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# Global Orbit Corrections

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## I. Introduction.

There are various reasons for preferring local (e.g., three-bump) orbit correction methods to global corrections. One is the difficulty of solving the mN equations for the required mN correcting bumps, where N is the number of superperiods and m is the number of bumps per superperiod. The latter is not a valid reason for avoiding global corrections, since, as shown in Ref. [1], we can take advantage of the superperiod symmetry to reduce the mN simultaneous equations to N separate problems, each involving only m simultaneous equations.

Since the method depends on the fact that the linearized equations have N superperiod symmetry, these equations must be taken to be the linear equations for the idealized lattice. Presumably the linear equations will be good enough so that a few iterations of the correction scheme will converge, though this point needs to be tested. Otherwise, some other correction scheme will be needed to bring the orbit within range of the linear correction equations.

The prescription in Ref. [1] amounts essentially to applying a Fourier point transform to each index which labels the superperiods. The mN linear equations then separate into N sets of m equations each, one set for each Fourier index. Since this is a decomposition into wavenumbers, it may have other advantages. For example, we will see that the dominant terms are those having Fourier indices closest the betatron tune $\nu_x$ or $\nu_z$.

In Reference [2], I will show how to solve the general problem when the machine contains unknown magnet errors of known probability distribution, we make measurements of known precision of the orbit displacements at a set of points, and we wish to apply correcting bumps to minimize the weighted rms orbit deviations. In this report, we will consider two simpler problems, using similar methods. In Section II, we consider the case when we make M beam position measurements per superperiod, and we wish to apply an equal number M of orbit correcting bumps to reduce the measured position errors to zero. In Section III, we consider the problem when the number of correcting bumps is less than the number of measurements, and we wish to minimize the weighted rms position errors. We will see that the latter problem involves solving equations of a different form, but involving the same matrices as the former problem.

## II. Reducing the Position Errors to Zero.

Let the beam position monitors be located at the points

$$s = s_m + kS, \; m = 1,\ldots, M, \; k = 0,\ldots,N-1 \; , \tag{2.1}$$

where s is the distance measured along the reference orbit, $s_m$ are a set of M points within a superperiod, k is the superperiod index and S is the length of the reference orbit in a superperiod. Note that we assume the beam position monitors are located at periodic points in all superperiods. Let the measured orbit errors (horizontally, or vertically) be $-Y_{km}$. We wish to find a set of orbit bumps $B_{\ell n}$ which reduce the measured orbit deviations to zero. The bumps are located at the points

$$s = s_n + \ell S, \; n = 1,\ldots,M, \; \ell = 0,\ldots,N-1, \tag{2.2}$$

where n labels the bumps within a single superperiod, and $\ell$ is the superperiod index. In linear approximation, we require

$$\Sigma_{\ell n} A_{km;\ell n} B_{\ell n} = Y_{km} \; , \tag{2.3}$$

where the matrix element $A_{km;\ell n}$ is the orbit deviation at $s_m + kS$ due to a unit bump at $s_n + \ell S$. The bumps $B_{\ell n}$ are measured in units such that a unit bump $B_{\ell n} = 1$ produces a unit increment in slope $Y'(s)$ at $s = s_n + \ell S$. In view of the symmetry, the matrix $A_{km;\ell n}$ may depend only on the difference of the superperiod indices:

$$A_{km;\ell n} = A_{(k-\ell)mn}. \tag{2.4}$$

The Fourier point transforms of the bumps $B_{\ell n}$ and the displacements $Y_{km}$ are defined by the equations

$$b^{\alpha}{}_n = N^{-1/2} \, \Sigma_{\ell} \, B_{\ell n} \, e^{-2\pi i \alpha \ell / N}, \tag{2.5}$$

$$B_{\ell n} = N^{-1/2} \, \Sigma_{\alpha} \, b^{\alpha}{}_n \, e^{2\pi i \alpha \ell / N}, \tag{2.6}$$

$$y^{\alpha}{}_m = N^{-1/2} \, \Sigma_k \, Y_{km} \, e^{-2\pi i \alpha k / N}, \tag{2.7}$$

$$Y_{km} = N^{-1/2} \, \Sigma_{\alpha} \, y^{\alpha}{}_m \, e^{2\pi i \alpha k / N}. \tag{2.8}$$

The exponentials in Eqs. (2.5)-(2.8) are periodic with period N in the indices $\alpha$, $\ell$, k. We take the coefficients $b^{\alpha}{}_{n}$, $B_{\ell n}$, $y^{\alpha}{}_{m}$, $Y_{km}$ to have the same periodicity in these indices. The sums in these equations may then be taken over any N consecutive values of the summation index. For many purposes, it will be convenient to let the Fourier index $\alpha$ run over the values $\alpha = 0$, $\pm 1, \ldots, \pm(N-2)/2$, $N/2$, in order emphasize the fact that

$$b^{-\alpha}{}_{n} = b^{\alpha}{}_{n}{}^{*}, \; y^{-\alpha}{}_{m} = y^{\alpha}{}_{m}{}^{*} \; . \tag{2.9}$$

The Fourier transform of Eq. (2.3) can be written in the form

$$\Sigma_{n} \, C^{\alpha}{}_{mn} \, b^{\alpha}{}_{n} = y^{\alpha}{}_{m}, \tag{2.10}$$

where

$$C^{\alpha}{}_{mn} = \Sigma_{r} \, A_{rmn} \, e^{-2\pi i r\alpha/N}. \tag{2.11}$$

We have to solve N separate sets of M simultaneous equations each, for the Fourier coefficients $b^{\alpha}{}_{n}$.

We now calculate explicitly the coefficients $C^{\alpha}{}_{mn}$. The orbit corresponding to a unit bump at $s = s_n + \ell S$ is given by

$$y(s) = \frac{w_n w(s)}{2\sin(\pi\nu)} \cos[\psi(s) - \psi_n - \ell\mu - \pi\nu], \tag{2.12}$$

where the phase in square brackets is to be adjusted by adding a multiple of $2\pi\nu$ so that it lies in the range $(-\pi\nu, \pi\nu)$, where

$$w_n = w(s_n) = [\beta(s_n)]^{1/2}, \tag{2.13}$$

and where $\psi_n = \psi(s_n)$ is the phase advance from $s = 0$ to $s = s_n$ in a superperiod. The phase advance through a superperiod is $\mu$, and the number of betatron oscillations per revolution is $\nu = N\mu/2\pi$. Because of the superperiod symmetry, $w(s_n)$ is periodic with period S, and $w_n$ does not require an index $\ell$. The matrix element (2.4) is therefore given by

$$A_{rnm} = \frac{w_n w_m}{2\sin(\pi\nu)} \cos[\psi_m - \psi_n + r\mu - \pi\nu]. \tag{2.14}$$

We substitute this value in Eq. (2.11). The sum is evaluated by expanding the cosine in complex exponential form, with the result

$$C^{\alpha}{}_{mn} = \frac{1}{4} w_m w_n \frac{\sin(\mu - |\psi_m - \psi_n|) + e^{2\pi i \Delta_{mn} \alpha/N} \sin|\psi_m - \psi_n|}{\sin\frac{\pi(\alpha-\nu)}{N} \sin\frac{\pi(\alpha+\nu)}{N}}, \tag{2.15}$$

where

$$\Delta_{mn} = \begin{cases} 1 \text{ if } \psi_m \geqslant \psi_n, \\ -1 \text{ if } \psi_m < \psi_n. \end{cases} \tag{2.16}$$

Note that $C^{\alpha}{}_{mn}$ has the symmetry:

$$C^{-\alpha}{}_{mn} = C^{\alpha}{}_{mn}{}^{*}. \tag{2.17}$$

We therefore need to solve Eqs. (2.10) only for $\alpha = 0,1,\ldots, N/2$, since for $\alpha = -1,\ldots, -(N-2)/2$, the solutions are the conjugates of the solutions for positive values of $\alpha$. Note the resonant dominators in Eqs. (2.15). Note also that if the bumps and BPM's are at the same locations, then the matrix $A_{km;\ell n}$ is symmetric, and the matrix $C^{\alpha}{}_{mn}$ is hermitian.

## III. Minimizing Position Errors.

If the number B of bumps per superperiod is less than the number M of beam position measurements ($n = 1,\ldots,B$), then we define the weighted mean square position error:

$$E = \Sigma_{km} W_m [\Sigma_{\ell n} A_{km;\ell n} B_{\ell n} - Y_{km}]^2, \tag{3.1}$$

where $W_m$ is the weight assigned to the position error at $s_m + kS$. Minimizing E results in the equations

$$\sum_{m=1}^{M} W_m C^{\alpha}_{mn} [\sum_{n'=1}^{B} C^{-\alpha}_{mn'} b^{\alpha}_{n'} - y^{\alpha}_{m}] = 0, \; n = 1,\ldots,B, \tag{3.2}$$

involving the same coefficients $C^{\alpha}_{mn}$ as Eq. (2.10). Equations (3.2) are to be solved for the Fourier coefficients $b^{\alpha}_{n}$. If $B = M$, then Eqs. (2.10) satisfy Eqs. (3.2).

## References.


1. K. Symon, "The Eigenvalue Equation for an N-Sector Ring". LS-98, July, 1987.
2. K. Symon, "Orbit Corrections Based on Bayes' Theorem". To appear as an LS report.