RADIATIVE COOLING OF RELATIVISTIC ELECTRON BEAMS*

Zhirong Huang, APS/ANL, Argonne, IL 60439
Ronald D. Ruth, SLAC, Stanford, CA 94309

Abstract

Radiative cooling is a natural and effective method of phase-space cooling for stored electron beams. In electron storage rings the average effects of synchrotron radiation from the bending magnets cause the beam emittances in all three degrees of freedom to damp towards equilibria, determined by the fluctuating nature of quantum emissions. In this paper, we show that the radiation damping in a focusing system is fundamentally different from that in a bending system. Quantum excitation to the transverse dimensions is absent in a straight, continuous focusing channel, and is exponentially suppressed in a focusing-dominated ring. Thus, the transverse normalized emittances in such systems can in principle be damped to the Compton wavelength of the electron, limited only by the Heisenberg Uncertainty Principle. In addition, we investigate methods of rapid damping such as radiative laser cooling. We propose a laser-electron storage ring (LESR) where the electron beam in a compact storage ring repetitively interacts with an intense laser pulse stored in an optical resonator. The laser-electron interaction gives rise to fast cooling of electron beams and can be used to overcome the space-charge effects encountered in a medium-energy circular machine. Applications to the designs of ultra-low-emittance damping rings and compact x-ray sources are also explored.

1 INTRODUCTION

Modern high-energy particle accelerators and synchrotron light sources demand smaller and smaller beam emittances in order to achieve greater luminosity or higher brightness. In electron synchrotrons and storage rings, radiative cooling is a natural and effective way to obtain low-emittance beams. Radiation damping in the longitudinal phase space occurs because higher energy electrons in a bunch lose more energy than lower energy electrons. Furthermore, the fact that electrons on average lose momenta in all three degrees of freedom to the radiation will contribute to the damping in the transverse phase space. Thus, both the longitudinal and the transverse damping rates are proportional to the energy damping rate, which is the ratio of the average radiation power P_γ to the electron energy $E = \gamma mc^2$:

$$\Gamma_b = \frac{\langle P_\gamma \rangle}{E} = \frac{2}{3} c r_e \gamma^3 \left\langle \frac{1}{\rho^2} \right\rangle_s , \tag{1}$$

where $r_e = 2.82 \times 10^{-15}$ m is the classical electron radius, ρ is the radius of the bending magnets, and the index s indicates averaging around the ring. In fact, the damping rates of the transverse and the longitudinal emittances are related through the Robinson theorem [1]:

$$\frac{d\varepsilon_i}{dt} = -\mathcal{J}_i \Gamma_b \varepsilon_i \quad \text{with} \quad \sum_i \mathcal{J}_i = 4, \tag{2}$$

where $i = x, y, s$ denotes horizontal, vertical, and longitudinal directions, respectively.

Nevertheless, particle motion does not contract to a point in phase space because synchrotron radiation occurs in quanta of discrete energies. Each time a photon is emitted the energy of the electron makes a small discontinuous jump. This quantum "noise" suddenly changes the off-energy orbit of the horizontal betatron oscillation and the instantaneous angle of the vertical betatron motion. The cumulative effect of many such disturbances introduces diffusion into oscillation modes. The amplitude of oscillation will grow until quantum excitation is, on average, balanced by the damping of the oscillations. In a smooth and separated-function storage ring, the equilibrium-normalized transverse emittances can be written as [2]

$$\begin{aligned} \varepsilon_x^{\mathrm{n}} &= \gamma \varepsilon_x \sim \lambda\!\!\!^{-}_c \frac{\beta_x^3}{(\rho/\gamma)^3} \sim \lambda\!\!\!^{-}_c \frac{\gamma^3}{\nu_x^3}, \\ \varepsilon_y^{\mathrm{n}} &= \gamma \varepsilon_y \sim \lambda\!\!\!^{-}_c \frac{\beta_y}{(\rho/\gamma)} \sim \lambda\!\!\!^{-}_c \frac{\gamma}{\nu_y}, \end{aligned} \tag{3}$$

where $\lambda\!\!\!^{-}_c = \hbar/mc = 3.86 \times 10^{-13}$m is the Compton wavelength of the electron, β_x, β_y and ν_x, ν_y are the average beta functions and the tunes of the machine. In the longitudinal direction, the equilibrium energy spread is [2]

$$\sigma_\delta \sim \gamma \sqrt{\frac{\lambda\!\!\!^{-}_c}{2\rho}}. \tag{4}$$

While the normalized longitudinal emittance $\varepsilon_s^{\mathrm{n}} \equiv \gamma \sigma_\delta \sigma_s$ can be changed by adjusting the bunch length σ_s through the rf systems, the equilibrium-normalized transverse emittances (sometimes called the natural emittances) are more or less fixed for a given lattice. Equation (3) indicates that lower emittances can be obtained by going to a lower energy and higher tune machine whenever possible. In practice, the tunes are much smaller than γ and the quantities $\frac{\beta_{x,y}}{(\rho/\gamma)}$ are always much larger than 1, resulting emittances are many orders of magnitude larger than the Compton wavelength, and the horizontal emittance is much larger than the vertical.

The above consideration suggests that a smaller equilibrium emittance could be obtainable by reducing the rate of quantum excitation and/or increasing the rate of radiation damping. If one considers an ideal limit $\rho \to \infty$ with γ

* Work supported by U.S. Department of Energy.

and $\beta_{x,y}$ fixed, then the equilibrium emittance would vanish according to Eq. (3). In Section 2 we show that quantum excitation to the transverse oscillations is absent in this straight focusing channel and is exponentially suppressed in a focusing-dominated damping ring. The normalized transverse emittances in such systems do not vanish but are limited by the Compton wavelength. In Section 3 we study a fast damping mechanism by employing a high peak or a high average power laser in an optical resonator to cool an electron bunch in a compact storage ring. This laser-electron storage ring (LESR) can be configured either for the production of low-emittance electron beams or as a high-intensity x-ray source.

2 SUPPRESSION OF QUANTUM EXCITATION

In deriving Eq. (3), the photon emissions are considered to be instantaneous and modeled as statistical noise [2]. Such a quasiclassical picture of quantum excitation is valid as long as the time associated with the emission of radiation quanta is short compared with the periods of the classical modes of oscillation in all three degrees of freedoms. The typical radiation formation length is on the order of ρ/γ and is much smaller than the betatron oscillation wavelengths $\beta_{x,y}$ in normal storage rings. However, as the strength of the transverse focusing increases or as the bending field gradually decreases, the radiation formation length and the betatron oscillation wavelengths may become comparable. The validity of the quasiclassical approximation along with Eq. (3) is suspect. In Refs. [3, 4], two such cases were investigated and an interesting regime for ultra-low emittance generation was found. The basic results are summarized here.

2.1 A straight focusing channel

Let us consider an ideal focusing channel: a continuous focusing force $(-K_x x)$ in the x direction and a free, relativistic longitudinal z motion. The Hamiltonian is then

$$\begin{aligned} H &= \sqrt{m_e^2c^4 + p_z^2c^2 + p_x^2c^2} + \frac{K_x x^2}{2} \\ &\approx E_z + J_x\omega_x, \end{aligned} \tag{5}$$

where $E_z \equiv \sqrt{m_e^2c^4 + p_z^2c^2}$ is the longitudinal energy, $\omega_x = \sqrt{K_x c^2/E_z}$ is the transverse oscillation frequency, and J_x is the transverse action. It is obvious that both p_z and J_x are constants of motion because their conjugate coordinates are absent in the Hamiltonian. The transverse action, averaged over the beam distribution, is related to the normalized transverse emittance by

$$\varepsilon_x^{\rm n} = \frac{\langle J_x\rangle_{\rm beam}}{mc}. \tag{6}$$

In the event of a photon emission, the total energy and the total longitudinal momentum between the electron and the photon are conserved. Suppose a photon of energy $\hbar\omega$ is emitted with an angle θ relative to the longitudinal direction (Fig. 1), then the change of electron energy and longitudinal momentum are given by

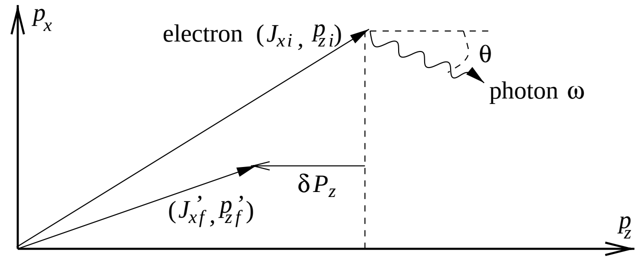

Figure 1: The initial and the final states of the electron in a focusing channel after a random photon emission.

$$\delta E = -\hbar\omega, \quad \delta p_z c = -\hbar\omega\cos\theta. \tag{7}$$

From Eq. (5), we obtain the change of the transverse action

$$\delta J_x = -\frac{\hbar\omega}{\omega_x}\left(1 - \beta\cos\theta + \frac{\theta_p^2}{4}\right) < 0, \tag{8}$$

where $\beta = p_z c/E_z$ is the average velocity, and $\theta_p^2 = 2J_x\omega_x/E_z$ is the pitch angle of the electron. Thus, the transverse action (as well as the transverse emittance) always decreases in spite of random photon direction of emissions. Electron does not recoil directly against the photon because of the existence of the transverse focusing environment.

The lack of quantum excitation in a focusing channel leads to a classical point in the transverse phase space for the electron. In quantum mechanics, this ground state is described by a Gaussian wave packet that has the minimum action set by the Heisenberg uncertainly principle:

$$J_x = \delta x\delta p_x \geq \frac{\hbar}{2}. \tag{9}$$

A beam of N electrons occupying different longitudinal phase space or spin states can all be damped to their transverse ground states, reaching the fundamental emittance

$$\varepsilon_0^n = \frac{\hbar/2}{mc} = \frac{\lambda\!\!\!^{-}_c}{2} = 1.93\times10^{-13}\mathrm{m}. \tag{10}$$

The channeling radiation damping rate can be obtained most easily by going to the longitudinal comoving frame of the channeled electron when the transverse oscillation amplitude is small ($\gamma\theta_p \ll 1$). The longitudinal translational invariance guarantees that the electron sees a one-dimensional (transverse) harmonic potential with the focusing strength $K_x^\star = \gamma K_x$ in this frame (denoted by a star). The electron gives away transverse energy $E_x^\star = \langle K_x^\star x^2\rangle$ through dipole radiation with a rate

$$\frac{1}{E_x^\star}\frac{dE_x^\star}{dt^\star} = \frac{1}{\langle K_x^\star x^2\rangle}\frac{2}{3}\frac{r_e}{mc}\langle (K_x^\star x)^2\rangle = \frac{2}{3}\frac{r_e}{mc}K_x^\star. \tag{11}$$

When transforming back to the lab frame, we obtain an energy-independent damping rate for the focusing channel

$$(\Gamma_c)_x = \frac{2}{3}\frac{r_e}{mc}K_x. \tag{12}$$

A more detailed analysis of channeling radiation damping when the oscillation amplitude is large and when the focusing strength is periodic (such as in a FODO lattice) can be found in Ref. [5]. As an example, suppose the quadrupole field gradient is about $g = 100$ T/m, corresponding to a focusing strength $K_x \simeq 30$ GeV/m^2. The damping constant $\Gamma_c \sim 30$ s^{-1}, which is a negligible effect for linear accelerators. However, the focusing strength for a typical crystal channel is $K_x \sim 10^{11}$ GeV/m^2, resulting in $\Gamma_c \sim 10$ ns^{-1} and the damping distance on the order of meters. Of course, a crystal channel is far from ideal, and the multiple Coulomb scattering is the primary excitation mechanism competing against the radiation damping.

2.2 A focusing-dominated damping ring

However, it is not necessary to have a straight channel. In a bent focusing system where the radiation formation length is comparable to the betatron oscillation wavelength, the quasiclassical picture of the instantaneous radiation is insufficient to describe the radiation reaction. By using a quantum mechanical perturbation approach, we have analyzed a continuous focusing and bending combined system, and obtain the rate of change for the transverse emittances [5]

$$\frac{d\varepsilon_x^{\mathrm{n}}}{dt} = -\Gamma_b\left[\left(\chi_x^2 - 1\right)\left(\varepsilon_x^{\mathrm{n}} - \frac{\lambda\!\!\!^-_c}{2}\right) - \lambda\!\!\!^-_c\frac{F_x(\chi_x)}{96\chi_x^3}e^{-2\sqrt{3}\chi_x}\right],$$
$$\frac{d\varepsilon_y^{\mathrm{n}}}{dt} = -\Gamma_b\left[\left(\chi_y^2 + 1\right)\left(\varepsilon_y^{\mathrm{n}} - \frac{\lambda\!\!\!^-_c}{2}\right) - \lambda\!\!\!^-_c\frac{F_y(\chi_y)}{96\chi_y}e^{-2\sqrt{3}\chi_y}\right], \tag{13}$$

where $\chi_x = \frac{\rho/\gamma}{\beta_x}$ and $\chi_y = \frac{\rho/\gamma}{\beta_y}$, and

$$F_x(\chi_x) = 55\sqrt{3} + 330\chi_x + 262\sqrt{3}\chi_x^2 + 300\chi_x^3 + 48\sqrt{3}\chi_x^4,$$
$$F_y(\chi_y) = 13\sqrt{3} + 30\chi_y + 12\sqrt{3}\chi_y^2. \tag{14}$$

Equation (13) describes the general results of radiation damping (the first term) and quantum excitation (the second term) to the transverse emittances in this combined-function system. The relative amount of damping and excitation in each transverse plane is determined from a single dimensionless parameter $\chi_{x,y}$, which is the measure of the radiation formation length in units of the average beta function. For separated-function systems, it is expected that the average effect of the bending magnets in the horizontal plane is damping ($\mathcal{J}_x = 1$) instead of anti-damping ($\mathcal{J}_x = -1$). In the limit of $\chi_{x,y} \ll 1$ or $\rho/\gamma \ll \beta_{x,y}$, setting these rates equal to zero reproduces the equilibrium transverse emittances of storage rings (i.e., Eq. (3)). In the opposite limit where $\chi_{x,y} \gg 1$, or $\rho \to \infty$, we have $\Gamma_b\chi_{x,y}^2 = (\Gamma_c)_{x,y}$ from Eq. (12), and the quantum excitation term vanishes, the basic results for a straight focusing channel.

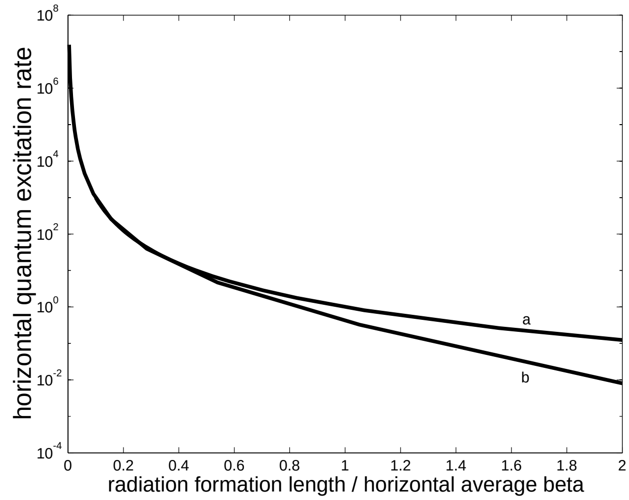

Figure 2: Horizontal quantum excitation rate in units of $\Gamma_b\lambda\!\!\!^-_c$, predicted by (a) the quasi-classical model, and (b) the quantum mechanical perturbation approach.

In the intermediate regime where the radiation formation length is on the order of the average beta function ($\rho/\gamma \sim \beta_{x,y}$ or $\chi_{x,y} \sim 1$), the transverse damping comes from both the bending and the focusing fields. The rates of quantum excitation in both transverse dimensions are exponentially suppressed according to Eq. (13) and start to depart from the results based on the quasiclassical approach (see Fig. 2). Thus, the fundamental emittance can be approached very closely in such a focusing-dominated system. The reason for the suppression of quantum excitation can be interpreted as follows: The transverse energy levels of the electron are well separated as a result of the strong focusing forces. Radiative transition to higher transverse levels becomes impossible for the electron with almost all photon emissions, and hence the quantum excitation is suppressed by the focusing environment.

In the same regime, the average radiation power comes predominately from the bending field rather than from the focusing field, and the longitudinal damping rate remains the same as the storage ring limit. In addition, since the synchrotron oscillation period is always much longer than the betatron ones, the radiation formation length is always much smaller than the synchrotron oscillation wavelength. Thus, the instantaneous picture of quantum emission is still valid in the longitudinal phase space and Eq. (4) still holds. The total phase-space volume of an N-electron beam is limited by the Fermi statistics, i.e.,

$$\varepsilon_x^{\mathrm{n}}\varepsilon_y^{\mathrm{n}}\varepsilon_s^{\mathrm{n}} \geq \frac{N}{2}\left(\frac{\lambda\!\!\!^-_c}{2}\right)^3. \tag{15}$$

When $\varepsilon_{x,y}^{\mathrm{n}} \sim \lambda\!\!\!^-_c/2$, we have $\varepsilon_s^{\mathrm{n}} \geq N\lambda\!\!\!^-_c/4$. For $N \sim 10^{10}$, this gives a limit of about 1 mm for $\varepsilon_s^{\mathrm{n}}$, which is quite reasonable because the end use of the beam does not require ultra-short bunches with ultra-small energy spread.

A focusing-dominated damping ring can be designed with many repetitive FODO cells. As indicated from Eqs. (3) and (13), it is favorable to use high-gradient focusing quads and low-energy electron beams. For instance, suppose that permanent-magnet quads have a field gradient 4 Tesla/cm, and that the electron energy is 25 MeV. We can have a ring with an average radius of about 2 m, an average beta function of 4 cm in both transverse planes, and a cell length of 2 cm with 60-degree phase advance. In principle, the equilibrium-normalized transverse emittances can reach the Compton wavelength while the energy spread is on the order of 10^{-5}. The damping time is inevitably long, about 30 s in all three degrees of freedom. Thus, the intensity of the ultra-cold beam is limited by the collective effects such as space charge and wakefields. It is conceivable to operate this ring below the transition energy $\gamma_t \approx \nu_x$, so that a six-dimensional phase space equilibrium may exist due to the effect of intrabeam scattering [6]. These effects have yet to be studied in this new regime.

3 RADIATIVE LASER COOLING

In this section, we investigate another route to low-emittance generation: a fast radiative cooling method. Traditionally, the increase of the damping rate in a storage ring is achieved through the insertion wigglers [7]. The effect of these damping wigglers is to generate a lot more radiation while keeping quantum excitation in check. This can be done by placing the wigglers in the dispersion-free region of the storage ring. Recently, Telnov pointed out [8] that with a sufficiently intense laser pulse, a high-energy electron beam can be cooled significantly during a single collision with the laser pulse. The electrons radiate energy in the form of scattered photons, and hence the term "radiative laser cooling." Later, we proposed [9] a compact laser-electron storage ring (LESR) where radiative laser cooling is used to overcome the space-charge effects encountered in a medium-energy circular machine for electron beam cooling or x-ray generation.

The basic idea of a laser-electron storage ring is shown in Fig. 3. An electron beam is injected into a storage ring and at the same time an intense laser pulse is built up inside a high-finesse optical resonator. The laser light path is chosen to match exactly the time it takes for the electron to circulate once around the ring so that a focused electron beam repeatedly encounters the short light pulse at the focus of the resonator each turn. Normally, in the absence of the laser the electron beam would damp at the rate determined by the time it takes to radiate its complete energy in the bending magnets in the ring. In the LESR, the laser pulse acts like an extremely strong damping undulator, and the fast radiative laser cooling leads to a very low-emittance beam for very moderate electron energy (around 100 MeV). As the beam circulates around the ring, the lost energy is restored by an rf accelerating system, as in a normal storage ring. This cooling effect can also be utilized as a stabilization mechanism to maintain a dense bunch of electrons for the generation of intense x-rays.

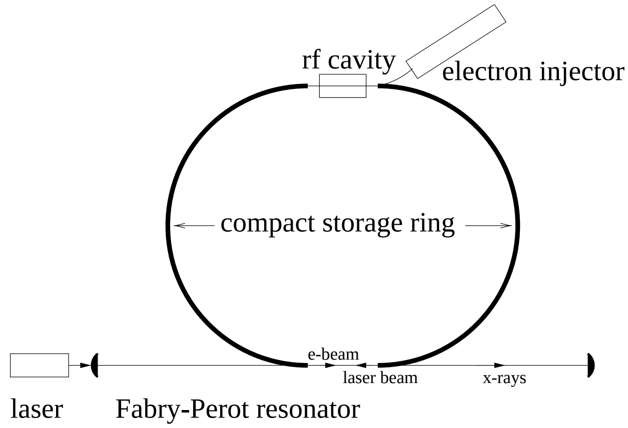

Figure 3: The schematic diagram of a laser-electron storage ring.

For electrons no more than a few hundred MeV, the normal radiation damping and quantum excitation from the bending magnets is negligible compared with those from the laser field. The laser field can be regarded as an undulator with an equivalent period one half of the laser wavelength λ_L, and an equivalent bending field given by

$$B_u = \frac{2}{c}\sqrt{2Z_0 I}, \tag{16}$$

where I is the laser intensity, and $Z_0 = (c\epsilon_0)^{-1} = 377$ Ω. The radiation damping rate can then be calculated from Eq. (1). Writing in terms of the laser flash energy E_L and the Rayleigh range Z_R, we have [9]

$$\Gamma_L = \frac{E_L[\mathrm{J}]E[\mathrm{MeV}]}{1.6\times10^5\lambda_L[\mu\mathrm{m}]Z_R[\mathrm{mm}]}\frac{1}{T_0}, \tag{17}$$

where T_0 is the revolution period. The damping partition numbers are

$$\mathcal{J}_x = \mathcal{J}_y = 1, \quad \mathcal{J}_s = 2. \tag{18}$$

The laser-electron interaction also gives rise to quantum excitation of the transverse emittances the same way as in a bending magnet. It can be shown [9] that the minimum-normalized transverse emittances are

$$(\varepsilon^{\mathrm{n}}_{x,y})_{\min} = \frac{3\pi}{5}\frac{\lambda\!\!\!^{-}_c}{\lambda_L}\beta^*_{x,y}, \tag{19}$$

where $\beta^*_{x,y}$ are the beta functions at the interaction region. The minimum energy spread is determined by the rms fluctuation of the scattered photons and is given by [5]

$$(\sigma_\delta)_{\min} = \sqrt{\frac{7\pi}{5}\frac{\lambda\!\!\!^{-}_c}{\lambda_L}\gamma}. \tag{20}$$

In addition to quantum excitation, intrabeam scattering [6] provides an intensity-dependent diffusion in the phase

Table 1: Two laser-electron storage ring configurations

LESR mode	transient	steady state
Laser and resonator para.		
wavelength [μm]	1	1
flash energy in resonator	2 J	20 mJ
Rayleigh range [mm]	5	8
focal spot size [μm]	20	25
Electron storage ring para.		
energy [MeV]	100	8
number of electrons	1×10^{10}	1×10^{10}
average ring radius [m]	1	0.5
horizontal/vertical tune	$\sim$10	$\sim$10
energy loss per turn	25 keV	1 eV
trans. damping time	84 μs	84 ms
equil. energy spread	1.8%	2.3%
rf frequency [MHz]	2856	1428
rf peak voltage	1 MV	60 kV
momentum acceptance	10%	23%
rms bunch length [mm]	5.8	6.6
norm. long. emit. [mm]	21	2.4
norm. trans. emit. [m]	1×10^{-7}	6×10^{-6}
X-ray parameter		
wavelength	6.25 pm	1 nm
photon energy [keV]	200	1.24
photon flux [sec^{-1}]	2×10^{20}	8×10^{14}

space of the electron beam. The equilibrium emittances and energy spread can be obtained by balancing the rate of radiative laser cooling against the combined rate of intrabeam scattering and quantum excitation.

Two configurations will be discussed to demonstrate some LESR design considerations for various applications. One is a transient mode device that is capable of producing very low emittance electron beams. The other is a steady-state operation for the generation of intense soft x-rays. Table 1 lists some typical parameters. In both configurations we assume that the two half circles of storage rings consist of identical FODO cells and that the Fabry-Perot resonators are made of mirrors with total reflectivity $R = 99.99\%$. To simplify the intrabeam scattering calculations, we consider a round beam for both cases.

In the transient mode, a 200-GW peak power, 10-ps-long laser pulse is built up inside the resonator. Suppose that a 100-MeV electron beam is injected into this ring with both normalized transverse emittances initially at 1×10^{-5} m. The laser pulse scatters off the electron bunch each round trip with little change of intensity because of negligible laser depletion and internal loss. From Table 1, 1×10^{10} electrons are cooled rapidly to the equilibrium-normalized transverse emittances 1×10^{-7} m. At the same time, very bright, energetic x-rays are also produced. After the extraction of the cold electron beam, a new laser pulse can be built up, and a new electron beam can be injected to repeat the process. The normalized transverse emittances achieved in the LESR are much smaller than those of the SLC damping ring, and are also well below present rf gun technology. The relatively large energy spread can also be made much smaller by adiabatic acceleration.

For the steady-state configuration, in order to sustain the energy level of the laser pulse in the Fabry-Perot resonator, a 200-W average power, mode-locked Nd:YAG laser is resonantly coupled to the resonator. From Table 1, when the accumulated laser pulse scatters off an 8-MeV electron bunch in the resonator, the interaction not only gives rise to soft x-rays with wavelength around 1 nm, but also provides a cooling and stabilization mechanism to maintain the intense compact bunch (1×10^{10} electrons) so that all electrons participate in each laser pulse collision. As a result of the radiative laser cooling, an average flux of 8×10^{14} x-ray photons per second is generated. The intensity and the compactness of this x-ray source may be suitable for x-ray lithography.

4 CONCLUSION

In summary, we have found two new approaches to generate very low-emittance electron beams. One is the suppression of quantum excitation through focusing environment. This approach can in principle reach the fundamental beam emittance with probably a rather limited intensity. The other is the radiative laser cooling in a laser-electron storage ring, which provides rapid damping for a dense electron beam. Generation of low-emittance electron beams is an interesting subject in its own right, and the two methods discussed here may have potential applications in linear colliders, novel accelerators, or light sources.

5 ACKNOWLEDGMENTS

We would like to thank Dr. Pisin Chen for his early collaboration and many members of the Accelerator Research Departments at SLAC for useful discussions.

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