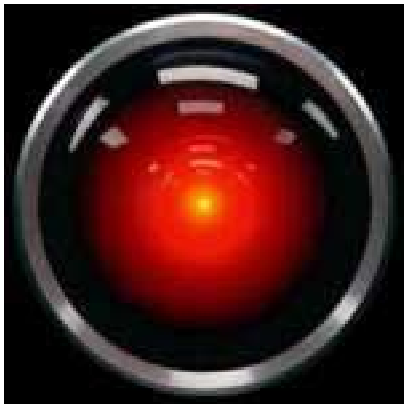
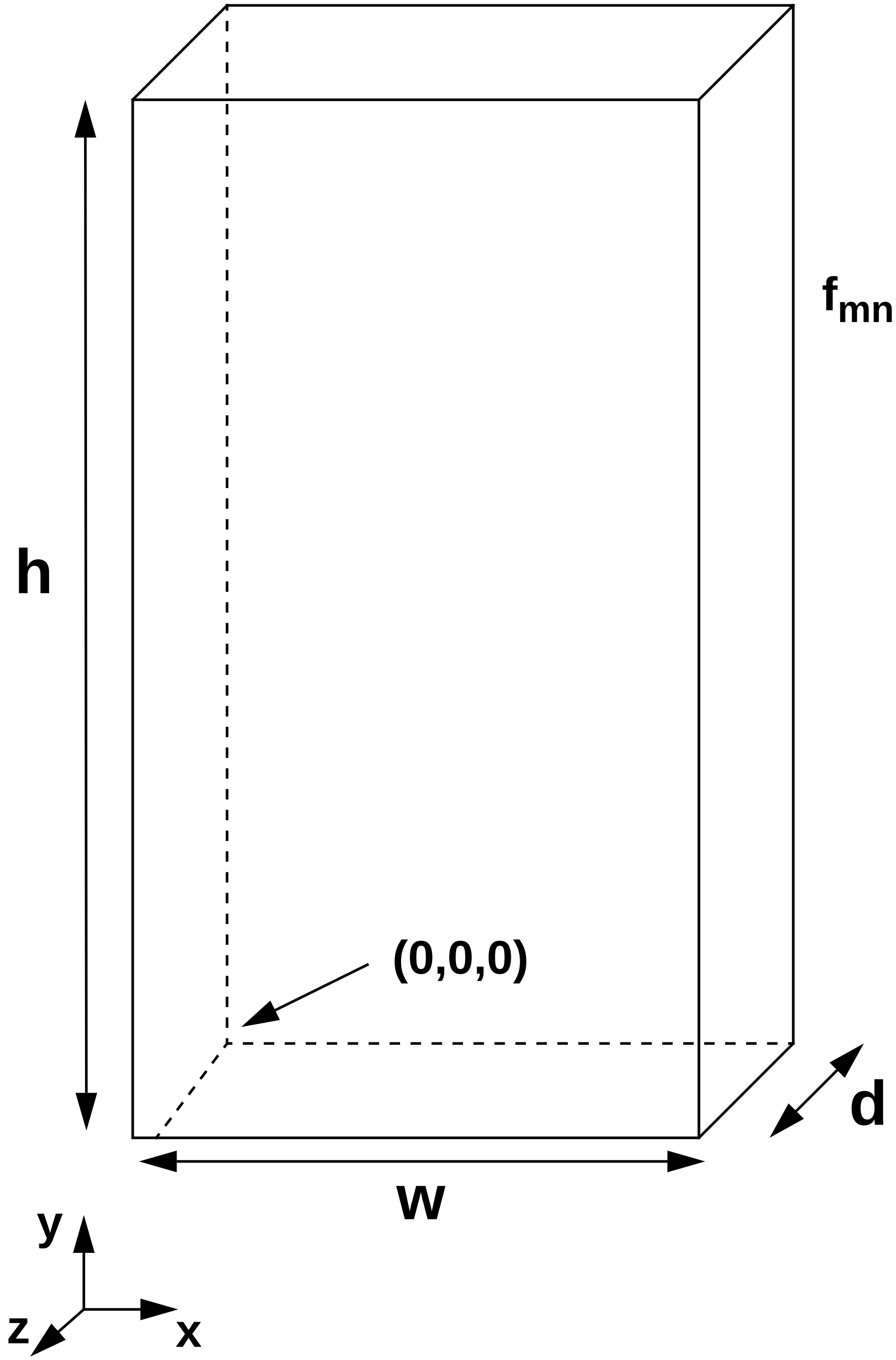
The Monolith



Dave Bowman: Open the pod bay doors, HAL.
HAL: I’m sorry, Dave. I’m afraid I can’t do that.
Dave Bowman: What’s the problem?
HAL: I think you know what the problem is just as well as I do.
Dave Bowman: What are you talking about, HAL?
HAL: This mission is too important for me to allow you to jeopardize it.



Rectangular Cavity Modes

$$f_{mnp} = \frac{c}{2}\left[\left(\frac{m}{w}\right)^2 + \left(\frac{n}{h}\right)^2 + \left(\frac{p}{d}\right)^2\right]^{1/2}$$

Pillbox TM modes p = 0;
independent of z:

$$f_{mn0} = \frac{c}{2}\left[\left(\frac{m}{w}\right)^2 + \left(\frac{n}{h}\right)^2\right]^{1/2}$$

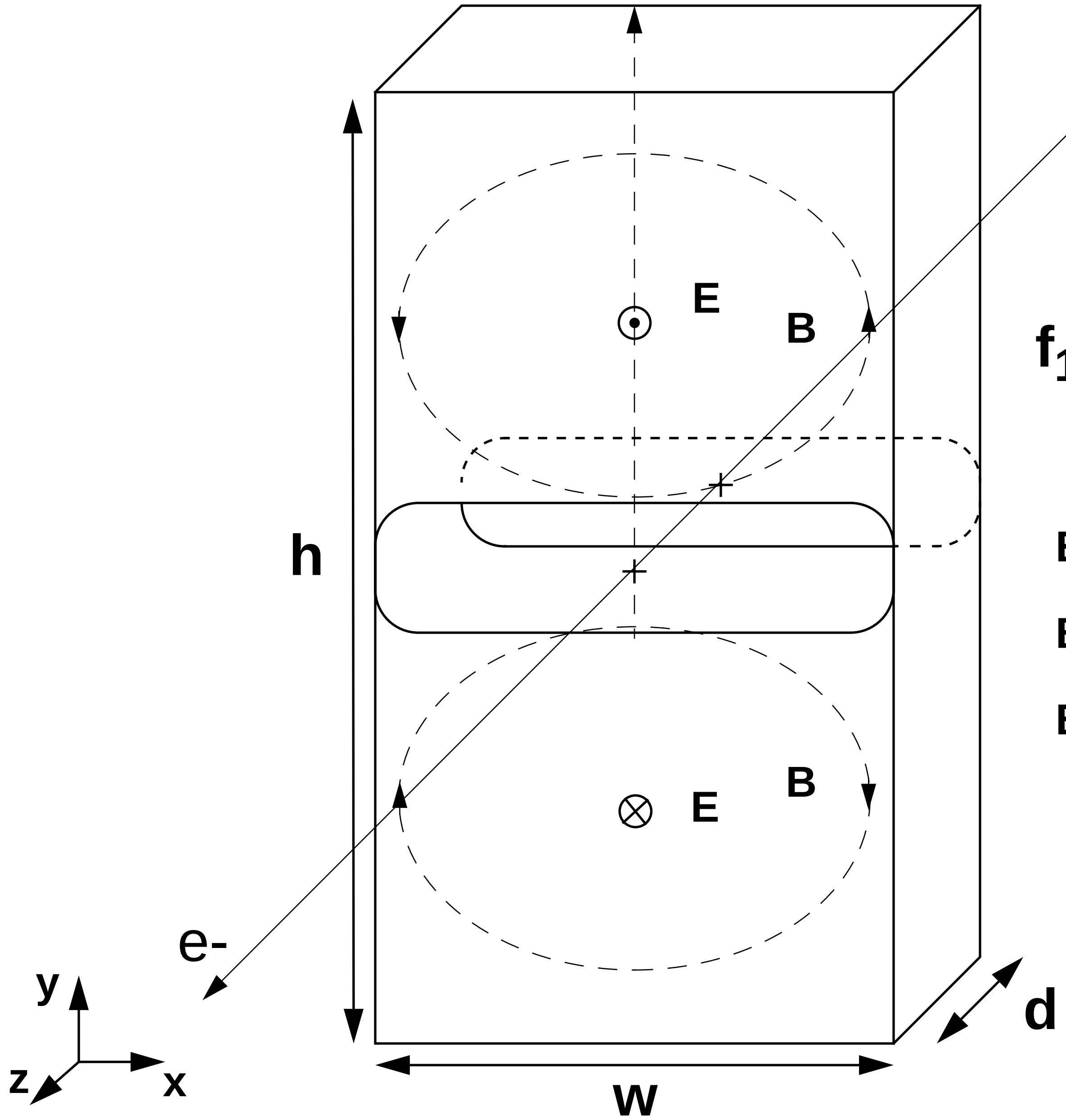
$E_z = E_{z0}\ \sin(m\pi x/w)\ \sin(n\pi y/h)$

$B_x = B_{x0}\ \sin(m\pi x/w)\ \cos(n\pi y/h)$

$B_y = B_{y0}\ \cos(m\pi x/w)\ \sin(n\pi y/h)$

Second Lowest Mode 120

Vertical BPM / Tilt Monitor

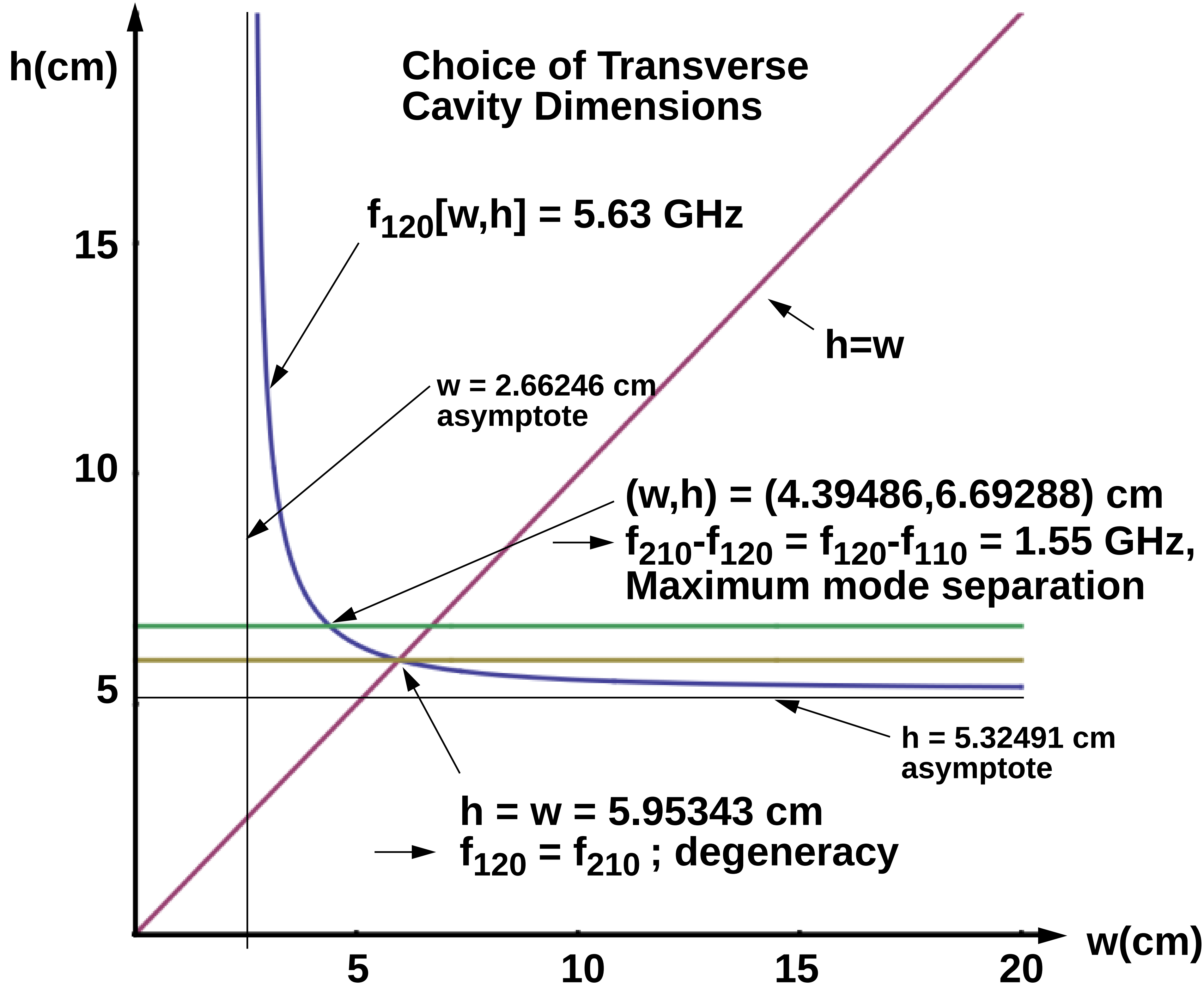


$$f_{120} = 5.63 \text{ GHz}$$
$$= 2 * \text{SPX frequency}$$

$$E_z = E_0 \sin(\pi x/w) \sin(2\pi y/h)$$

$$B_x = B_{x0} \sin(\pi x/w) \cos(2\pi y/h)$$

$$B_y = B_{y0} \cos(\pi x/w) \sin(2\pi y/h)$$


Choice of Transverse Cavity Dimensions
h(cm)
w(cm)
15
10
5
5
10
15
20
f120[w,h] = 5.63 GHz
h=w
w = 2.66246 cm asymptote
(w,h) = (4.39486,6.69288) cm
f210-f120 = f120-f110 = 1.55 GHz,
Maximum mode separation
h = 5.32491 cm asymptote
h = w = 5.95343 cm
f120 = f210 ; degeneracy

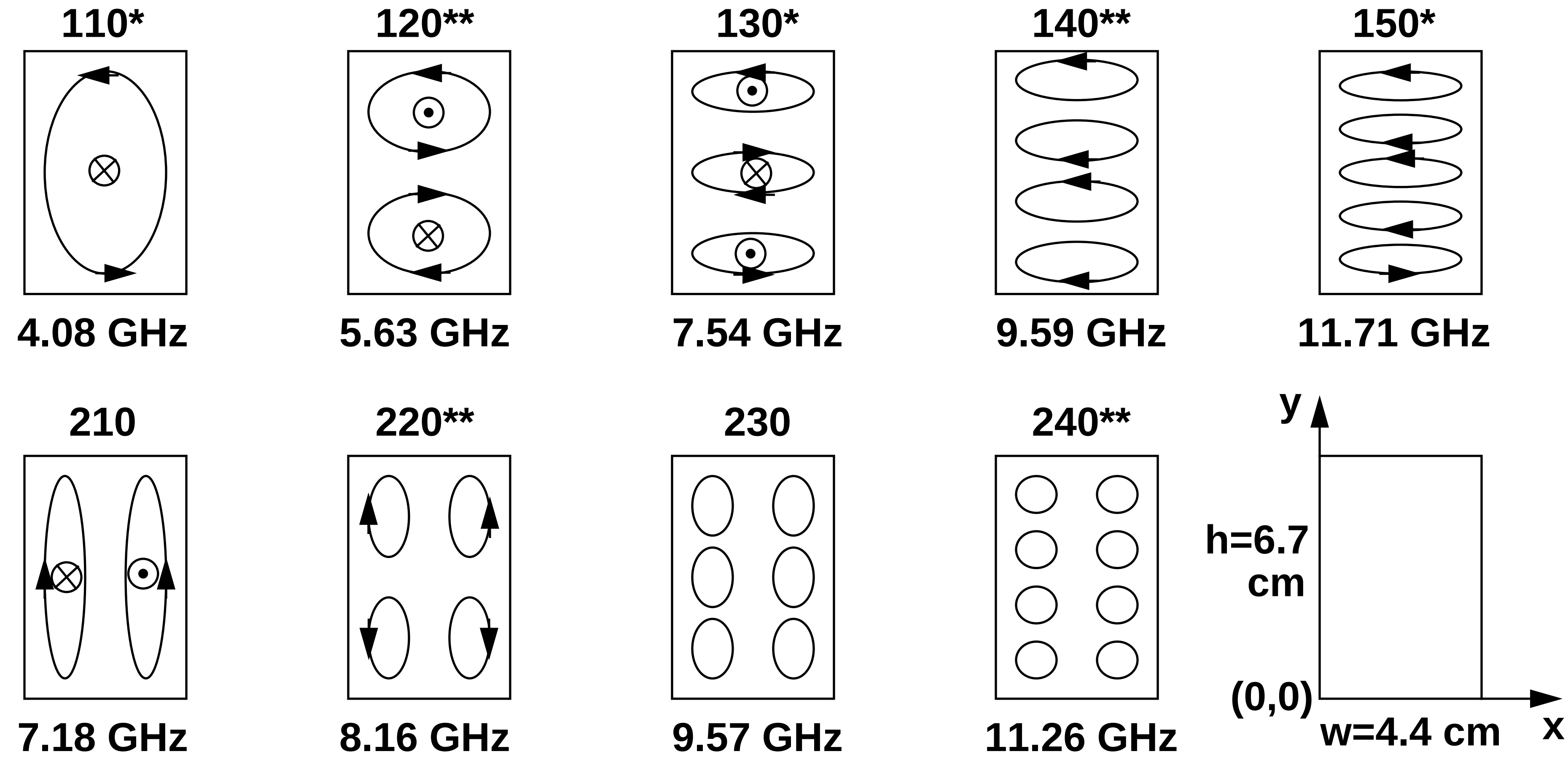
(w, h) = (4.4, 6.7) cm

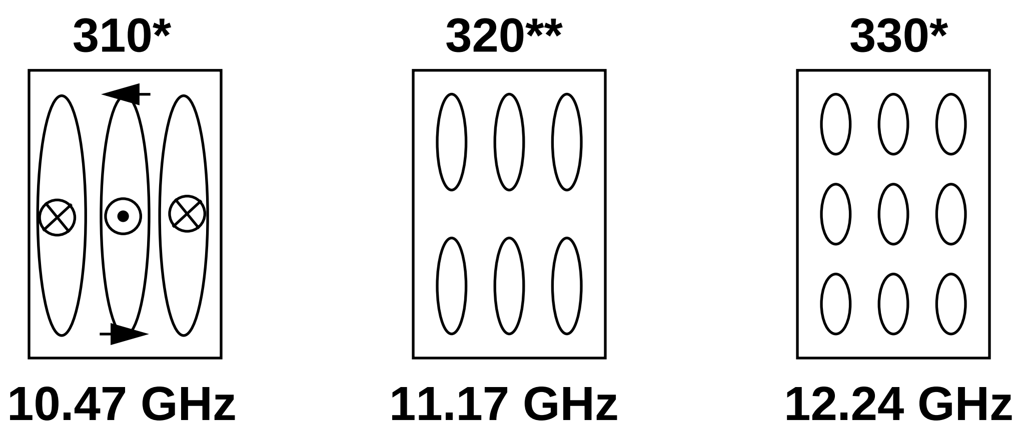
$f_{210} - f_{120} = f_{120} - f_{110} = 1.55$ GHz,

Maximum mode separation

Mode	Frequency (GHz)
110	4.08
120	5.63 ← BPM / Tilt Mode
210	7.18
130	7.54
220	8.16
230	9.57
140	9.59
310	10.47
320	11.17
240	11.26
150	11.71
330	12.24

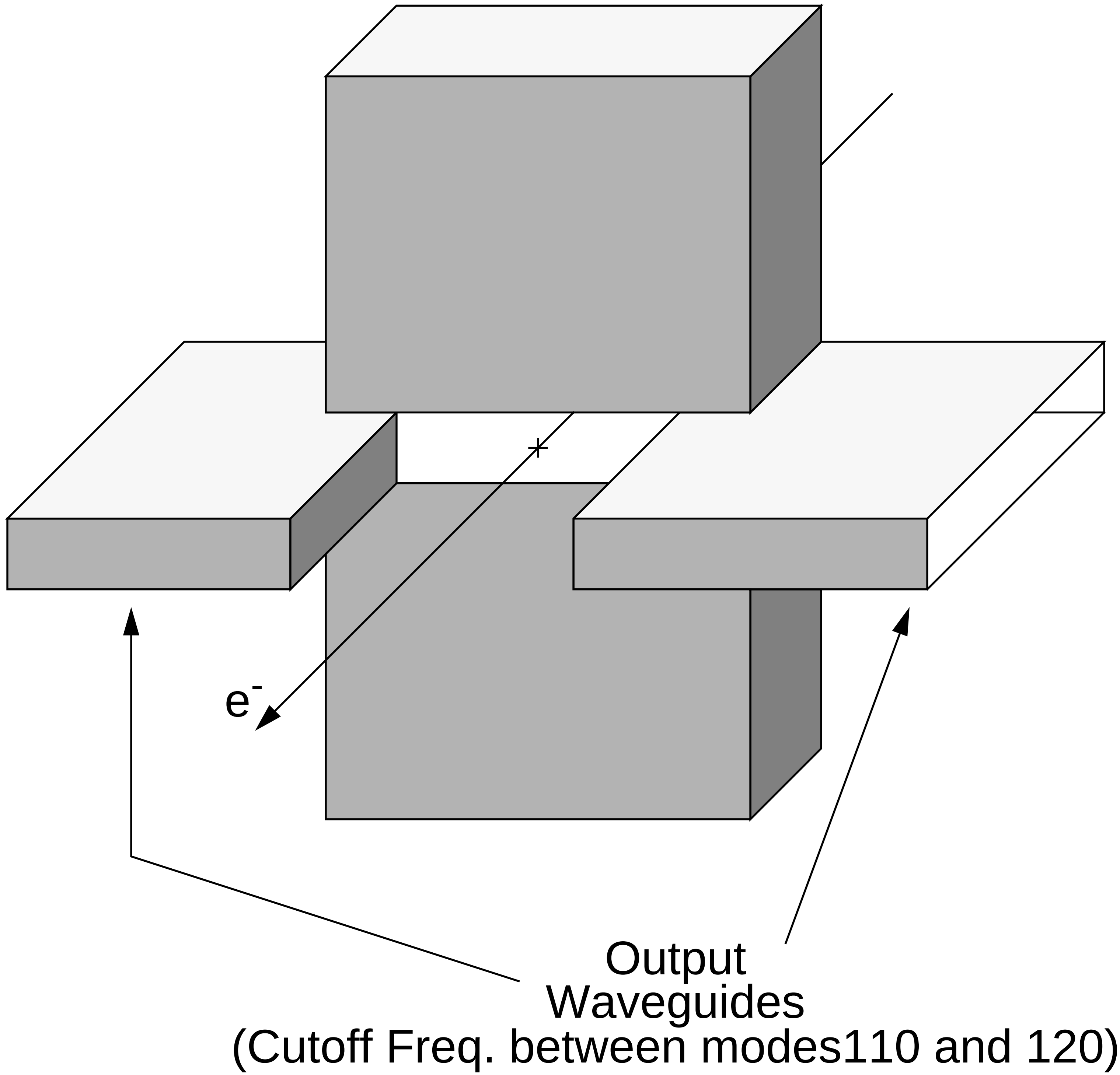
Pillbox Mode Symmetries



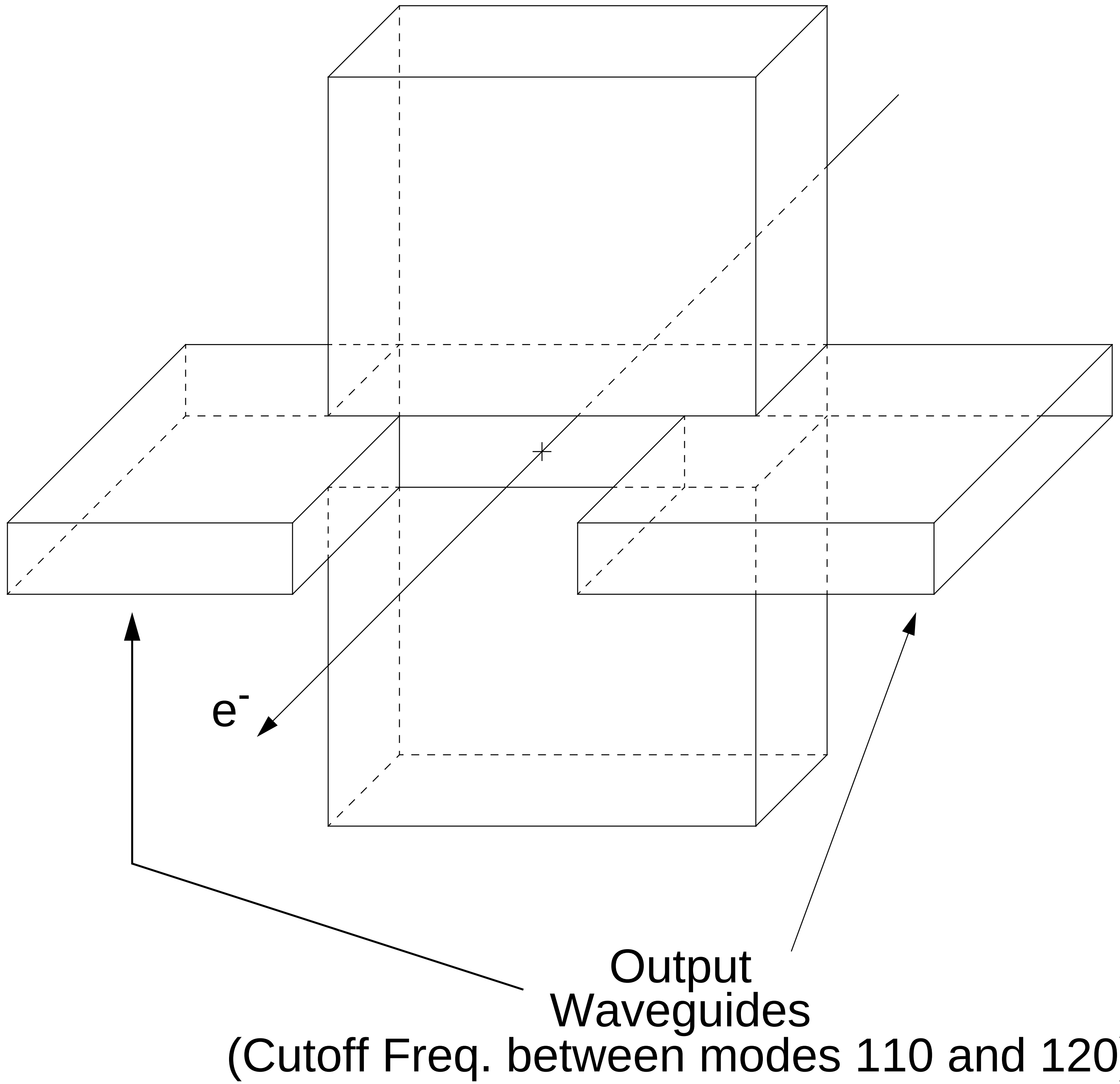


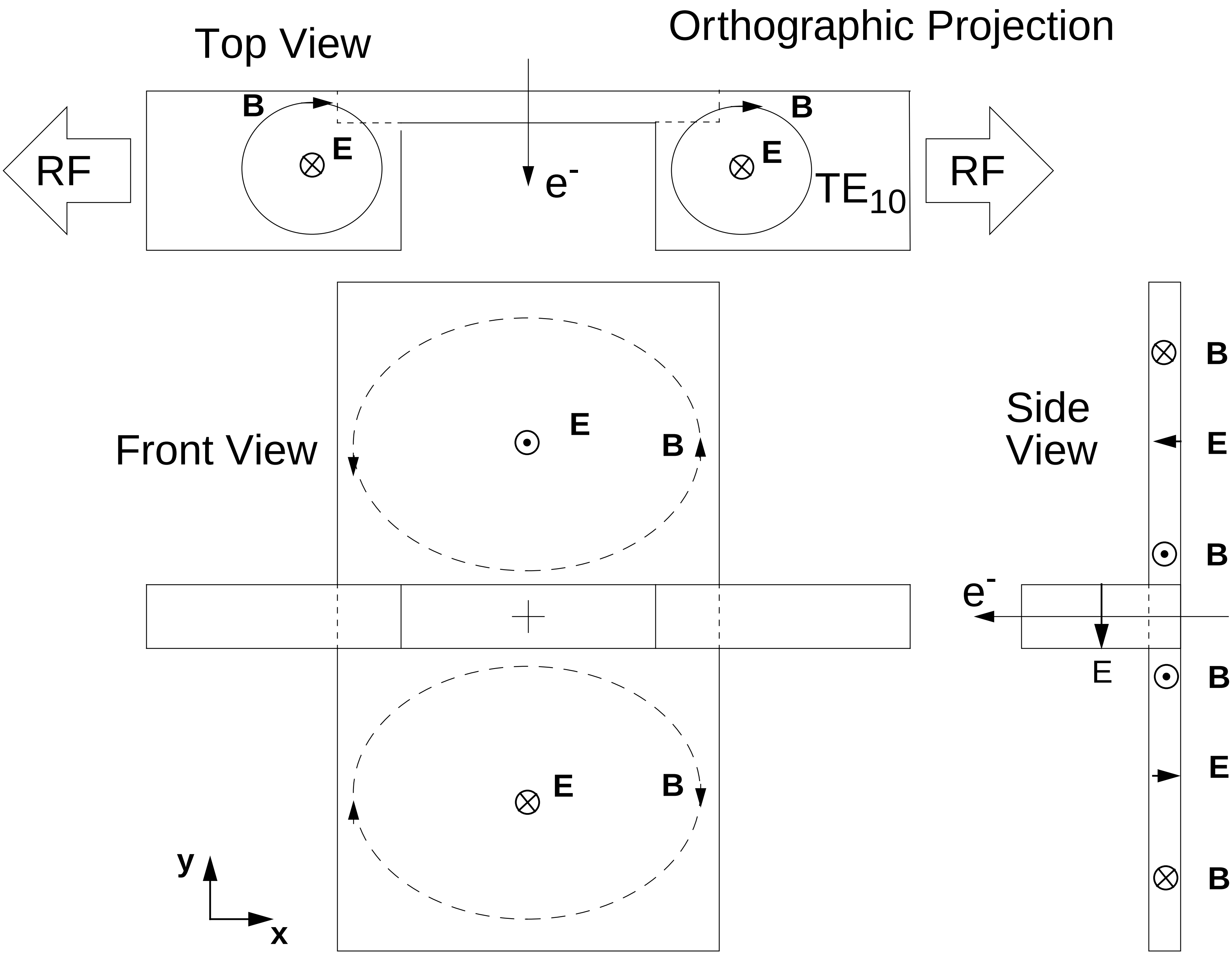
*Beam-driven modes;
$B_x(h/2+\delta y) = - B_x(h/2-\delta y)$,
$B_x(y=h/2) = 0$

** Vertical BPM-like Modes:
$B_x(h/2+\delta y) = B_x(h/2-\delta y)$.
$B_x(y=h/2)$ non-zero



+
e-
Output
Waveguides
(Cutoff Freq. between modes110 and 120)


e-
Output
Waveguides
(Cutoff Freq. between modes 110 and 120)



Two-sided Waveguide Couplers

Fields

The expressions for the fields shown in figure 1 represent the modes TM_{mn0}. In general, the expressions for the fields for modes TM_{mnp} with non-zero p are [4]

$$\begin{aligned}
E_z &= D\, sin\frac{m\pi x}{a} sin\frac{n\pi y}{b} cos\frac{p\pi z}{d} \\
E_x &= -\frac{D}{k_c^2}\left(\frac{p\pi}{d}\right)\left(\frac{m\pi}{a}\right) cos\frac{m\pi x}{a} sin\frac{n\pi y}{b} sin\frac{p\pi z}{d} \\
E_y &= -\frac{D}{k_c^2}\left(\frac{p\pi}{d}\right)\left(\frac{n\pi}{b}\right) sin\frac{m\pi x}{a} cos\frac{n\pi y}{b} sin\frac{p\pi z}{d} \\
H_x &= \frac{j\omega\epsilon_0 D}{k_c^2}\left(\frac{n\pi}{b}\right) sin\frac{m\pi x}{a} cos\frac{n\pi y}{b} cos\frac{p\pi z}{d} \\
H_y &= -\frac{j\omega\epsilon_0 D}{k_c^2}\left(\frac{m\pi}{a}\right) cos\frac{m\pi x}{a} sin\frac{n\pi y}{b} cos\frac{p\pi z}{d} \qquad (2)
\end{aligned}$$

where

$$k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \qquad (3)$$

and

$$\frac{\omega^2}{c^2} = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2 \qquad (4)$$

Voltage, Energy

The voltage gain of a test particle travelling along the z-axis through the field E_z is [5]

$$V = \int_0^d E_z(x,y,z) e^{-j\omega z/c} dz \tag{5}$$

For the fields of equation 2, this results in

$$V = \frac{2D\omega}{ck_c^2} sin\frac{m\pi x}{a} sin\frac{n\pi y}{b} \exp\frac{-j\omega d}{2c} \begin{cases} sin\frac{\omega d}{2c} & \text{p even} \\ -jcos\frac{\omega d}{2c} & \text{p odd} \end{cases} \tag{6}$$

For the case $p = 0$, this reduces to

$$V = \frac{2D}{\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}} sin\frac{m\pi x}{a} sin\frac{n\pi y}{b} \exp\frac{-j\omega d}{2c} sin\frac{\omega d}{2c} \tag{7}$$

Total energy stored in cavity with peak longitudinal field D:

$$U = \frac{\epsilon_0 D^2}{8} abd.$$

Bunching Factor
Untilted Beam

Implicit in everything described up until this point is the fact that the fields and voltages have sinusoidal time dependence $\exp[j\omega t]$. Defining the scalar quantity in equation 7 to be V_0, a phasor $\tilde{V}$ can be defined as

$$\tilde{V} = V_0 \exp j\omega t \tag{14}$$

Equation 3.21 in reference [6] relates the average voltage loss of a longitudinal charge distribution:

$$\tilde{V}_{avg} = \frac{1}{q} \int_{-\infty}^{\infty} V_0(x, y) e^{j\omega t} I(t) dt \tag{15}$$

where $I(t)$ is the time-dependent beam current flowing past a fixed point. Now for an untilted Gaussian bunch,

$$I(t) = \frac{q}{\sqrt{2\pi}\sigma_t} \exp[-(t - t_0)^2/2\sigma_t^2] \tag{16}$$

and so

$$\tilde{V}_{avg} = V_0(x, y) e^{j\omega t_0} \exp[-\omega^2 \sigma_t^2/2] \tag{17}$$

Bunching Factor
Tilted Beam

For a vertically tilted bunch, the differential element of beam current can be written

$$dI(t) = \frac{q}{\sqrt{2\pi}\sigma_t} \exp[-(t-t_0)^2/2\sigma_t^2]\,\delta(\delta y - \theta ct)\,d(\delta y) \tag{18}$$

where $\delta(\delta y - \theta ct)$ is the Dirac delta function and θ is the angle of tilt in the $y - z$ plane. (Apologies for the cumbersome notation $d(\delta y)$, but this is at least consistent - note that δy is not a differential element.)

Then evaluating equation 15 but now integrating over both time t and vertical displacement δy, we have

$$\begin{aligned}
\tilde{V}_{avg} &= \frac{1}{q}\int_{-\infty}^{\infty} d(\delta y)\int_{-\infty}^{\infty} dt\; V_0(x,y)e^{j\omega t}\frac{1}{\sqrt{2\pi}\sigma_t}\exp[-(t-t_0)^2/2\sigma_t^2]\,\delta(\delta y - \theta ct)\,d(\delta y) && (19)\\
&= \frac{V_0(x,y)}{\delta y}\frac{q}{\sqrt{2\pi}\sigma_t}\int_{-\infty}^{\infty} dt\;(\theta ct)e^{j\omega t}\exp[-(t-t_0)^2/2\sigma_t^2] && (20)\\
&= \frac{V_0(x,y)}{\delta y}\,\theta\,\left[j\left(\frac{\omega}{c}\right)(c\sigma_t)^2 + ct_0\right]\,e^{j\omega t_0}\;\exp[-\omega^2\sigma_t^2/2] && (21)
\end{aligned}$$

Loss Factor

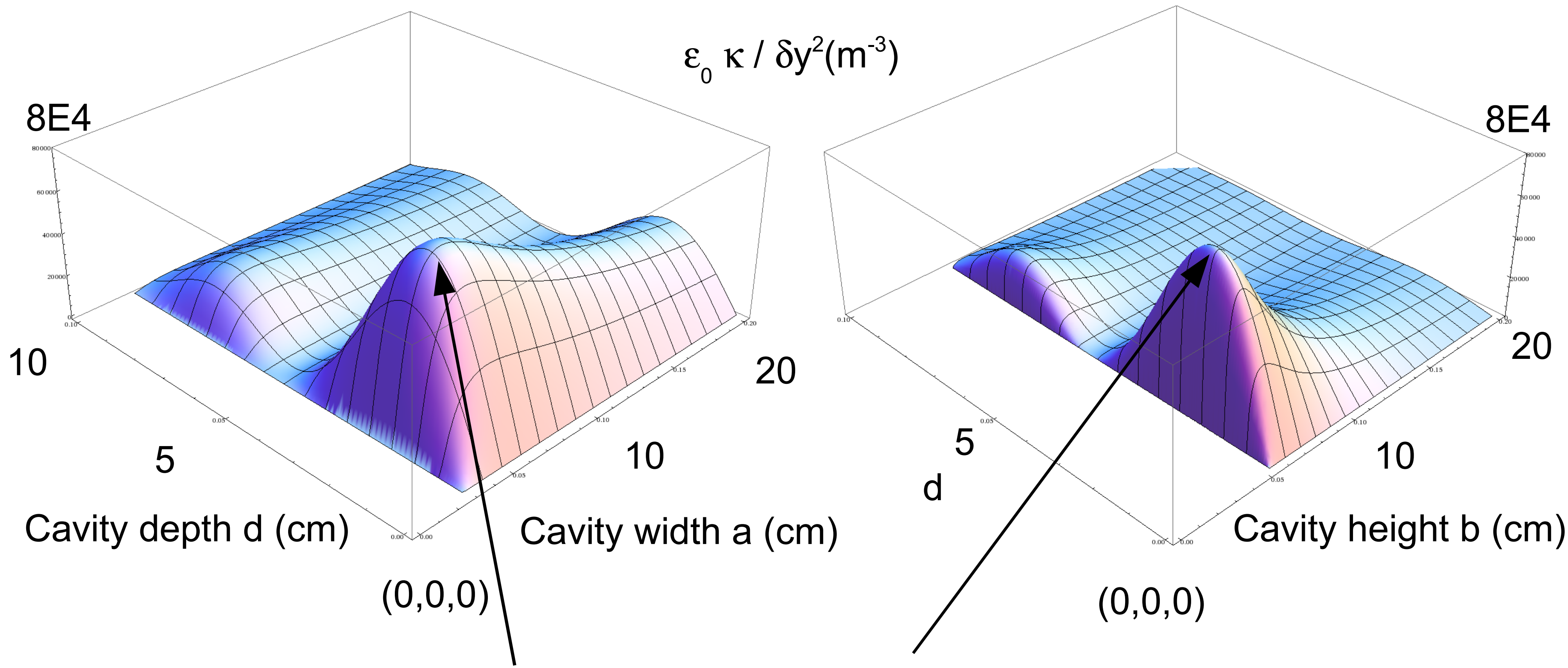
The loss factor which relates the amount of energy deposited into the cavity by a relativistic charge q traversing it is given by

$$\kappa = \frac{|V|^2}{4U} \tag{10}$$

$$= \frac{8}{\pi^2 \epsilon_0 d} \frac{ab}{(m^2 b^2 + n^2 a^2)} sin^2 \frac{m\pi x}{a} sin^2 \frac{n\pi y}{b} sin^2 \frac{\omega d}{2c} \tag{11}$$

The energy deposited into mode TMmn0 by a point charge q is κq^2

Loss Factor



Peak at (a, b, d) = (w, h, d) = (5.33, 6.15, 1.98) cm

Loss Factor, BPM

Finally, note that for $(\omega d)/(2c)$ small compared to one, the TM_{mn0} loss factor for m odd and n even is linear in d, taking on the form

$$\begin{aligned}\kappa_{mn0} &\approx \frac{8}{\pi^2\epsilon_0}\frac{ab}{(m^2b^2+n^2a^2)}\left(\frac{n\pi\delta y}{b}\right)^2\frac{1}{d}sin^2\frac{\omega d}{2c}\exp\left(-\omega^2\sigma_t^2\right)\\ &\approx \frac{2d}{ab\epsilon_0}\left(\frac{n\pi\delta y}{b}\right)^2\exp\left(-\omega^2\sigma_t^2\right) \qquad (13)\end{aligned}$$

Loss Factor, Tilt Monitor

Ultimately, what is desired is the power deposited into the cavity from a pure tilt. The simple replacement

$$\delta y \Rightarrow \theta\omega c\sigma_t^2 = [\theta\sigma_z][\omega\sigma_z/c] \qquad (22)$$

in equation 13 will yield the loss factor for a purely tilted beam:

$$\kappa_{mn0}^{tilt} \approx \frac{2d}{ab\epsilon_0}\left(\frac{n\pi}{b}\right)^2\left(\theta\sigma_t^2\omega c\right)^2\exp\left(-\omega^2\sigma_t^2\right) \qquad (23)$$

Power deposited into mode TM120 including bunching factor and resonant enhancment:

$$\begin{aligned} P_{120} &= \frac{\kappa_{120} q^2}{T_{bunch}} [2F_R(\tau, \delta = 0)] \\ &= \frac{\kappa_{120} q^2}{T_{bunch}} \left(\frac{1 + e^{-\tau}}{1 - e^{-\tau}} \right) \\ &\approx \frac{q^2}{T_{bunch}} \frac{2d}{ab\epsilon_0} \left(\frac{2\pi\delta y}{b} \right)^2 \exp\left(-\omega^2\sigma_t^2\right) \left(\frac{1 + e^{-\tau}}{1 - e^{-\tau}} \right) \end{aligned} \tag{26}$$

where $\tau = (\omega T_{bunch})/(2Q_L)$. The quantity δ is a measure of mode's phase slippage from one bunch to the next, and is zero for this mode. The quantity Q_L is the loaded Q, $Q_L = Q_0/(1+\beta)$, and β is the ratio of power coupled out to wall loss power. In this case the power coupled out is the signal to be sent to the processing electronics, equal to β multiplied by equation 26. Q_0 is the unloaded Q resulting from wall losses alone.

Making the substitution equation 22 into equation 26 yields the power deposited into mode TM_{120} as a consequence of a pure vertical / longitudinal tilt θ:

$$P_{120}^{tilt} \approx \frac{q^2}{T_{bunch}} \frac{2d}{ab\epsilon_0} \left(\frac{2\pi}{b} \right)^2 (\theta\sigma_t^2\omega c)^2 \exp\left(-\omega^2\sigma_t^2\right) \left(\frac{1 + e^{-\tau}}{1 - e^{-\tau}} \right) \tag{27}$$

Results

For 102 mA 24-bunch mode, $\sigma_t = 33.5$ ps, $d = 5$ mm, $a = 4.4$ cm, $b = 6.7$ cm, and using a tilt angle equal to 30 μrad (the resolution requirement for this detector), equation 27 indicates that the total power dumped into mode TM_{120} as a result of this tilt amounts to 172 nanowatts, otherwise known as -37.6 dBm. This is assuming no resonant enhancement, $\tau \gg 1$. If for example 10% of this power is coupled to the external electronics, (reducing the Q by about 10%), then -47.6 dBm will be seen in the external waveguides. It is important to note, to get a sense of scale, that 30 μrad of tilt will produce the same amount of power as a parallel displacement of only 0.36 μm, i.e. 360 nanometers, using equation 22. At the opposite extreme, supposing that the SPX cavities are completely mistuned such that a tilt as large as 100 mrad (6 degrees) occurs. In this case, due to the quadratic dependence on θ, the power dumped into TM_{120} goes up by +70.5 dB to +33 dBm, i.e. 2 watts. This amount of tilt produces the same amount of power as a parallel displacement of 1.2 mm.

Q

$Q_0 = \omega U/W_L$:

$$Q_{mn} = \frac{\pi\eta}{2R_s} \frac{[m^2b^2 + n^2a^2]^{3/2}d}{[n^2a^3(b+2d) + m^2b^3(a+2d)]} \tag{30}$$

The quantity $\pi\eta/2R_s$ is approximately 22673 for copper. Also, the index $p = 0$ for this formula, which covers all the modes shown in figure 4. For mode TM_{120}, with $(a, b, d) = (w, h, d) = (4.4, 6.7, 0.5)$ cm, the unloaded Q is 3614. Mode TM_{110} has $Q = 2563$.

LOM, HOM Power

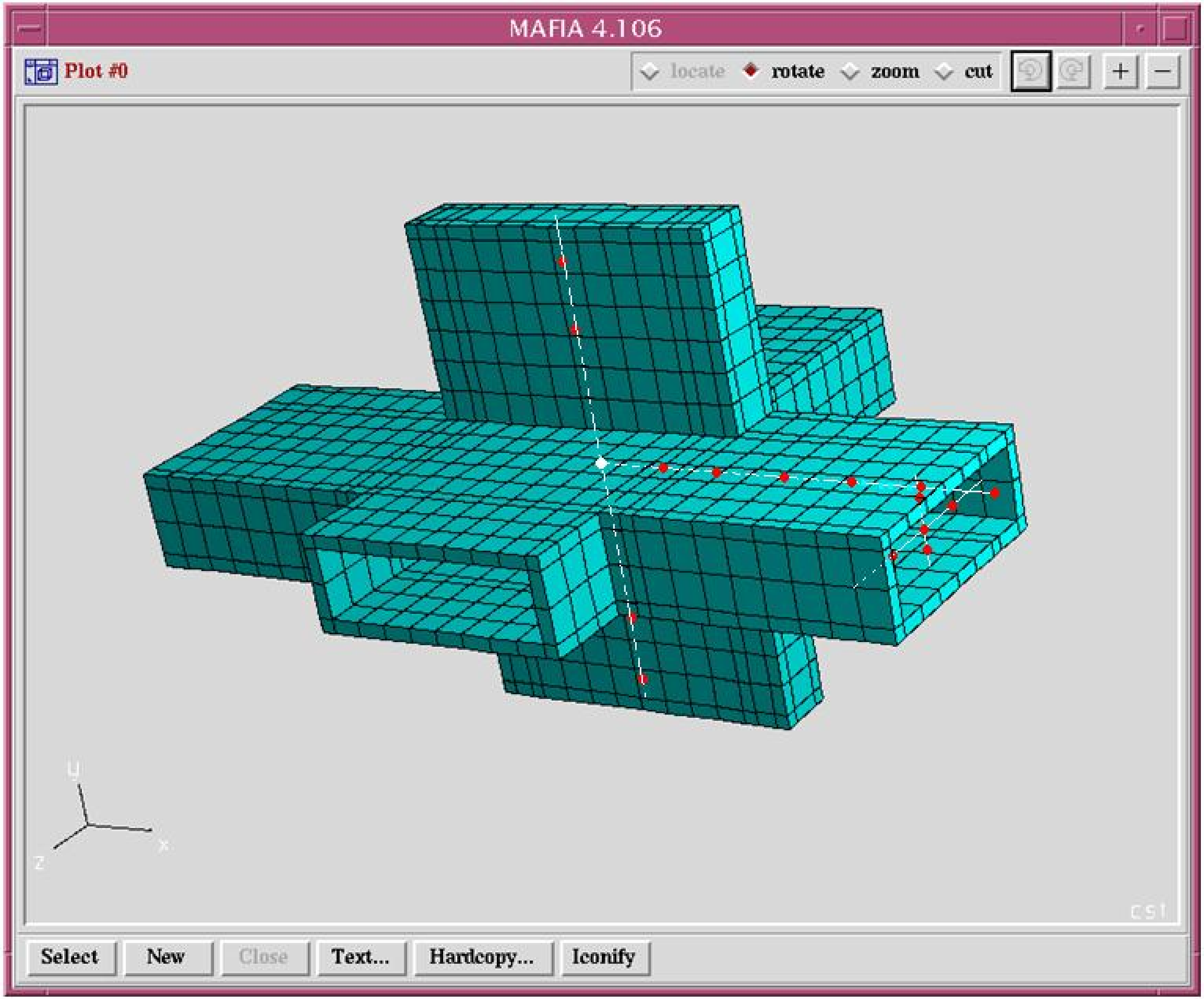
Equation 13 relates the energy deposited in modes sensitive to vertical beam offsets δy. From an impedance standpoint, however, the modes with both m and n odd will be strongly driven and ultimately will require significant damping. For m and n both odd, the loss factor to lowest order is independent of transverse offsets δx and δy, and equation 11 becomes

$$\begin{aligned}\kappa_{mn0} &\approx \frac{8}{\pi^2\epsilon_0}\frac{ab}{(m^2b^2+n^2a^2)}\frac{1}{d}sin^2\frac{\omega d}{2c}\left[1-\left(\frac{m\pi\delta x}{a}\right)^2\right]\left[1-\left(\frac{n\pi\delta y}{b}\right)^2\right]\exp\left(-\omega^2\sigma_t^2\right) \\ &\approx \frac{2d}{ab\epsilon_0}\exp\left(-\omega^2\sigma_t^2\right) \qquad (24)\end{aligned}$$

To put this in perspective, using $a = 4.4$ cm, $b = 6.7$ cm, and supposing $d = 5$ mm, for 102 mA stored in 24 bunches in the APS storage ring, equation 24 indicates that 300 watts of power will be deposited into the lowest order TM_{110} mode. Clearly this could get out of control pretty quickly if not carefully handled. For completeness, to compute the power deposited starting from the loss factor, the desired formula is for example

$$P_{110} = \frac{\kappa_{110}q^2}{T_{bunch}} = \kappa_{110}I_{bunch}^2T_{rev}N_b = \kappa_{110}I_{total}^2T_{rev}/N_b, \qquad (25)$$

Mafia Model



MAFIA 4.106
Plot #0
locate
rotate
zoom
cut
+
−
y
z
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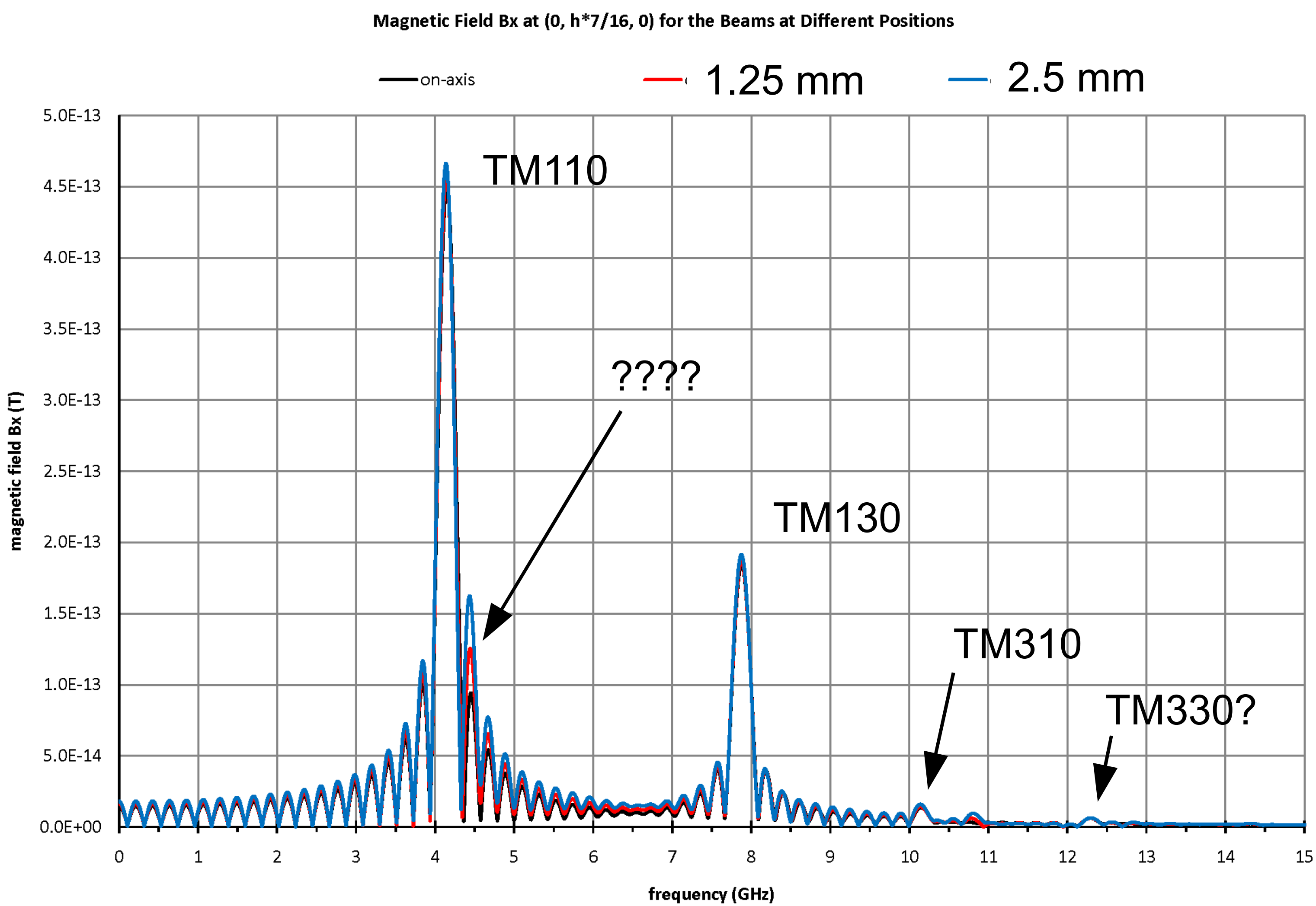
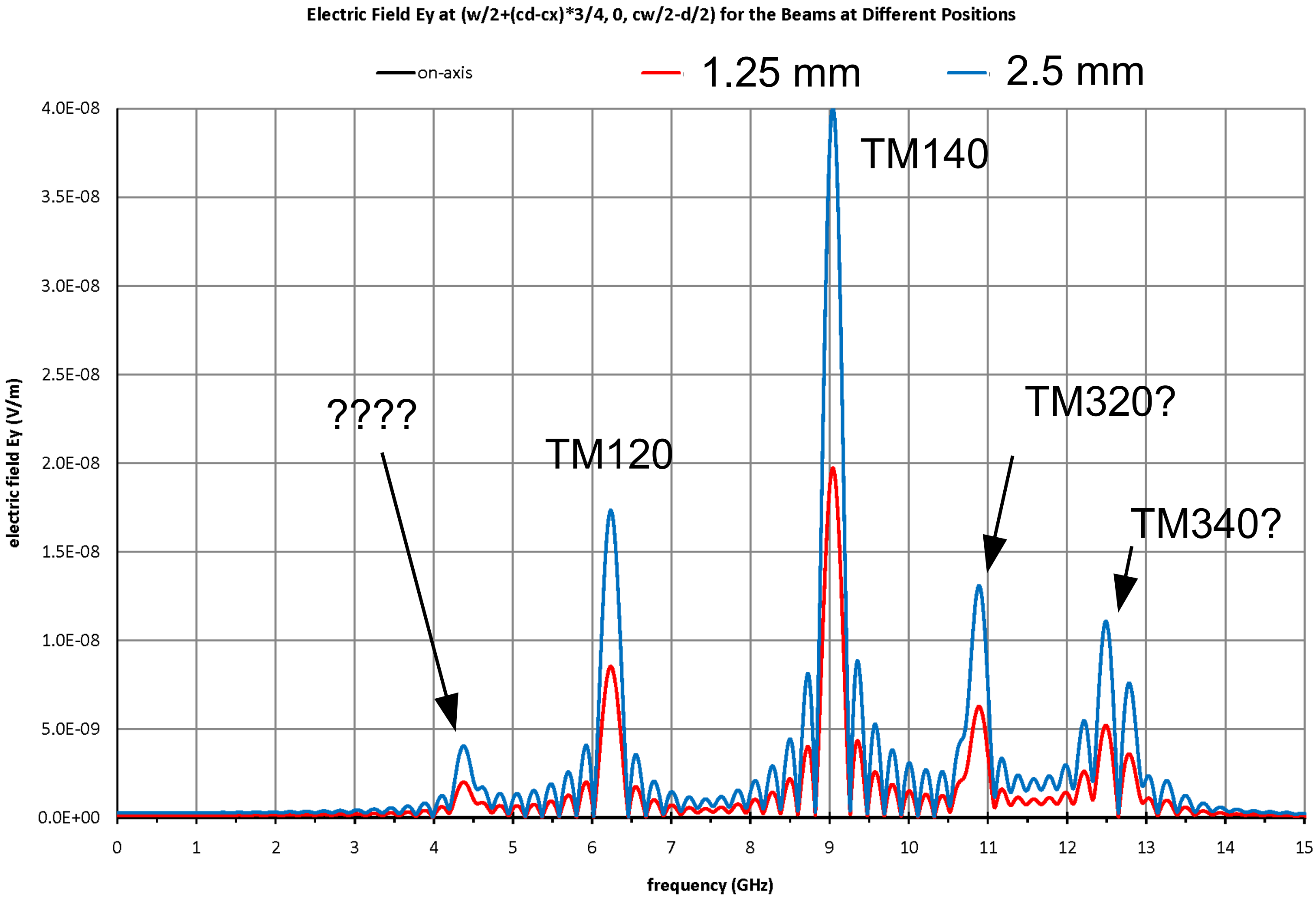
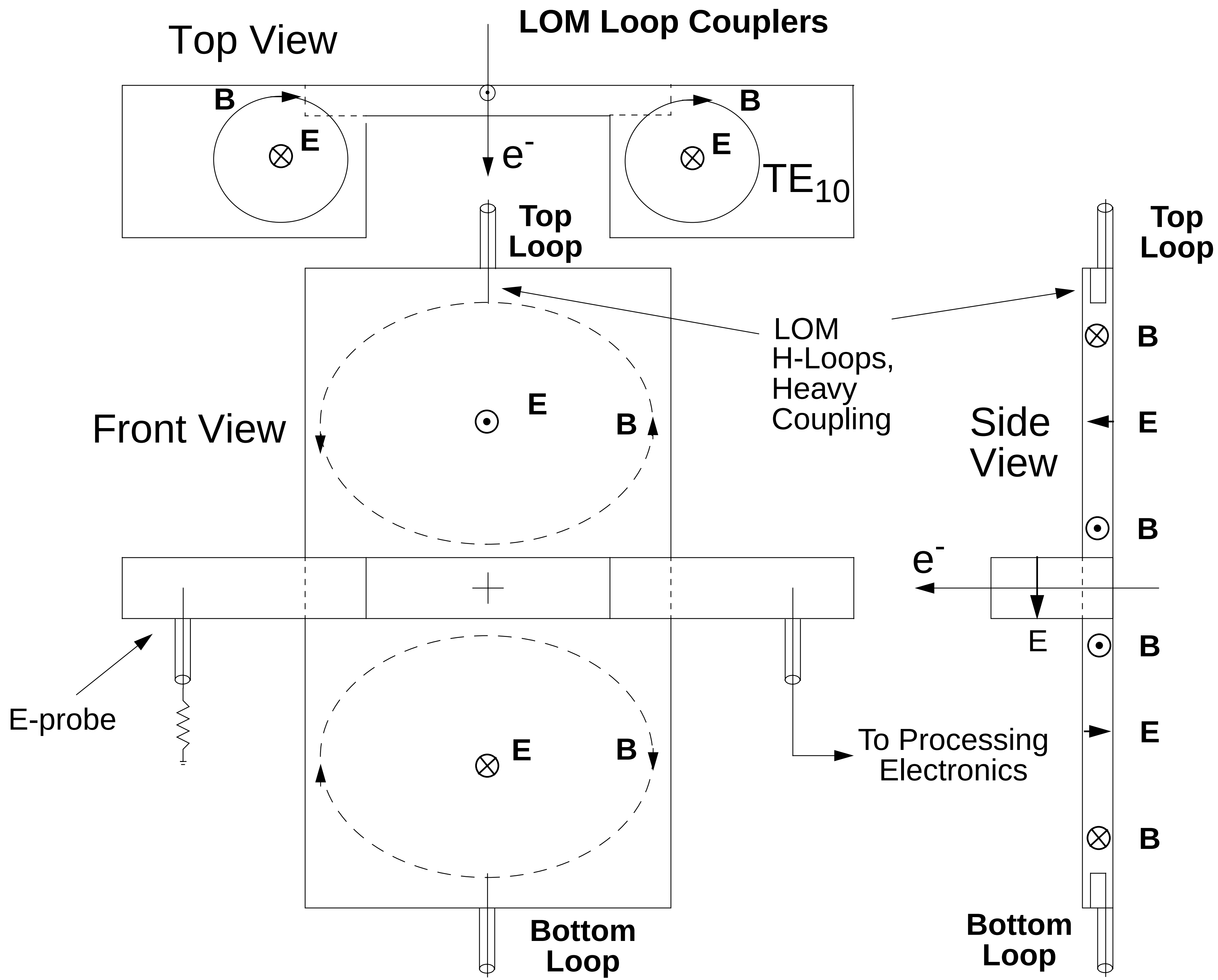
cst
Select
New
Close
Text...
Hardcopy...
Iconify

Mafia Model, Bx at top of Cavity, $\sigma_t = 33$ ps

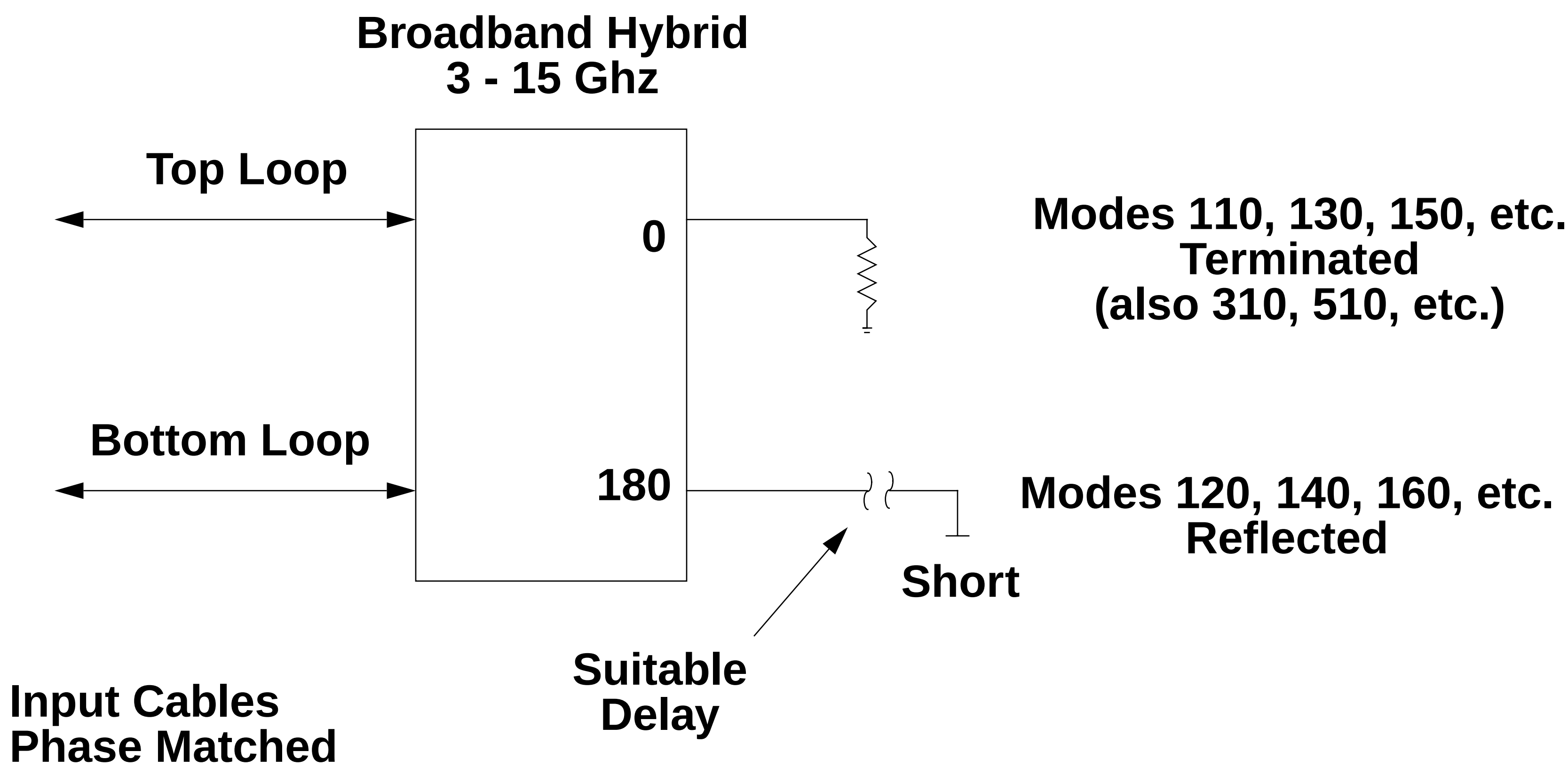


Mafia Model, Ey in External Waveguide σ_t = 33 ps

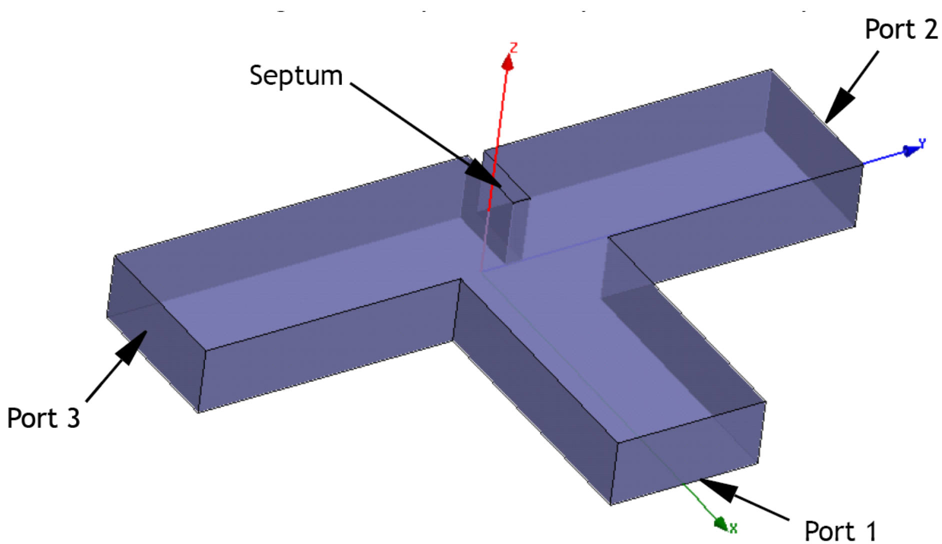


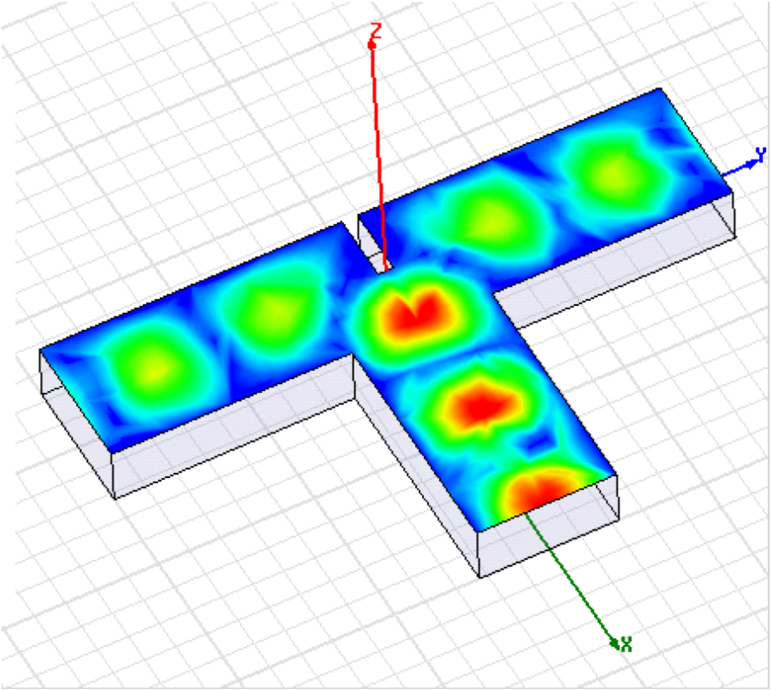

LOM Loop Couplers
Top View
B
E
B
E
e-
TE10
Top Loop
Front View
E
B
LOM H-Loops, Heavy Coupling
Side View
B
E
B
e-
E
B
E
B
E
E-probe
To Processing Electronics
Bottom Loop
Top Loop
Bottom Loop

Odd LOM / HOM Damping Scheme



Waveguide Tee (H-Plane)

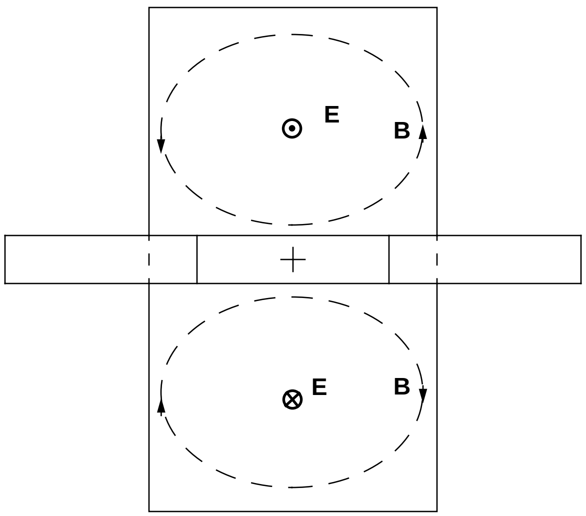


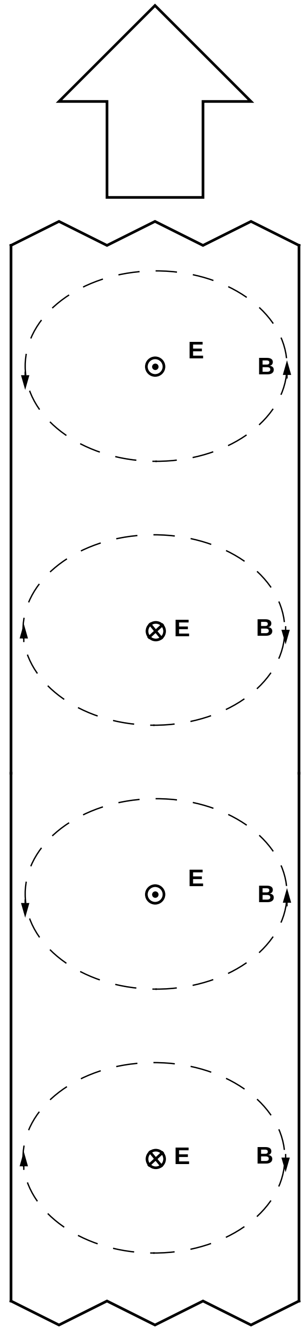


Ez ~ Surface Charge Density

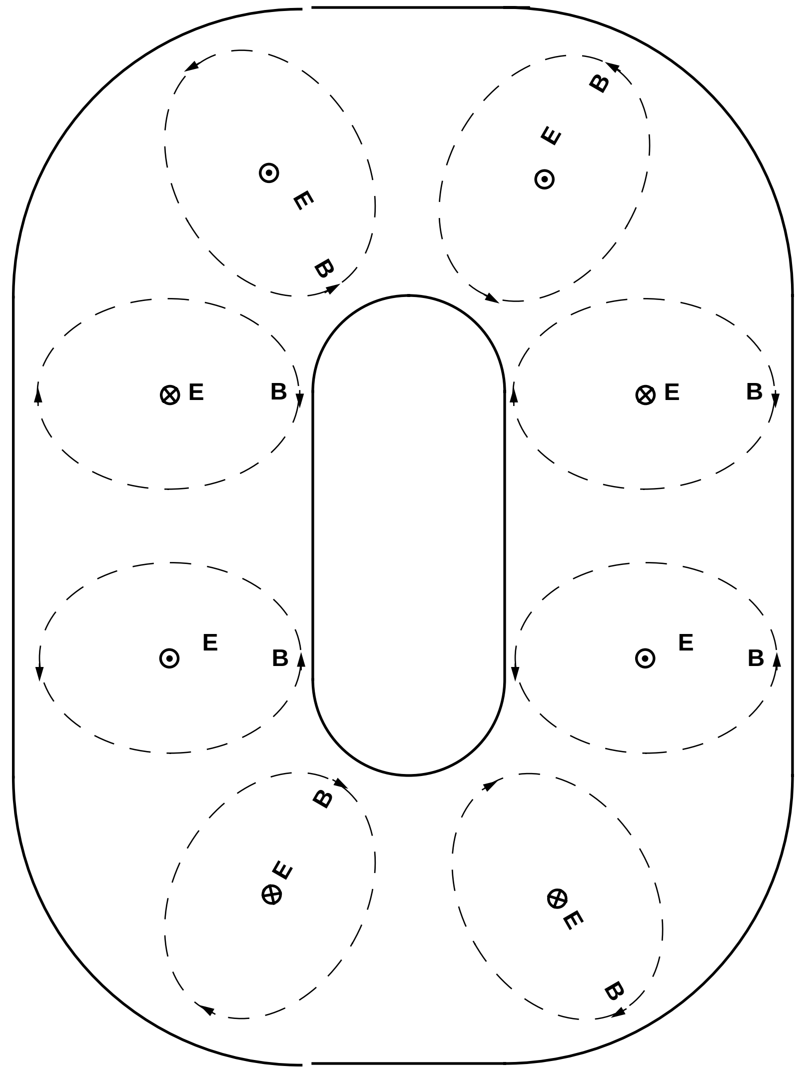
http://www.sunist.org/shared%20documents/References/TeeGSG.pdf

TM_{120}
Cavity Mode



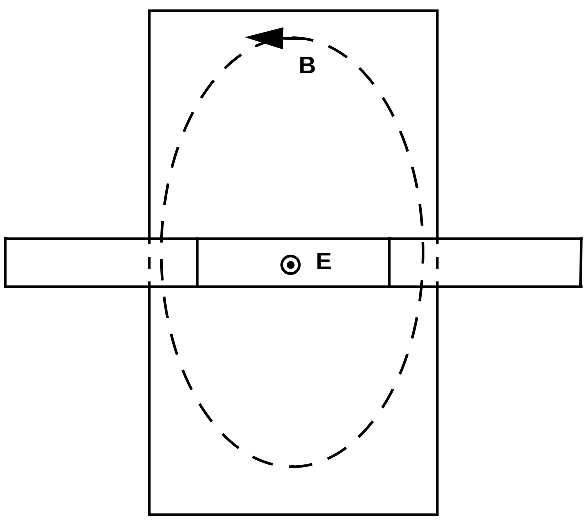


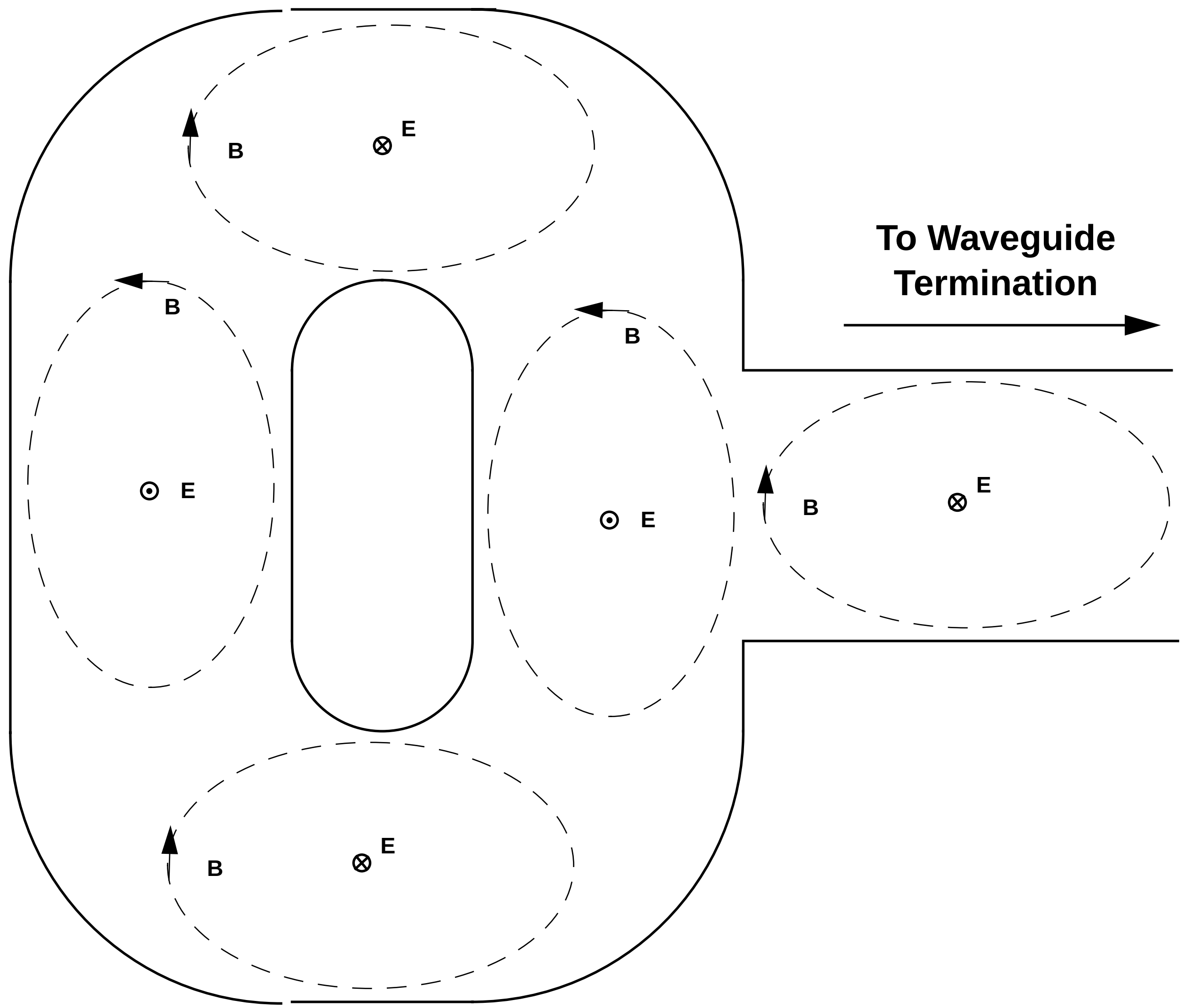
TE_{10}
Waveguide Mode



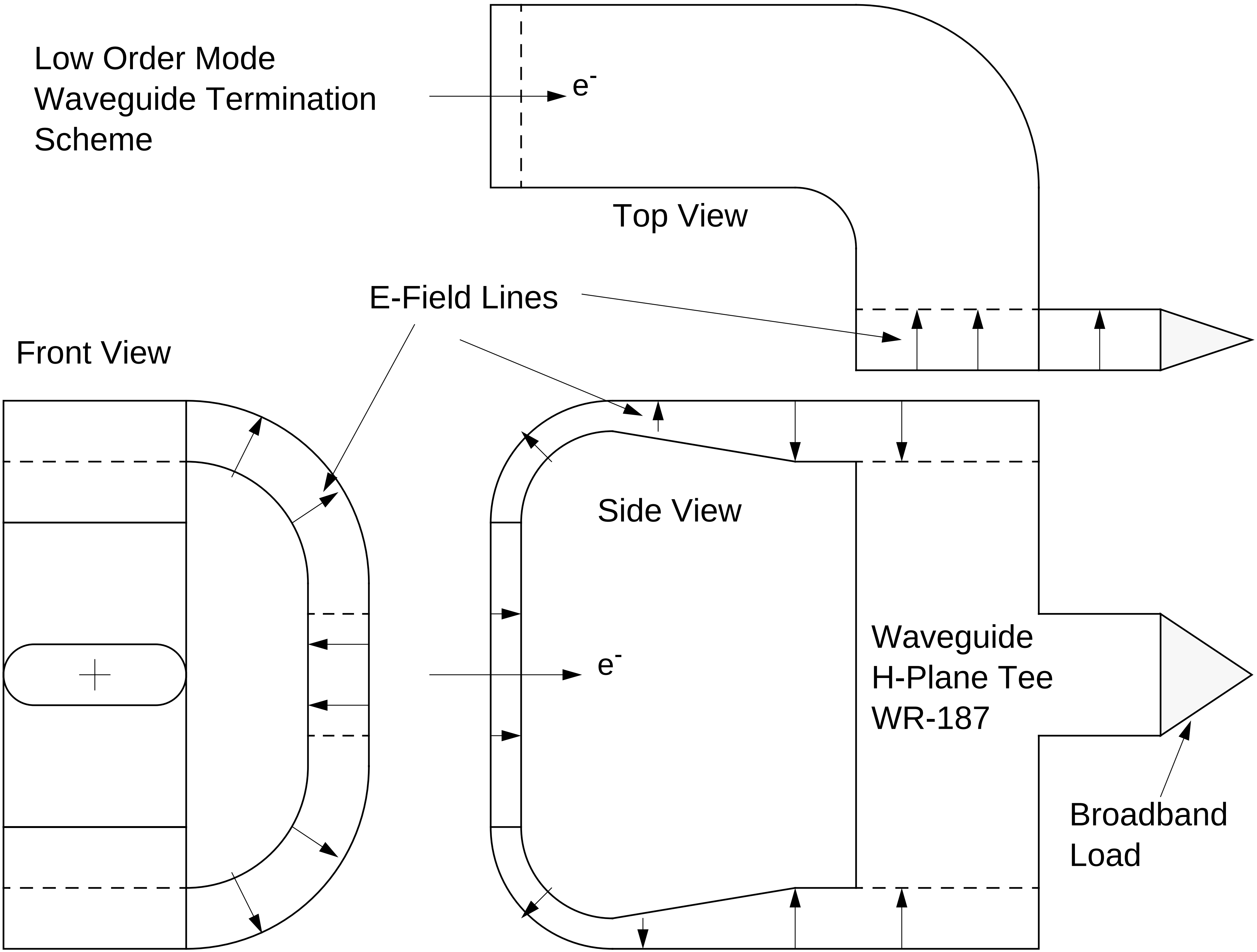
Waveguide Ring
Resonator

TM_{110}
Cavity Mode





Waveguide Ring
Resonator



Low Order Mode
Waveguide Termination
Scheme
e-
Top View
E-Field Lines
Front View
Side View
e-
Waveguide
H-Plane Tee
WR-187
Broadband
Load

Summary

- A C-band cavity beam position / tilt monitor design concept has been completed.
- Dimensions are 4.4 cm wide by 6.7 cm high to maximize LOM / HOM mode separation (a bit wider will improve signal strength).
- Operating frequency is 5.63 GHz = 2 * SPX crab cavity frequency.
- Output waveguide prevents strongly beam-driven LOM, HOM modes from getting to processing electronics.
- Sensitivity amounts to -47 dBm = 30 microradians = 360 nanometers (with 24 bunches, 102 mA total stored beam current, 5 mm cavity depth).
- LOM power deposited at 4.08 GHz is 300 watts, scales with cavity depth.
- Mafia simulations support design concept, sensitivity to mode TM_{140} is strongest. Everything coupled to output waveguide scales with vertical position / tilt.
- LOM, HOM mode damping scheme using waveguide appears feasible.