Homework Solutions Set#3

Problem 1.

$$\alpha = -4np - m^{-1}$$

$$\gamma^2 = (\alpha + j\beta)^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

$$\left(\alpha^2 - \beta^2\right) + j(2\alpha\beta) = -\omega^2\mu\varepsilon + j\omega\mu\sigma = -\omega^2\mu_0\left(18.5\varepsilon_0\right) + j\omega\mu_0\sigma$$

$$\left(\alpha^2 - \beta^2\right) = -\omega^2\mu_0\left(18.5\varepsilon_0\right)$$

$$\Rightarrow \beta = \sqrt{\alpha^2 + 18.5\omega^2\mu_0\varepsilon_0} \approx 9.86\,rad - m^{-1}$$

$$and\ \ 2\alpha\beta = \omega\mu_0\sigma \Rightarrow \sigma = \frac{2\alpha\beta}{\omega\mu_0} \approx 0.1Sm^{-1}$$

Homework Solutions Set#3

Problem 1. Cont.

$$\eta_c = |\eta_c| e^{j\phi_n} = 74.2 e^{j0.386}$$

$$So \;\; \mathcal{E}_x(z,t) = 2e^{-4z} \cos(\omega t - 9.86z) V - m^{-1}$$

$$\mathcal{H}_y(z,t) = 2.7\times 10^{-2} \cos(\omega t - 9.86z - 0.386) A - m^{-1}$$

$$S_{av} = \hat{z}\left(2.7\times 10^{-2}\right)e^{-bz} \cos(0.386) \approx \hat{z}\left(2.5\times 10^{-2} e^{-8z}\right) W - m^{-2}$$

$$P_{entering} = (\mathit{Area\ of\ xy\ plane})\left|S_{av}(z=0)\right|$$

$$= (ab)\left(2.5\times 10^{-2}\right) = 1.25\times 10^{-2} W$$

$$P_{exiting} = (\mathit{Area\ on\ z = d\ plane})\left|S_{av}(z=d)\right|$$

$$= (ab)\left(2.5\times 10^{-2}\right)\left(e^{-8}\right) = 4.19\times 10^{-6} W$$

Homework Solutions Set#3

Problem 1. Cont.

$$P_d = \bar{\mathcal{J}} \cdot \bar{\mathcal{E}} = \sigma \bar{\mathcal{E}} \cdot \bar{\mathcal{E}} = \sigma |Ec|^2 = 0.4e^{-8z} \cos^2(\omega t - 9.86z)$$

$$= 0.2e^{-8z} + 0.2e^{-8z} \cos[2(\omega t - 9.86z)]$$

$$\Rightarrow [P_d]_{av} = 0.2e^{-8z} W - m^{-3}$$

$$P_{total} = \int_V [P_d]_{av} dv = \int_0^1 (ab) 0.2e^{-8z} dz = \int_0^1 (.5) 0.2e^{-8z} dz \approx 1.25 \times 10^{-2} W$$

Note that $P_{total} = P_{entering} - P_{exiting}$

Homework Solutions Set#3

Problem 2.

Note that since the for normal incidence, **1+Γ=*τ*,** the reflection and transmission coefficients must be Γ**=-0.5** and ***τ***=0.5 respectively. For loss-less nonmagnetic case, the reflection and transmission coefficients can be simplified as

$$\Gamma = \frac{\sqrt{\varepsilon_{1r}} - \sqrt{\varepsilon_{2r}}}{\sqrt{\varepsilon_{1r}} + \sqrt{\varepsilon_{2r}}} = -0.5$$

$$t = \frac{2\sqrt{\varepsilon_{1r}}}{\sqrt{\varepsilon_{1r}} + \sqrt{\varepsilon_{2r}}} = 0.5$$

dividing these two equations with one another yields

$$0.5 - 0.5\frac{\sqrt{\varepsilon_{2r}}}{\sqrt{\varepsilon_{1r}}} = -1 \Rightarrow \frac{\varepsilon_{2r}}{\varepsilon_{1r}} = 9$$

Problem 3.

At 3GHz, the skin depth of copper is $\delta \approx 1.15$ μm which means that we can easily reflect multiple reflections for a copper foil of 10 μm thickness. The reflection coefficient at air-copper interface is given

$$\Gamma = \frac{\eta_c - \eta_\circ}{\eta_c + \eta_\circ} \quad (\eta_c \text{ *is the interinsic impedance of copper*})$$

where

$$\eta_c = \sqrt{\frac{j\omega\mu_\circ}{\sigma}} = \sqrt{\frac{\omega\mu_\circ}{\sigma}}e^{j45^\circ} = \frac{2\pi\sqrt{f}}{\sqrt{5.8}}(1+j)$$

$$\eta_c \approx 2.61\times10^{-7}\sqrt{f}(1+j)\Omega$$

$$\Gamma = \frac{\eta_c - \eta_\circ}{\eta_c + \eta_\circ} = \frac{2.61\times10^{-7}\sqrt{f}(1+j)\Omega - 377\Omega}{2.61\times10^{-7}\sqrt{f}(1+j)\Omega + 377\Omega}$$

Homework Solutions Set#3

Problem 3. Cont.

$$\Gamma = \frac{\eta_c - \eta_\circ}{\eta_c + \eta_\circ} \quad \left(\eta_c \textit{ is the interinsic impedance of copper}\right)$$

where

$$\eta_c = \sqrt{\frac{j\omega\mu_\circ}{\sigma}} = \sqrt{\frac{\omega\mu_\circ}{\sigma}} e^{j45^\circ} = \frac{2\pi\sqrt{f}}{\sqrt{5.8}}(1+j)$$

$$\eta_c \approx 2.61\times10^{-7}\sqrt{f}(1+j)\Omega$$

$$\Gamma = \frac{\eta_c - \eta_\circ}{\eta_c + \eta_\circ} = \frac{2.61\times10^{-7}\sqrt{f}(1+j)\Omega - 377\Omega}{2.61\times10^{-7}\sqrt{f}(1+j)\Omega + 377\Omega}$$

Problem 3. Cont.

At 3GHz

$$\Gamma \cong 0.9993623 e^{j179.997^\circ}$$

$$(S_{av})_r = |\Gamma^2| (S_{av})_i = 998.72 W - m^{-2}$$

$$(S_{av})_t = (1 - |\Gamma^2|)(S_{av})_i = 1.274993 W - m^{-2}$$

for copper with $\sigma = 5.78 \times 10^7 \, S - m^{-1}$, *at* $3GHz$, *we have*

$$\alpha \approx 827593.621 \; np - m^{-1}$$

$$(S_{av})_{emerging} = \left[(S_{av})_t \left(e^{-2\alpha z} \right) \right]_{z = 10\mu m} \approx 82.626 n W - m^{-2}$$

Thus 10um sheet of copper is extremely effective in shielding 3 GHz radiation.

Homework Solutions Set#3

Problem 4.

At the center of L-band (f-1.5 GHz), the min. thickness of fiberglass needed that causes no reflection is

$$d_{min} = \frac{\lambda_2}{2} = \frac{\lambda_{air}}{2\sqrt{\varepsilon_r}} = \frac{c/f}{2\sqrt{\varepsilon_r}} \approx \frac{3\times 10^{10}/1.5\times 10^{9}}{2\sqrt{4.6}}$$

$$d_{min} \approx 4.66cm$$

At $1GHz$,

$$\lambda_2 = \frac{\lambda_{air}}{\sqrt{\varepsilon_r}} \approx 1.0cm$$

Phase constant β *follows as*

$$\beta_2 = \frac{2\pi}{\lambda_2} \approx 0.449\,rad - cm^{-1}$$

Homework Solutions Set#3

Problem 4. Cont.

$$\eta_{23} = \eta_2 \frac{\eta_3 + j\eta_2 \tan(\beta_2 d_{min})}{\eta_2 + j\eta_3 \tan(\beta_2 d_{min})}$$

substituting for $\eta_2 = \dfrac{377\Omega}{\sqrt{4.6}}$, *and* $\eta_3 = 377\Omega$

$$\eta_{23} \approx 126 e^{j36^\circ}\,\Omega$$

The effective reflection coefficient is

$$\Gamma_{eff} = \frac{\eta_{23} - \eta_1}{\eta_{23} + \eta_1} \approx 0.6 e^{j156^\circ}$$

Problem 4. Cont.

Hence the % of the time - average incident power that transmits to through the radome at $1GHz$ can be calculated as

$$\frac{\left|S_{av}\right|_t}{\left|S_{av}\right|_i}\times 100=\left(1-\left|\Gamma_{eff}\right|^2\right)\times 100=64\%$$

Repeating the same at $2GHz$

$$\eta_{23}\approx 126e^{-j36.0}\Omega$$

$$\Gamma_{eff}\approx 0.6e^{-j156^\circ}\Rightarrow\left(1-\left|\Gamma_{eff}\right|^2\right)\times 100=64\%$$