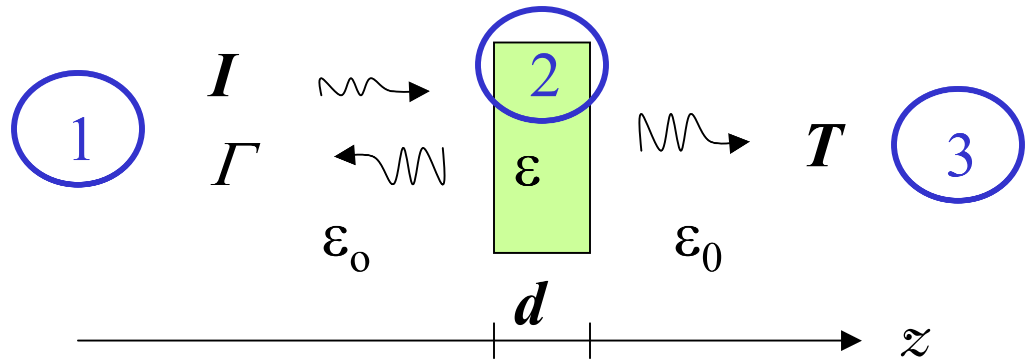
Solutions to Homework Problems Set 2

Problem 1.

$$d = \lambda_0 / 4\sqrt{\varepsilon_r}$$



We start by writing general plane wave fields in each region:

$$\bar{E}^i = \hat{x} e^{-jk_0 z} \qquad \bar{H}^i = \frac{\hat{y}}{\eta_0} e^{-jk_0 z} \quad \text{for } z < 0$$

$$\bar{E}^r = \hat{x}\Gamma e^{jk_0 z} \qquad \bar{H}^r = \frac{-\hat{y}}{\eta_0}\Gamma e^{jk_0 z} \quad \text{for } z < 0$$

$$\bar{E}^s = \hat{x}\left(Ae^{-jkz} + Be^{jkz}\right) \qquad \bar{H}^s = \frac{\hat{y}}{\eta}\left(Ae^{-jkz} - Be^{jkz}\right) \quad \text{for } 0 < z < d$$

$$\bar{E}^t = \hat{x} T e^{-jk_0 (z-d)} \qquad \bar{H}^t = \frac{\hat{y}}{\eta_0} T e^{-jk_0 (z-d)} \qquad z > d$$

Solutions to Homework Problems Set 2

Problem 1. Cont.

Match E_x and H_y at $z=0$ and $z=d$

$$z=0 \quad 1+\Gamma = A+B \qquad \frac{1}{\eta_0}(1-\Gamma)=\frac{1}{\eta}(A-B)$$

$$z=d \quad j(-A+B)=T \qquad \frac{j}{\eta}(-A-B)=\frac{T}{\eta_0} \quad \left(\text{since } d=\lambda_0/4\sqrt{\varepsilon_r}\right)$$

Solving for Γ:

$$\Gamma = \frac{\eta^2 - \eta_0^2}{\eta^2 + \eta_0^2}$$

Solutions to Homework Problems Set 2

Problem 2. (a) Since Al_2o_2 is a low-loss material,i.e., $tan\delta_c = \frac{\varepsilon''}{\varepsilon'} = 2\times10^{-4} << 1$

Using ε_r=9.7 and μ_r=1, the attenuation constant α at 10 GHz can be computed as

$$\alpha \approx \frac{\sigma_{eff}}{2}\sqrt{\frac{\mu_0}{\varepsilon'}} = \frac{\omega\varepsilon'\varepsilon_0(tan\,\delta_c)}{2}\sqrt{\frac{\mu_0}{\varepsilon'_r\varepsilon_0}}$$

$$\approx \frac{2\pi\left(10^{10}\right)9.7\left(8.85\times10^{-12}\right)\left(2\times10^{-4}\right)}{2}\frac{377}{\sqrt{9.7}}$$

$$\approx 0.0653\,np-m^{-1}$$

Solutions to Homework Problems Set 2

Problem 2. (b) The penetration depth *d* can be found from α as

Using ε_r=9.7 and μ_r=1, the attenuation constant α at 10 GHz can be computed as

$$d = \alpha^{-1} \approx 15.3m$$

(c) For the thickness of 1 cm, the total dB attenuation is found to be

$$\sim 20 \log e^{-(0.0653)(0.01)} \approx -0.00567 dB$$

(c) And for the thickness of 1 m, the total dB attenuation is found to be

$$\sim 20 \log e^{-(0.0653)(1)} \approx -0.567 dB$$

Solutions to Homework Problems Set 2

Problem 3. (a) Using σ=0, ε= ε'- jε'', and μ= μ'- jμ'', we rewrite the wave equation (see class notes)

$$\nabla^2 E - j\omega(\mu' - j\mu'')j\omega(\varepsilon' - j\varepsilon'')E = 0$$

After some algebraic manipulations, this equation can be simplifies as

$$\nabla^2 E - j\omega^2[\mu'\varepsilon'' + \mu''\varepsilon' + j(\mu'\varepsilon' - \mu''\varepsilon'')]E$$
$$= \nabla^2 E - j\omega^2\mu_0\varepsilon_0[\delta''_r + j\delta'_r]E = 0$$

Where $\delta''_r = \mu'_r\varepsilon''_r + \mu''_r\varepsilon'_r$ and $\delta'_r = \mu'\varepsilon' - \mu''\varepsilon''$ respective ly.

Comparing this equation with $\nabla^2 E - \gamma^2 E = 0$ in the case of a uniform medium characterized by real constants σ, ε and μ where the loss tangent was defined as $tan\,\delta_c = \dfrac{\sigma}{\omega\varepsilon}$

Solutions to Homework Problems Set 2

Problem 3.

We conclude that the corresponding loss tangent in this case is given by

$$tan\,\delta_c = \frac{\delta''_r}{\delta'_r}$$

(b) The attenuation constant α is defined as the real part of the propagation constant γ. The propagation constant We conclude that the corresponding loss tangent in this case is given by γ is given by

$$\gamma = \alpha + j\beta = \sqrt{j\omega(\mu' - j\mu'')j\omega(\varepsilon' - j\varepsilon'')}$$

Calculate $\alpha^2+\beta^2$ and $\alpha^2-\beta^2$ and add.

Solutions to Homework Problems Set 2

Problem 3.

$$\gamma = \alpha + j\beta = \sqrt{j\omega(\mu' - j\mu'')j\omega(\varepsilon' - j\varepsilon'')}$$

$$\gamma^2 = (\alpha + j\beta)^2 = \left[\sqrt{j\omega(\mu' - j\mu'')j\omega(\varepsilon' - j\varepsilon'')}\right]^2 = j\omega(\mu' - j\mu'')j\omega(\varepsilon' - j\varepsilon'')$$

$$\alpha^2 + \beta^2 + j2\alpha\beta = \omega^2(\mu''\varepsilon'' - \mu'\varepsilon') + j\omega(\mu''\varepsilon' + \mu'\varepsilon'')$$

$$\Rightarrow \alpha^2 + \beta^2 = \omega^2(\mu''\varepsilon'' - \mu'\varepsilon')$$

$$and \;\; 2\alpha\beta = \omega(\mu''\varepsilon' + \mu'\varepsilon'')$$

Do the same for $\alpha^2 - \beta^2$ After some algebra!

$$\Rightarrow \alpha = \frac{\pi f\sqrt{2\delta'_r}}{c}\left[\sqrt{1 + tan^2\,\delta_c} - 1\right]^{1/2}$$

Solutions to Homework Problems Set 2

Problem 4.

$$\overline{D}_1 = \varepsilon_1 \overline{E}_1 = 2\varepsilon_0 \left(2\overline{a}_x - 3\overline{a}_y + 5\overline{a}_z\right)$$

$$= \left(4\overline{a}_x - 6\overline{a}_y + 10\overline{a}_z\right)\varepsilon_0$$

$$\overline{E}_{1t} = \overline{E}_{2t}$$

$$\overline{E}_{2x} = 2, \quad \overline{E}_{2y} = -3$$

$$\overline{D}_{2x} = \varepsilon_2 \ \overline{E}_{2x} = 10\varepsilon_0$$

$$\overline{D}_{2y} = \varepsilon_2 \ \overline{E}_{2y} = -15\varepsilon_0$$

$$\overline{D}_{1n} = \overline{D}_{2n}$$

$$\overline{D}_{2z} = \overline{D}_{1z} = 10\varepsilon_0$$

$$\overline{E}_{2z} = \frac{\overline{D}_{2z}}{\varepsilon_2} = \frac{10\varepsilon_0}{5\varepsilon_0} = 2$$

$$\overline{E}_2 = 2\overline{a}_x - 3\overline{a}_y + 2\overline{a}_z$$

$$\overline{D}_2 = \left(10\overline{a}_x - 15\overline{a}_y + 10\overline{a}_z\right)\varepsilon_0$$