RF Systems for the 3rd Generation Synchrotron Radiation Facilities

Lecture 11

Linac & Accumulation Ring

February 4, 2003

Topics:

- Structure Electrical Properties
- Longitudinal Dynamics
- Beam Loading
- Wakefield
- **P**ositron (**E**lectron) **A**ccumulation **R**ing (PEAR)

Structure Electrical Properties:

Shunt impedance per unit length r: is a measure of the accelerating quality of a structure

$$r = -\frac{E_a^2}{dp/dz}$$

Unit of MΩ/m or Ω/m

Where E_a is the synchronous accelerating field amplitude and dP/dz is the RF power dissipated per unit length.

$$R = \frac{V^2}{P_d}$$

Unit of MΩ or Ω

Structure Electrical Properties:

Factor of merit Q, which measures the quality of an EF structure as a resonator.

For a traveling wave structure $Q = -\frac{\omega W}{dP/dz}$ where W is the rf energy stored per unit length and ω is the angular frequency and dP/dz is the power dissipated per unit length.

For standing wave structure,

$$Q = \frac{\omega W}{P_d}$$

Structure Electrical Properties:

Group velocity V_g which is the speed of RF energy flow along the accelerator is given by

$$V_g = \frac{P}{W} = \frac{-\omega P}{Q \, {dP}/{dz}} = \frac{d\omega}{d\beta}$$

Attenuation factor τ of a constant-impedance or constant-gradient is

$$\frac{dE}{dz} = -\alpha E \qquad \frac{dP}{dz} = -2\alpha P$$

α Is the attenuation constant in nepers per unit length.

Structure Electrical Properties:

Attenuation factor τ for a traveling wave section is defined as

$$\frac{P_{out}}{P_{in}} = e^{-2\tau}$$

For a constant-impedance section, the attenuation is uniform,

$$\alpha = \frac{-\,{}^{dP}\!/_{dz}}{2P} = \frac{\omega}{2v_g Q} \qquad \tau = \alpha L = \frac{\omega L}{2v_g Q}$$

Structure Electrical Properties:

For non-uniform structures,

$$\tau = \int_0^L \alpha(z)\,dz$$

For a constant-gradient section, the attenuation constant α is a function of z: $\alpha = \alpha(z) = \omega / 2V_g(z)Q$

We have the following expression

$$\frac{dP}{dz} = -2\alpha(z)P = const = \frac{P_{in}\left(1 - e^{-2\tau}\right)}{L}$$

Structure Electrical Properties:

r/Q ratio is a measure of accelerating field for a certain stored energy. It only depends on the geometry and independent of material and machining quality.

$$\frac{r}{Q} = \frac{E^2}{\omega W}$$

Filling time t_F- For a traveling wave structure, the field builds up "in space". The filling time is the time which is needed to fill the whole section of either constant impedance or constant gradient. It is given by:

$$t_F = \int_0^L \frac{dz}{V_g} = \frac{Q}{\omega} \int_0^L \frac{-dp/dz}{P} dz = \frac{2Q}{\omega} \tau$$

Structure Electrical Properties:

The field in SW structures builds uo "in time". The filling time is the the time needed to build up the field to (1-1/e)=0.632 times the steady-state field:

$$t_F = \frac{2Q_L}{\omega} = \frac{2Q_0}{(1+\beta_c)\omega} \quad \beta_c = \frac{Q_0}{Q_{ext}}$$

Q_0 is the unloaded Q value

$$Q_0 = \frac{\omega W}{P_d}$$

Q_{ext} is the external Q value

$$Q_{ext} = \frac{\omega W}{P_{ext}} = \frac{Q_0}{\beta}$$

Q_L is the loaded Q value

$$Q_L = \frac{\omega W}{(P_d + P_{ext})} = \frac{Q_0}{(1+\beta)}$$

Structure Electrical Properties:

Traveling Wave Structures

Energy W stored in the entire section at the end of filling time is

$$CI : W = \frac{Q}{\omega}\int_0^L \frac{dP}{dz}dz = P_{in}\frac{Q}{\omega}\left(1-e^{-2\tau}\right)$$

$$CG : W = \int_0^L \frac{P}{v_g}dz = P_{in}\frac{Q}{\omega}\left(1-e^{-2\tau}\right)$$

Structure Electrical Properties:

Standing Wave Structures

Due to multi reflections, the equivalent input power is increased:

$$P = P_s + P_s e^{-2\alpha L} + P_s e^{-4\alpha L} + \ldots = \frac{P_s}{1 - e^{-4\alpha L}}$$

The slightly higher energy gain for SW is paid by field building up time (choice of length of SW structures).

For resonant cavity, power feed is related to the RF coupling:

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} \quad \beta_c = \frac{Q_0}{Q_{ext}}$$

Structure Electrical Properties:

Standing Wave Structures

As defined before, Q_L, $Q_0=\omega W/Pd$, $Q_{ext}= \omega W/P_{ext}$ are the loaded Q, cavity Q, and external Q values. β_c is the coupling coefficient between the waveguide and the structure.

The energy gain of a charged particle is given:

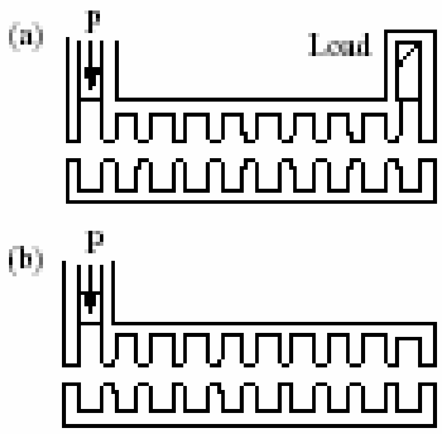
$$V = \left(1-e^{-t/t_F}\right)\frac{2\sqrt{\beta_c}}{1+\beta_c}\sqrt{P_{in} rL} = \left(1-e^{-t/t_F}\right)\sqrt{P_{disp} rL}$$

Structure Electrical Properties:

The choice of the operating frequency is of fundamental importance since almost all the basic RF parameters are frequency dependent.

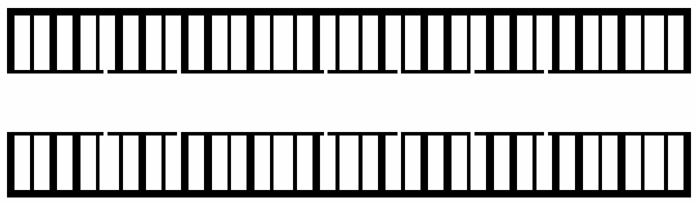
$$r \propto \sqrt{f} \quad size \propto \frac{1}{f} \quad Q \propto 1/\sqrt{f} \quad r/Q \propto f$$

Structure Types

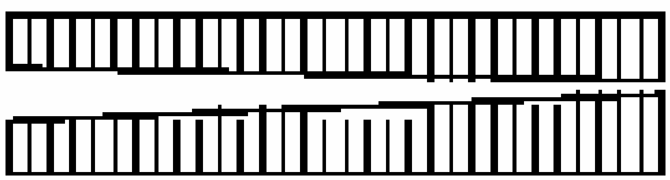


(a)Traveling Wave Structure (TW)

(b) Standing Wave Structure (SW)



Constant Impedance Structure (CI)



Constant Gradient Structure (CG)

Non-Resonance Perturbation

Reflected wave amplitude is

$$E_r(z) = K\frac{E^2(x,y,z)}{P(z)}E_i(z)$$

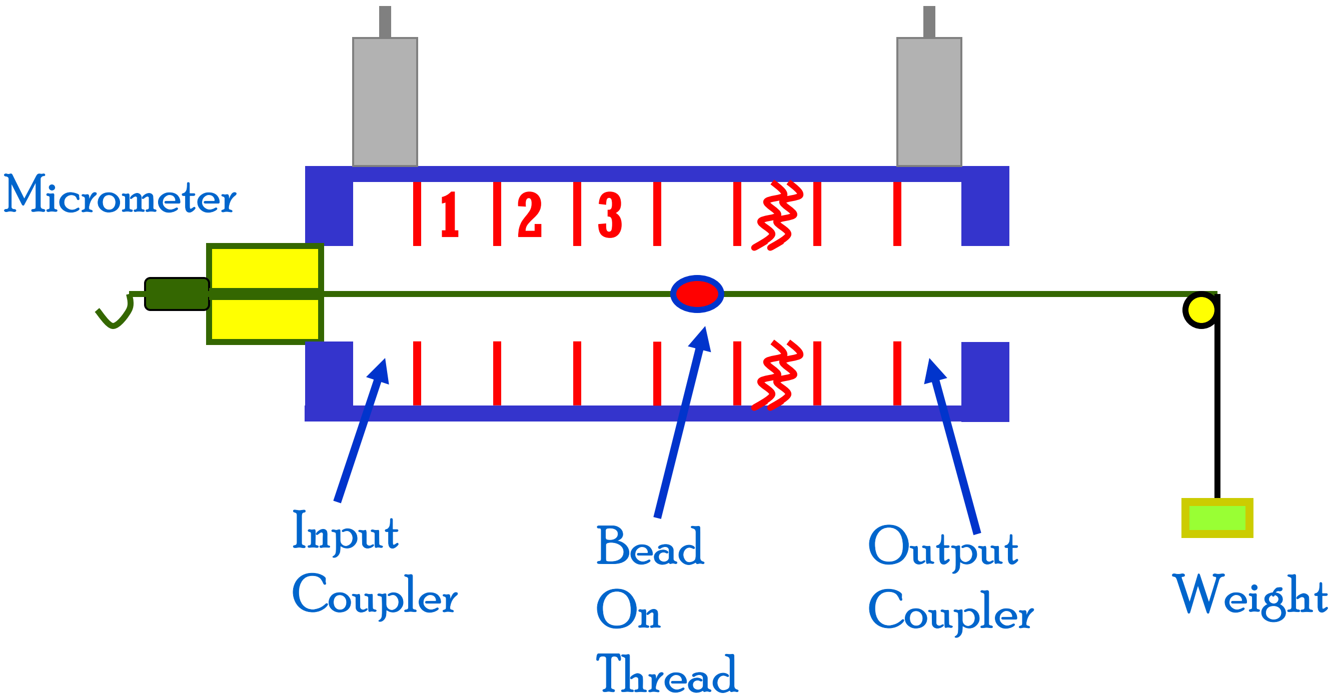
Where K is a constant which depends on the bead, E(x,y,z) is the forward power flowing across the structure at z, E_i is the incident wave amplitude.

The reflection coefficient is defined as:

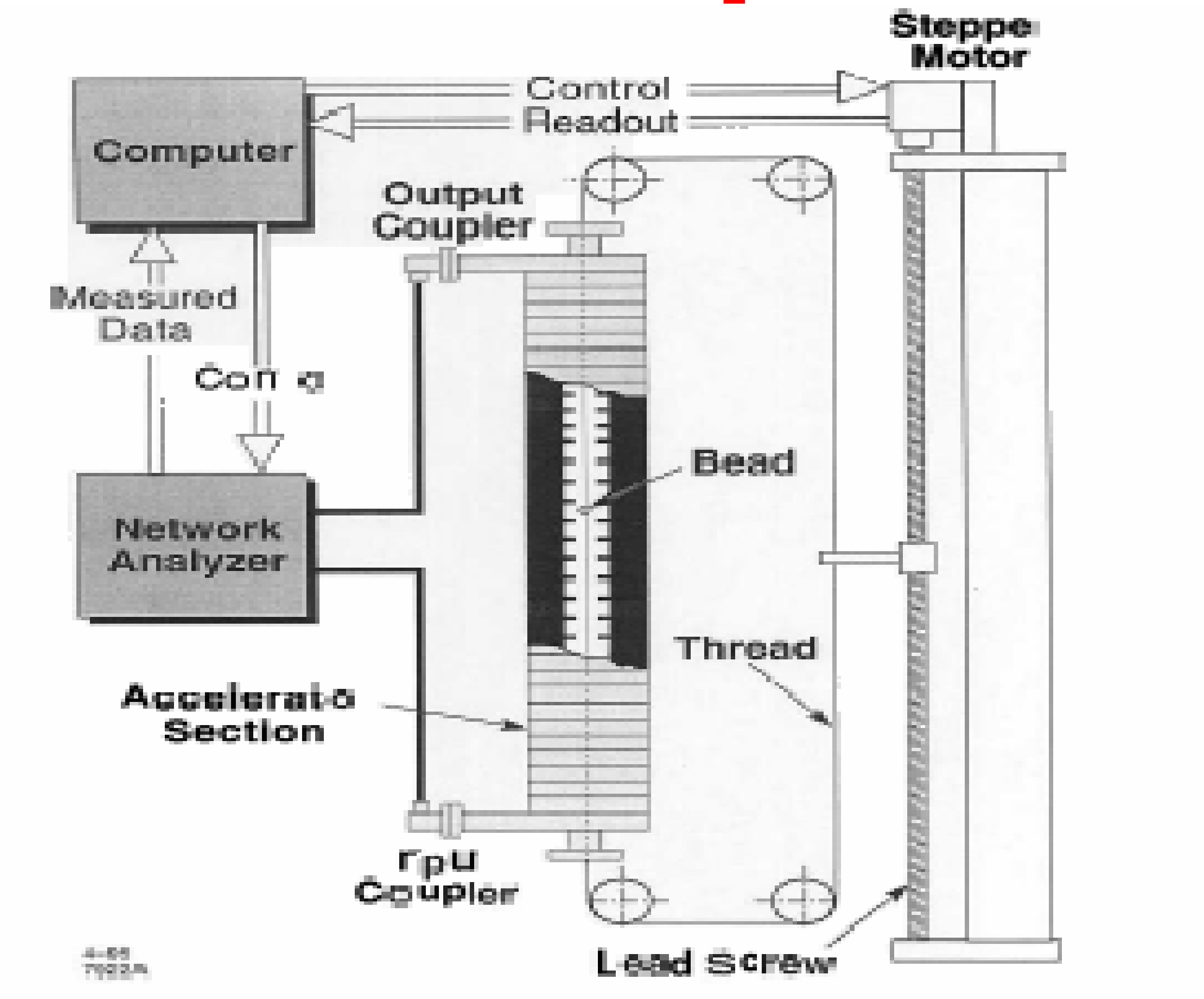
$$\rho(z) = \frac{E_r(z)}{E_i(z)}$$

For a constant gradient structure: $\rho(0) = \frac{E_r(0)}{E_i(0)} = K\frac{E^2(x,y,z)}{P(0)}$

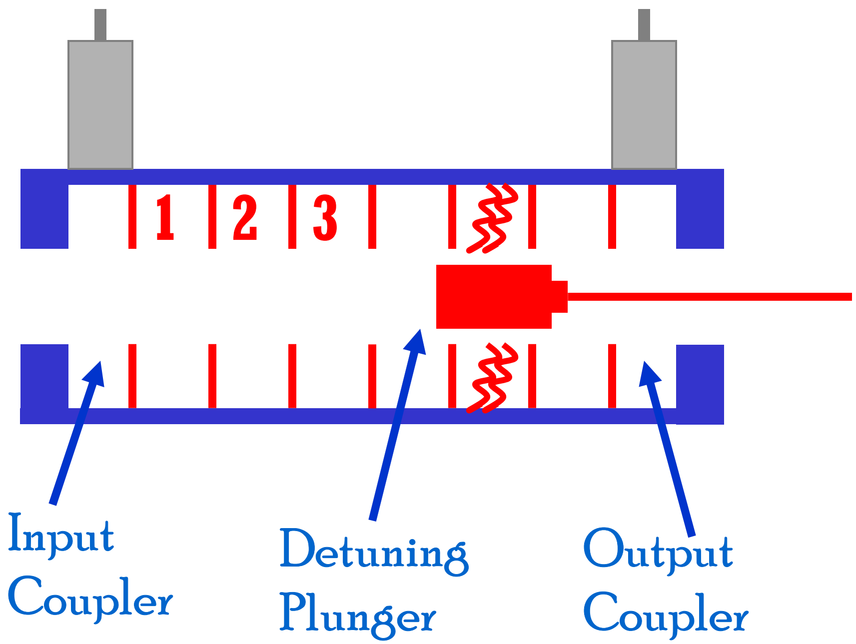
Non-Resonance Perturbation



Bead Pull Setup



Nodal Shift Technique



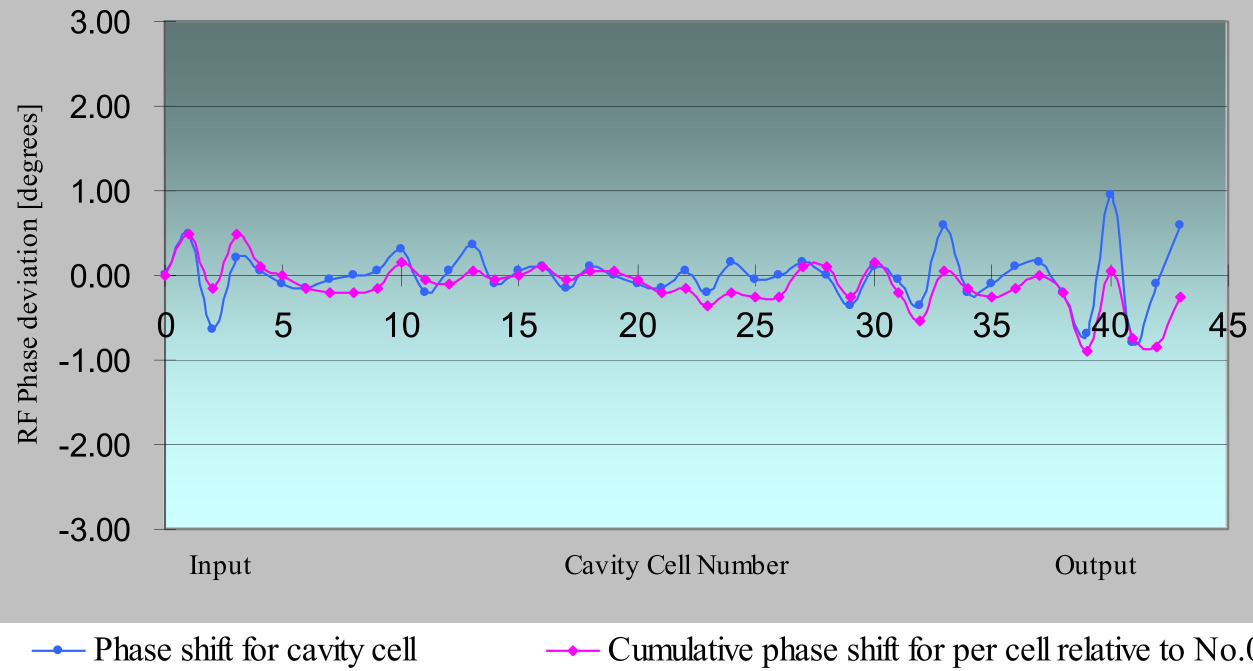
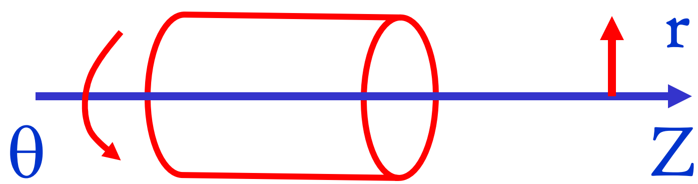


Fig 3. ANL Accelerating Structure Phase Shift Measurement

"Coupled Cavity" Structures

Linacs are coupled cavities in which many connected cavities are powered by one RF source.

Single cavity modes: ω_{mnp}:



TM_{mnp} (no H_z)

TE_{mnp} (no E_z)

M = # of full period azimuthal variations 0,1,2,...

N = # of half period radial variations 1,2,...

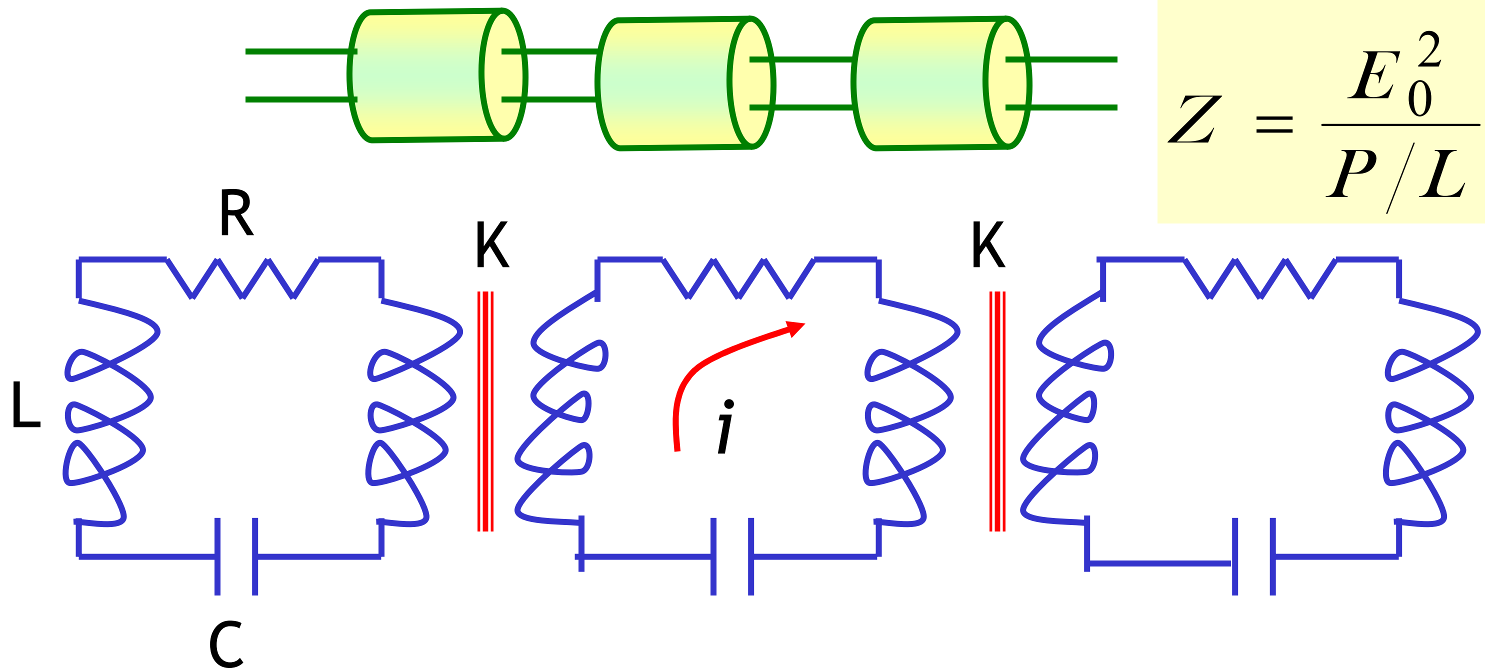
P = # of full period axial variations 0,1,2,...

"Coupled Cavity" Structures

We can describe the properties of coupled chain using separate modes of single cavity as they develop into "bands" of coupled systems.

$$Q = \frac{\omega U}{P} = \frac{\omega}{\Delta\omega}$$

$$Z = \frac{E_0^2}{P/L}$$



"Coupled Cavity" Structures

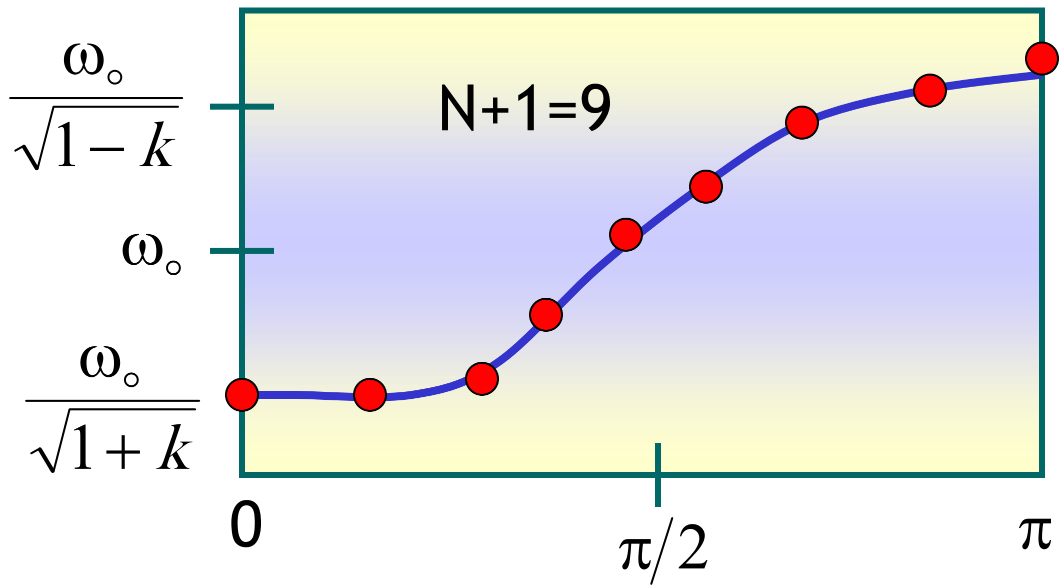
For a chain of N+1 cavities, each single cavity mode yields N+1 normal modes ($Q \rightarrow \infty$):

$$A_n^{(q)} = A_0 \cos\left(\frac{\pi q^n}{N}\right) e^{i\omega_q t}$$

Mode: q= 0,1,...N

$$\omega_q^2 = \frac{\omega_0^2}{1 + k \cos \frac{\pi q}{N}}$$

Cell: n= 0,1,...N



Mode spacing:

k/N^2 near $0, \pi$

k/N near $\pi/2$

Longitudinal Dynamics:

Energy of particle:

$$u = \frac{m_\circ c^2}{\sqrt{1-\beta_e^2}}$$

Energy change with time:

$$\frac{du}{dt} = -eE_z \frac{dz}{dt} \sin\theta$$

$$\theta = \frac{\omega z}{v_p} - \omega t$$

Where t is the time it takes particle to reach z

$$\theta = \omega \int \left(\frac{1}{v_p} - \frac{1}{v_e} \right) dz$$

Longitudinal Dynamics:

$$d\theta = \omega\left(\frac{1}{v_p} - \frac{1}{v_e}\right)dz$$

When a particle is faster ($v_e > v_p$), $d\theta > 0$ and vise versa.

It is convenient to use z as a variable and $\frac{d}{dt} = \frac{dz}{dt}\frac{d}{dz}$

The longitudinal motion is described by the following two equations:

$$\frac{du}{dz} = -eE_z \sin\theta$$

$$\frac{d\theta}{dz} = \frac{2\pi}{\lambda}\left(\frac{1}{\beta_p} - \frac{u}{\sqrt{u^2 - u_\circ^2}}\right)$$

Longitudinal Dynamics:

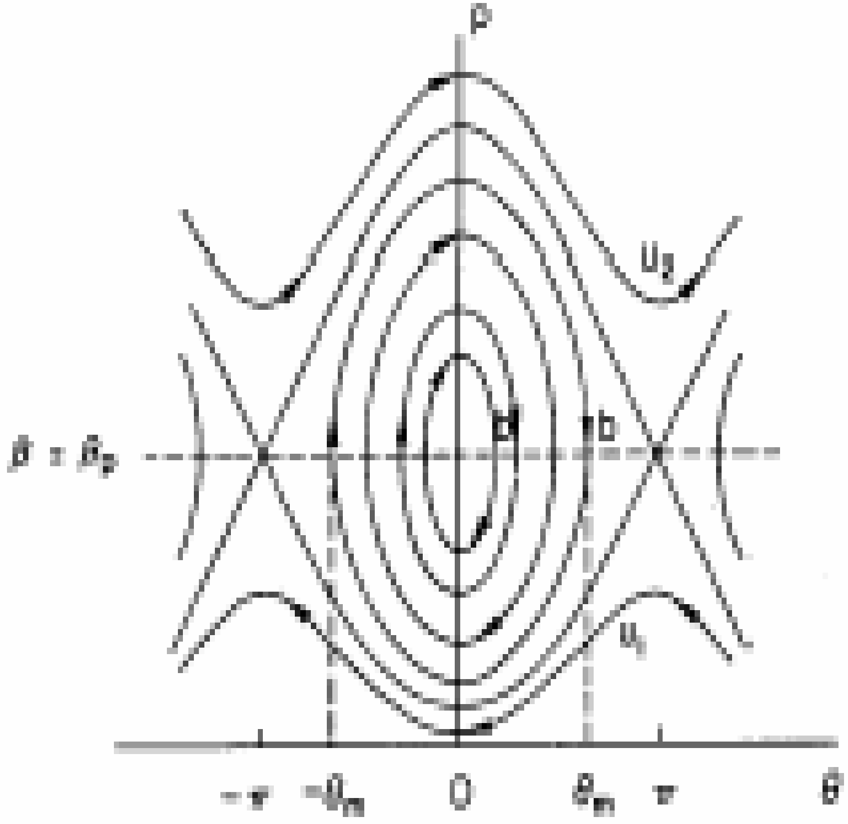
Using $p = \frac{mv_e}{m_\circ c} = \gamma\beta_e = \sqrt{\gamma^2 - 1}$ (normalized momentum)

Integration using variable substitution of $\gamma^2 = p^2 + 1,$

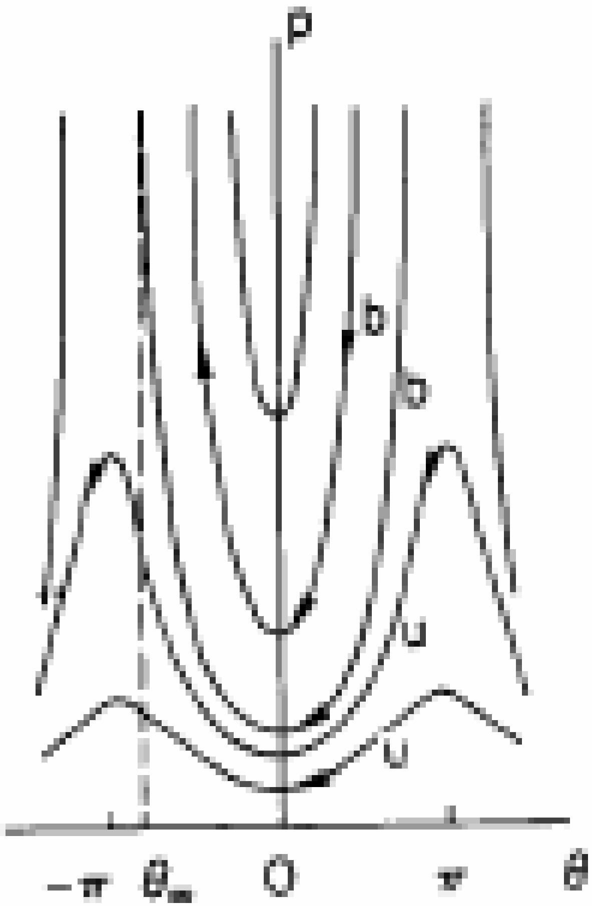
The equation for the orbit in phase space is

$$cos\,\theta - cos\,\theta_m = \frac{2\pi m_\circ c^2}{eE\lambda}\left[\sqrt{p^2+1} - \sqrt{1-\beta_p^2} - \beta_p p\right]$$

Longitudinal Dynamics:



Longitudinal phase space for $\beta_p<1$



Longitudinal phase space for $\beta_p=1$

Longitudinal Dynamics:

Phase velocity less than c (β_p<1)

When 1>cosθ>-1, the particles oscillate in p and θ plane with elliptical orbits centered around ($\beta = \beta_p$, $\theta=0$). If an assembly of particles with a relative large phase extent and small momentum extent enters such a structure, then after traversing ¼ of a phase oscillation it will have a small phase extent and large momentum extent, bunching.

Longitudinal Dynamics:

Phase velocity equals c (β_p=1)

When β_p=1, $d\theta/dz$ is always negative, and the orbits become open-ended. The orbit equation becomes

$$\cos\theta - \cos\theta_m = \frac{2\pi m_\circ c^2}{eE\lambda}\left[\sqrt{p^2+1} - p\right] = \frac{2\pi m_\circ c^2}{eE\lambda}\sqrt{\frac{1-\beta_e}{1+\beta_e}}$$

Where θ_m has been renamed to θ_∞ to emphasize that it corresponds to $p=\infty$. The threshold accelerating gradient for capture is $\cos\theta-\cos\theta_\infty=2$, or

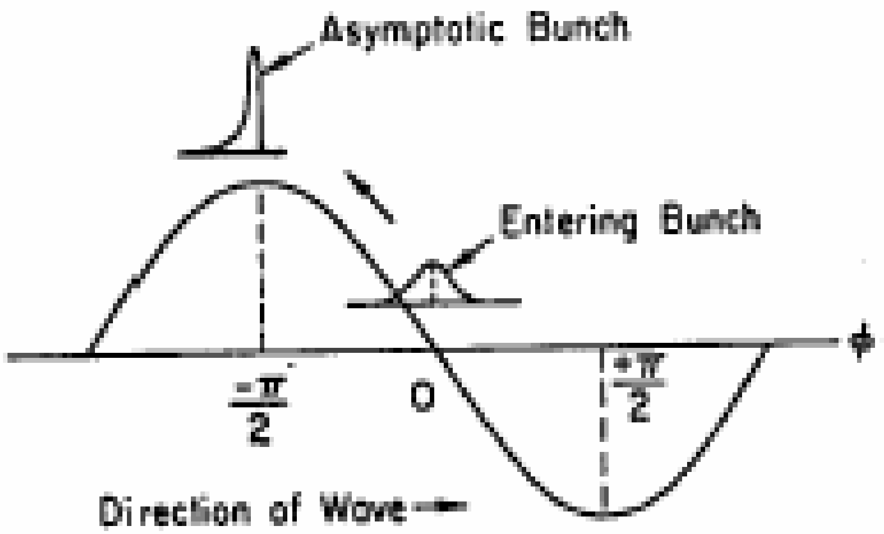
$$E_0(threshold) = \frac{\pi m_0 c^2}{e\lambda}\left[\sqrt{p^2+1} - p_0\right]$$

Longitudinal Dynamics:

Lets consider a particle entering the structure with a phase $\theta_0=0$, has an asymptotic phase $\theta_\infty=-\pi/2$, thus a assembly of particles will get maximum acceleration and minimum phase compression. For small phase extents $\pm\,\Delta\theta_0$ around $\theta_0=0$,

$$\theta_\infty = -\frac{\pi}{2} - \frac{(\Delta\theta_\circ)^2}{2}$$

Example:

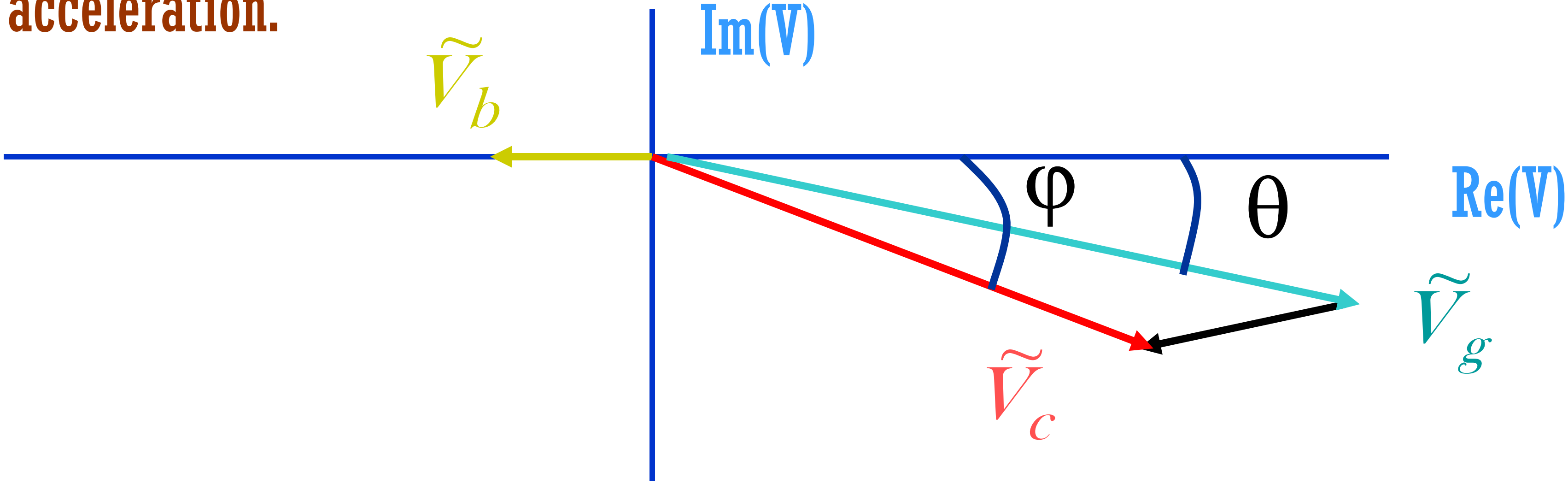


Beam Loading:

- Long train of bunches
- Bunches in from extract energy from linac
  - Lower gradient
  - Increase phase
- Effect on later bunches
  - Bunch placed directly ignoring beam loading
  - Bunch doesn't gain enough energy
  - If it gained enough energy, it would arrive at the same RF phase
  - Non-isochronous arc: bunch arrives in next linac late, sees higher gradient
  - Gaines excess energy

Beam Loading:

The effect of the beam on the accelerating field is called **BEAM LOADING.** The superposition of the accelerating field established by external generator and the beam-induced field needs to be studied carefully in order to obtain the net **P**hase and **A**mplitude of acceleration.



Beam Loading:

In order to obtain a basic physics picture, we will assume that the synchronized bunches in a bunch train stay in the peak of RF field for both TW and SW analysis.

The RF power loss per unit length is given by:

$$\frac{dP}{dz} = \left(\frac{dP}{dz}\right)_{wall} + \left(\frac{dP}{dz}\right)_{beam}$$

$$E^2 = 2\alpha r P$$

$$E\frac{dE}{dz} = rP\frac{d\alpha}{dz} + \alpha r\frac{dP}{dz} = r\frac{E^2}{2\alpha r}\frac{d\alpha}{dz} - \alpha r\left(\frac{E^2}{r} + EI\right)$$

$$\frac{dE}{dz} = -\alpha E\left(1 - \frac{1}{2\alpha^2}\frac{d\alpha}{dz}\right) - \alpha r I$$

Beam Loading:

For constant impedance structure:

$$\frac{dE}{dz} = -\alpha E - \alpha r I$$

$$E(z) = E(0)e^{-\alpha z} - Ir\left(1 - e^{-\alpha z}\right)$$

$$E(0) = \sqrt{2\alpha r P_{in}}$$

The total energy gain through a length L is

$$V = \int_0^L E(z)dz = \sqrt{2rP_0L}\,\frac{1-e^{-\tau}}{\sqrt{\tau}} - IrL\left(1 - \frac{1-e^{-\tau}}{e^{-\tau}}\right)$$

P_0 is input rf power in MW, r is the shunt impedance per unit length in MΩ/m, L is the structure length in meters, I is the average beam current in Ampere, and V is the total energy gain in MV

Beam Loading:

For constant gradient structures: $$\frac{dE}{dz} = -\alpha r I$$

$$E = E_0 + \frac{rI}{2} ln\left(1 - \frac{z}{L}\left(1 - e^{-2z}\right)\right)$$

The attenuation coefficient is

$$\alpha(z) = \frac{\left(1 - e^{-2\tau}\right)/2L}{1 - \left(1 - e^{-2\tau}\right)(z/L)}$$

Beam Loading:

The complete solution including transient can be expressed as

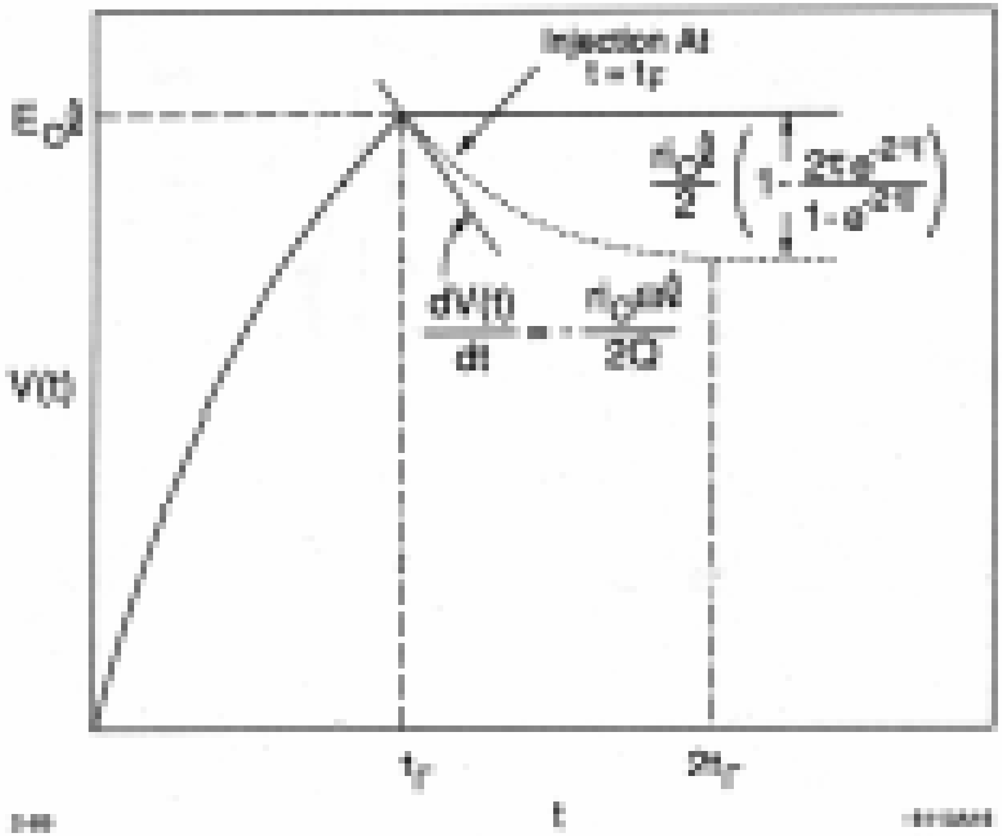
$t_F \le t \le 2t_F$:

$$V(t) = E_0 L + \frac{rI}{2}\left[\frac{\omega L e^{-2\tau}}{Q\left(1-e^{-2\tau}\right)}t - \frac{L}{1-e^{-2\tau}}\left(1-e^{-\frac{\omega}{Q}t}\right)\right]$$

$t \ge 2t_F$:

$$V(t) = E_0 L - \frac{rIL}{2}\left[1-\frac{2\tau e^{-2\tau}}{1-e^{-2\tau}}\right]$$

Beam Loading:



Transient beam loading in a TW constant gradient structure.

Beam Loading:

For a standing wave structure with a coupling coefficient β_c, the energy gain V(t) is

$$V = \left(1 - e^{-t/t_F}\right)\frac{2\sqrt{\beta_c}}{1+\beta_c}\sqrt{P_{in} rL} = \left(1 - e^{-t/t_F}\right)\sqrt{P_{disp} rL}$$

$$V = \left(1 - e^{-t/t_F}\right)\sqrt{P_{disp} rL} - \frac{IrL}{1+\beta}\left(1 - e^{t-t_b/t_F}\right)$$

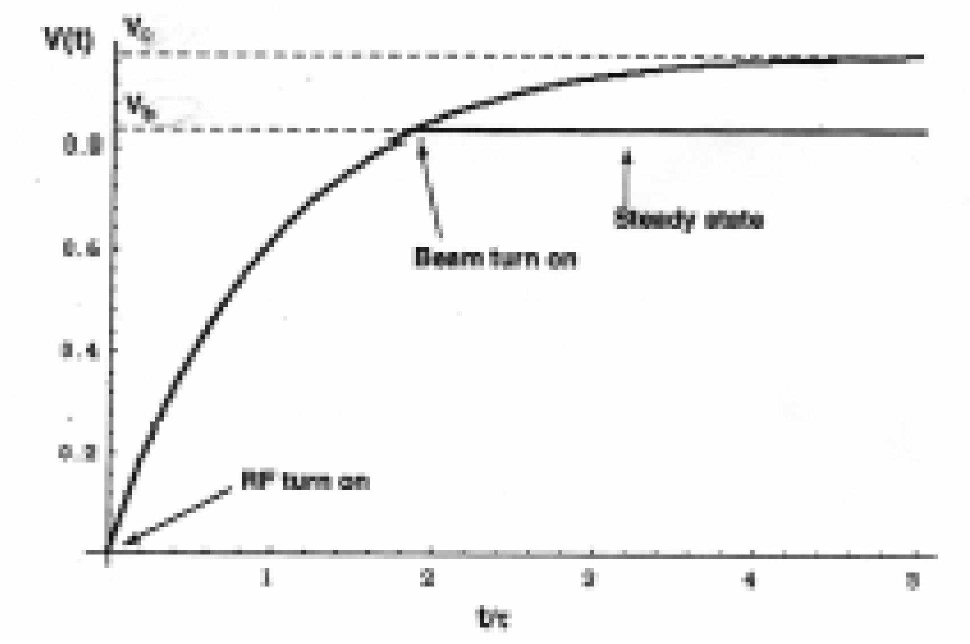
If the beam is injected at time t_b and the coupling coefficient meets the following condition:

$$\beta_c = 1 + \frac{P_0}{P_{disp}} \quad \Longrightarrow \quad P_{in} = P_{disp} + P_b$$

Beam Loading:

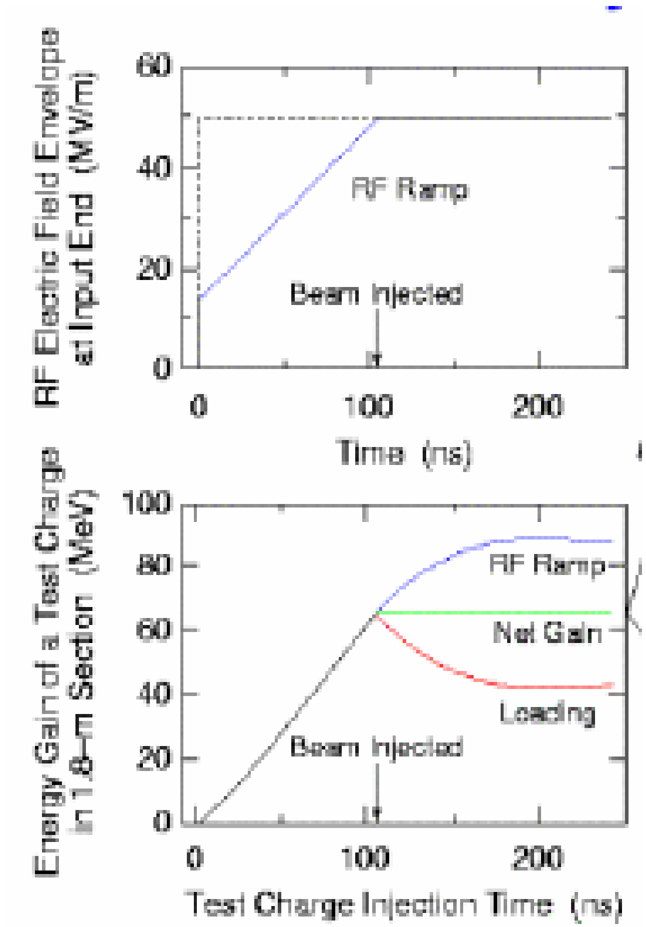
There is no reflection from the structure to power source with beam. The beam injection time can be obtained:

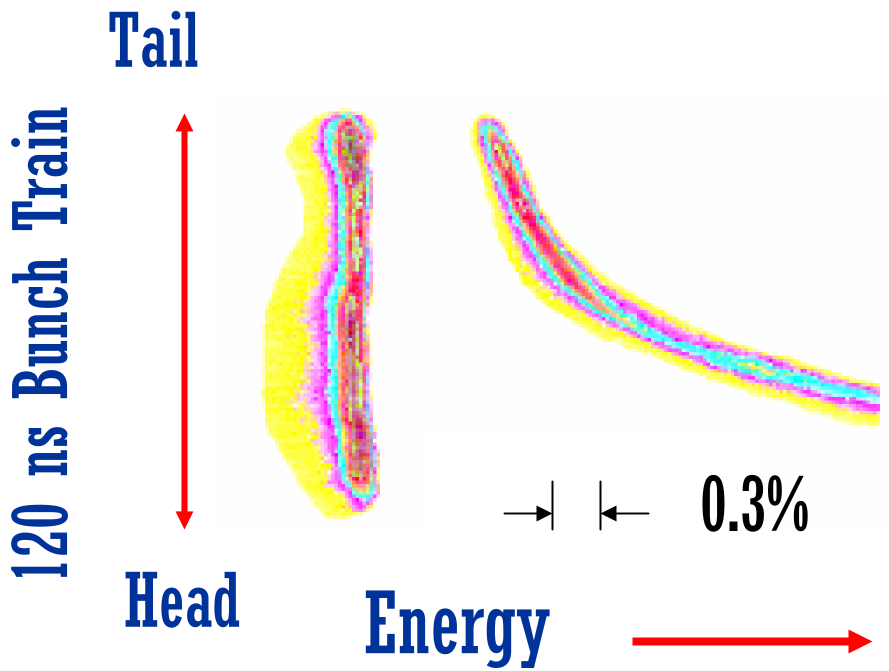
$$t_b = -t_F \, ln\left(1 - \frac{V_b}{V_0}\right) = t_F \, ln \frac{2\beta_c}{\beta_c - 1}$$



Beam Loading Compensation:

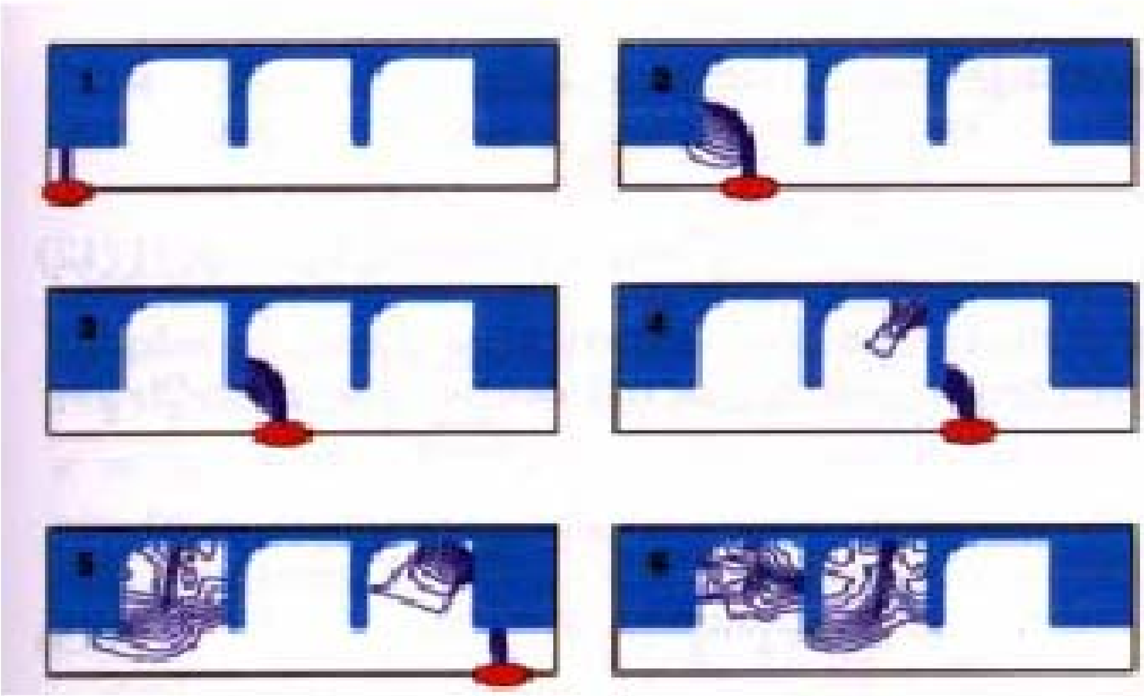
Using RF Amplitude Ramp during fill





Wakefields:

The wakefield is the scattered electromagnetic radiation created by relativistic moving charged particles in RF cavities, vacuum bellows, and othe bem line components.



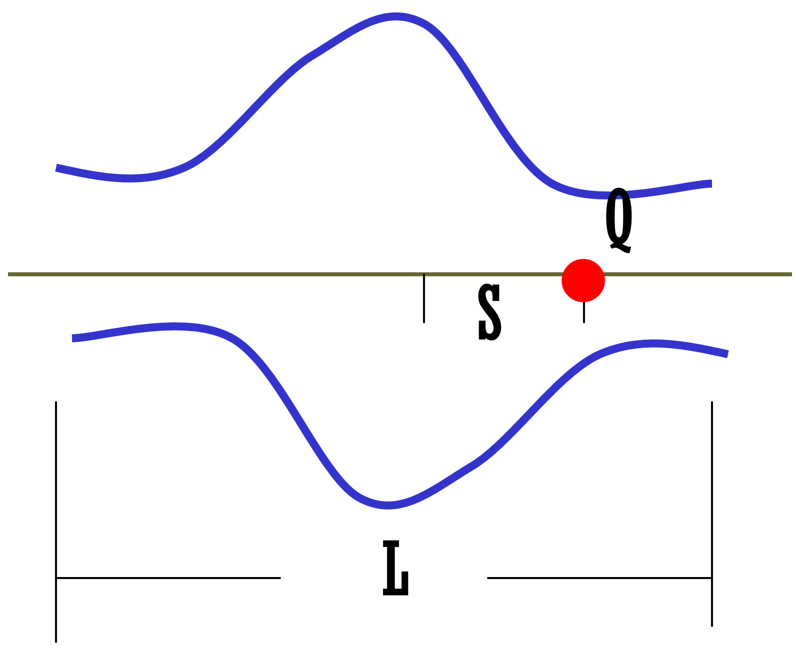
Electric field lines of a bunch traversing through a three-cell disk-loaded structure

Wakefields:

- No disturbance ahead of moving charge – **CAUSALTY**.
- Wakefields behind the moving charge vary in a complex way, in space and time.
- These fields can be decomposed into **MODES**.
- Each mode has its particular **FIELD PATTERN** and will oscillate wit its own frequency.
- For simplified analysis, the modes are orthogonal, i.e., the energy contained in a particular mode does not have energy exchange with the other modes.

Wakefields:

Lets consider a point charge with charge Q moving at the speed of light along a path in z direction through a discontinuity L:



Longitudinal Wakefields:

For practical purposes, all the bunches (driven and test bunch) are near the structure axis.

We define the longitudinal delta-function potential $W_z(s)$ as the potential (Volt/Coulomb) experienced by the test particle following along the same path at time τ (distance $s = \tau c$)behind the unit driving charge.

$$W_z(s) = \frac{1}{Q} \int_0^L dz E_z\left(z, \frac{z+s}{c}\right)$$

Longitudinal Wakefields:

The longitudinal wakefields are dominated by the m=0 modes, for example TM_{01}, TM_{02}, ...

$$W_z(s) = \sum_n k_n \cos\left(\frac{\omega_n s}{c}\right) \times \begin{matrix} 0(s<0) \\ 1(s=0) \\ 2(s>0) \end{matrix}$$

The loss factor k_n is:

$$k_n = \frac{|V_n|^2}{4U_n} = \frac{\omega_n}{4}\left(\frac{R_n}{Q_n}\right)$$

Where U_n is the stored energy in the n^{th} mode. V_n is the maximum voltage gain from the n^{th} mode for a unit test charge particle with speed of light.

Longitudinal Wakefields:

The total amount of energy deposited in all the modes by the driving charge is:

$$U = Q^2 \sum_n k_n$$

Longitudinal wakefields are approximately independent of the transverse position of both the driving charges and the test charges.

Short range longitudinal wakefield – Energy spread within a bunch.

Long range longitudinal wakefield – Beam loading effect.

Transverse Wakefields:

Transverse wakefield potential is defined as the transverse momentum kick experienced by a unit test charge following at a distance s behind the same path with a speed of light.

$$W_\perp = \frac{1}{Q}\int_0^L dz\left[\vec{E}_\perp + \left(\vec{v}\times\vec{B}\right)_\perp\right]_{\frac{z+s}{c}}$$

The transverse wakefields are dominated by the dipole mode (m>1), for example, HEM_{11}, HEH_{21},…

Transverse Wakefields:

An expression of the transverse wakefield is approximately:

$$W_{\perp} \cong \left(\frac{r'}{a}\right)\hat{x}\sum_{n}\frac{2k_{1n}}{\omega_{1n}\,a/c}\,sin\left(\frac{\omega_{1n}s}{c}\right)\quad (s \geq 0)$$

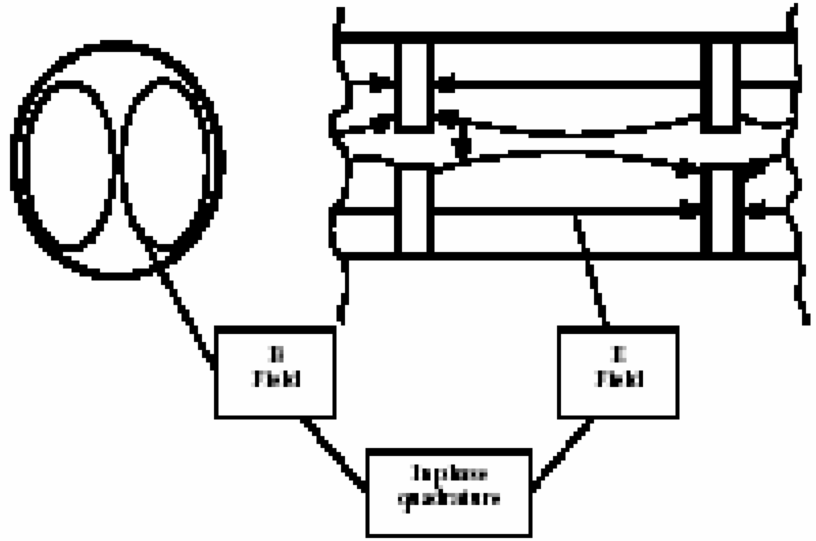
Where r' is the transverse offset of the driving charge, a is the tube radius of the structure, and k_{1n} for m=1 nth dipole mode has a similar definition as m=0 case. The unit of transverse potential is V/Coulomb.mm.

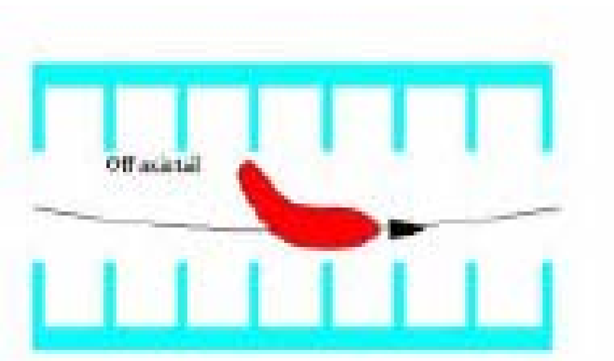
The transverse wakefields depend on the driving charge as the first power of its offset r', the direction of the transverse wake potential vector is decided by the position of the driving charge.

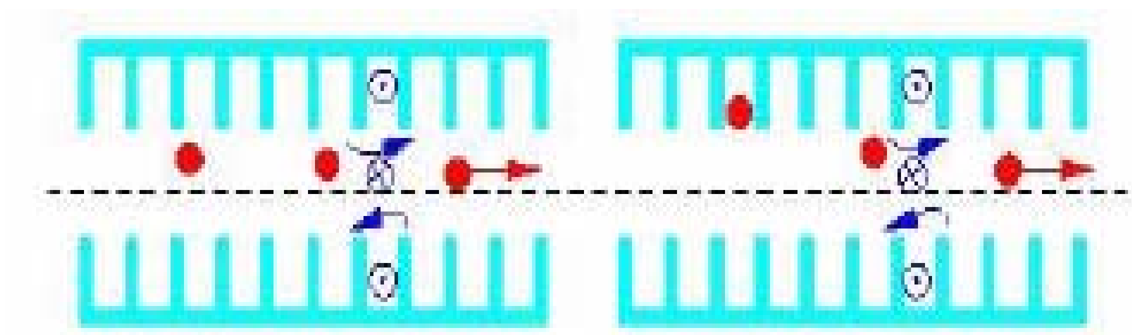
Transverse Wakefields:

Example:

Schematic of field Pattern for the lowest frequency mode – HEM_{11} Mode.







H-T Instability short range   Multi-Bunch Beam Breakup Long range.

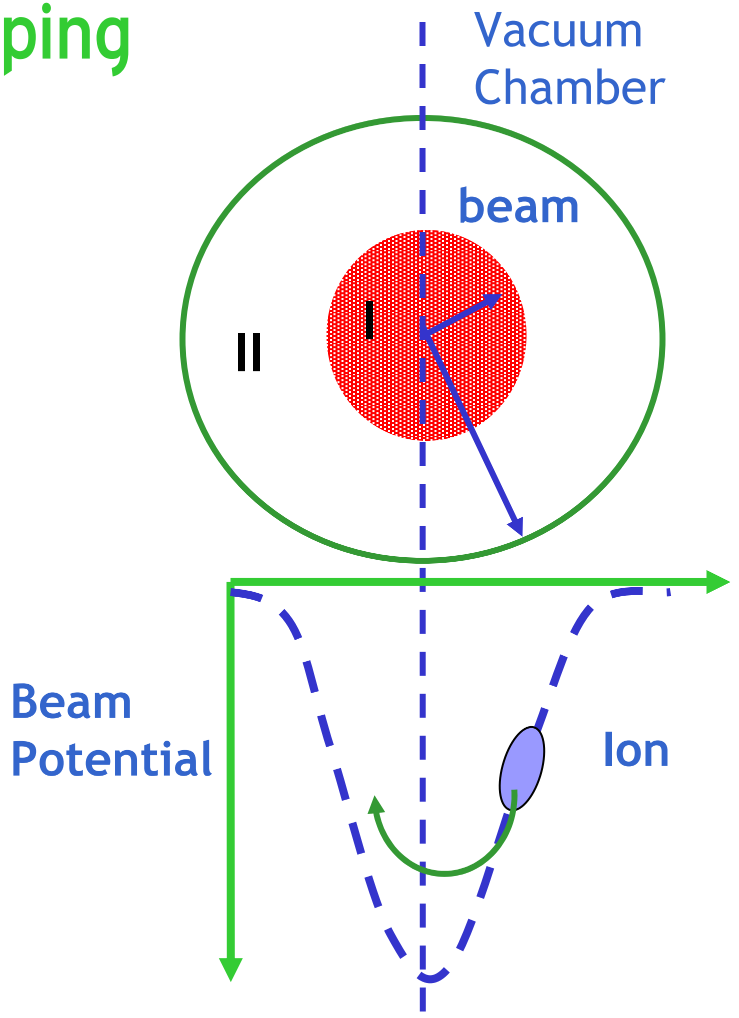
Positron Accumulation Ring

Why positrons?

Ion Trapping

It has long been recognized that ion trapping as a potential limitation in electron storage rings.

The ions, generated by beam-gas collisions, become trapped in the negative potential of the beam and accumulate over multiple beam passages.



Ion Trapping

The Lorentz Force acting on the ion is:

$$\bar{F} = qe\left(\bar{E} + \bar{v} \times \bar{B}\right)$$

With components

$$\begin{cases} F_r = qe\left(E_r - B \cdot \dot{z}\right) \\ F_\theta = 0 \\ F_z = qe\left(B \cdot \dot{r} + E_z\right) \end{cases}$$

Ion Trapping

From Gauss theorem, the electric field can be written as:

In region I
$$E_r(r) = \frac{I}{2\pi\varepsilon_\circ \beta c}\frac{1}{r}$$

In region II
$$E_r(r) = \frac{I}{2\pi\varepsilon_\circ \beta c}\frac{r}{a^2}$$

The magnetic field is simply:

$$B = \frac{\beta E_r}{c}$$

Ion Trapping

The beam potential at the center is

$$U = \frac{1}{2\pi\varepsilon_{\circ}\beta c}\left(ln\frac{r_c}{a} + \frac{1}{2} \right)$$

The forces on the ion reduces to:

$$F_r = A_i m_p \ddot{r} = \frac{qeI}{2\pi\varepsilon_{\circ}\beta c^2}\left(1 - \beta\frac{\dot{z}}{c} \right) r$$

$$F_z = A_i m_p \ddot{z} = \frac{qeI}{2\pi\varepsilon_{\circ}\beta c^2} \dot{r} r + qeE_z$$

Ion Trapping

As the ions can only have a maximum potential energy equal to the beam potential times their charge, i.e. typically up to few hundred eV, they are non-relativistic, so the equations can be decoupled by neglecting $\beta \dot{z}/c << 1$. The ion motion is thus transversally oscillatory with a frequency ("bouncy" frequency) of

$$\omega_i = 2\pi f_i = \left(\frac{I}{2\pi\varepsilon_\circ \beta c} \frac{1}{a^2} \frac{qe}{A_i m_p} \right)^{1/2}$$

$$r(t) = r_m \, cos(\omega_i t + \alpha)$$

Ion Trapping

The trapped ions are observed to cause effects such as:

- Increasing beam phase space
- Broadening and shifting of the beam transverse oscillation frequencies (tunes)
- Collective beam instabilities
- Beam lifetime reductions

Ion Trapping

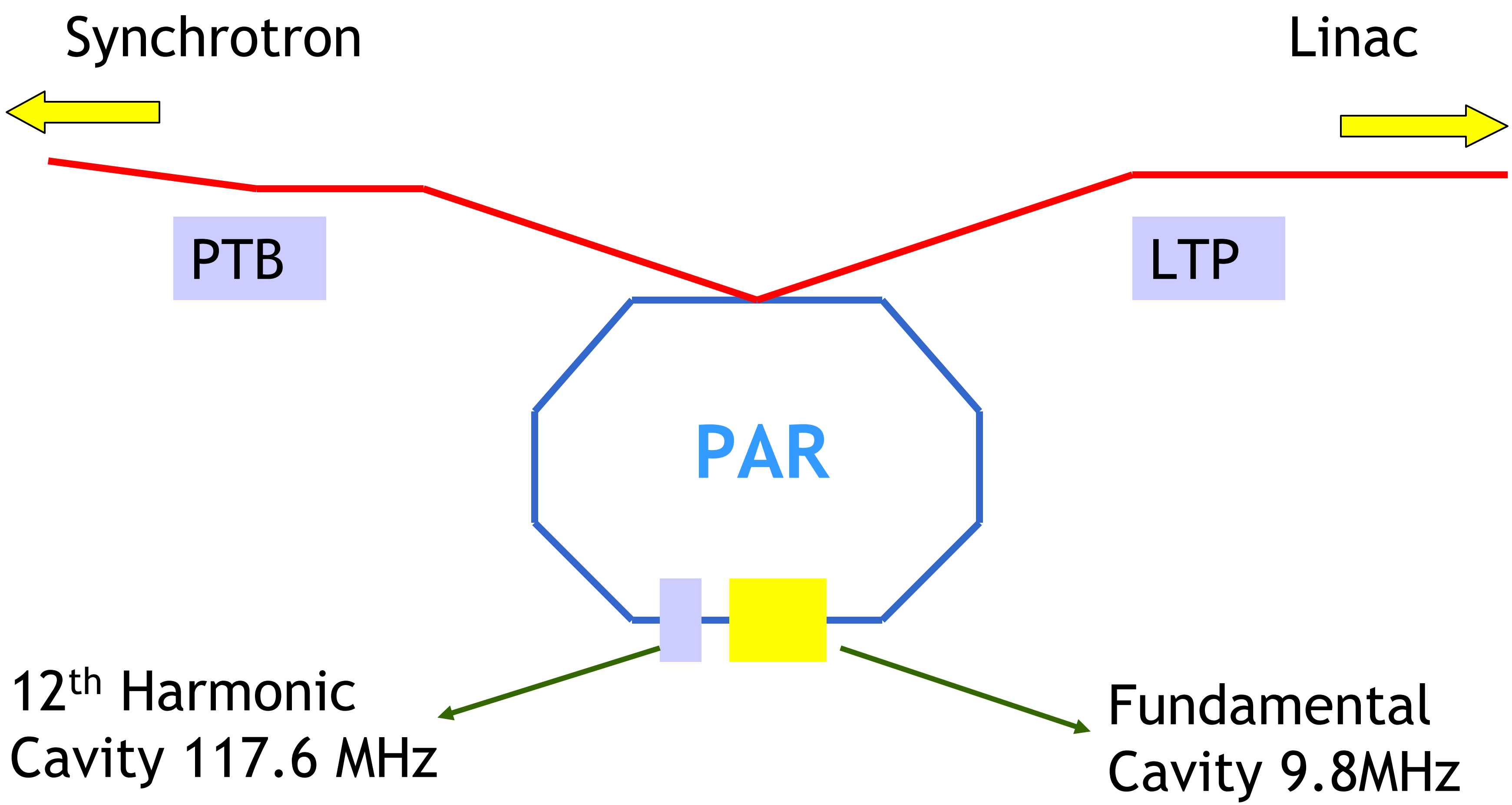
Standard cures:

1. Include a gap in the bunch train (long compared to the bunch spacing)
2. Insert clearing electrodes
3. Switch from electrons to positrons (Design Stage)

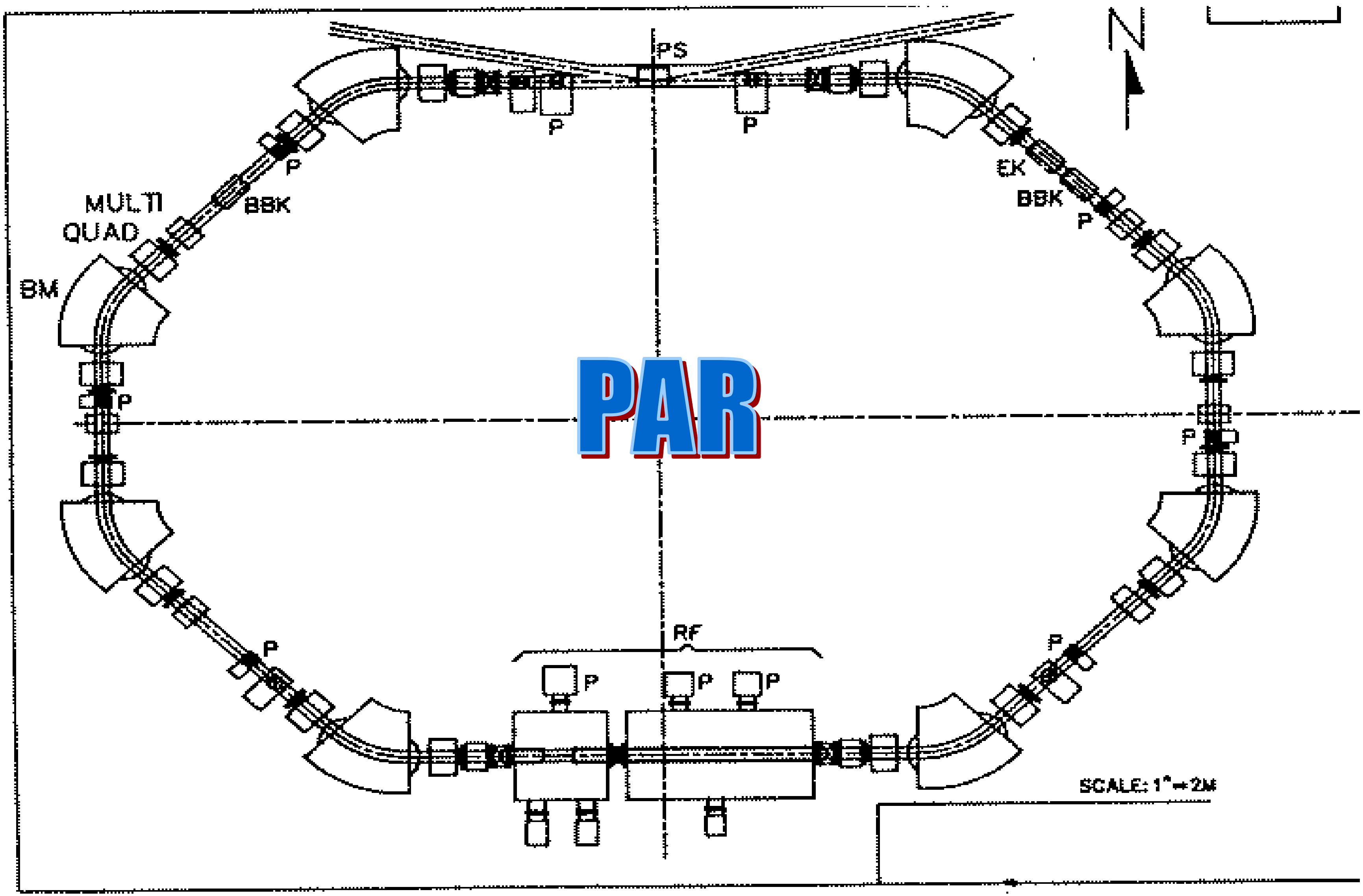
- More of problem for small rings (low energy) synchrotron rings (2nd generation).
- B-factories and damping rings for NLCs

VERY HIGH BEAM CURRENT AND CLOSELY SPACED BUNCHES, ULTRA–LOW EMMITANCES

PAR Beam Injection/Extraction



PAR Operation Cycle



PAR Operation Cycle

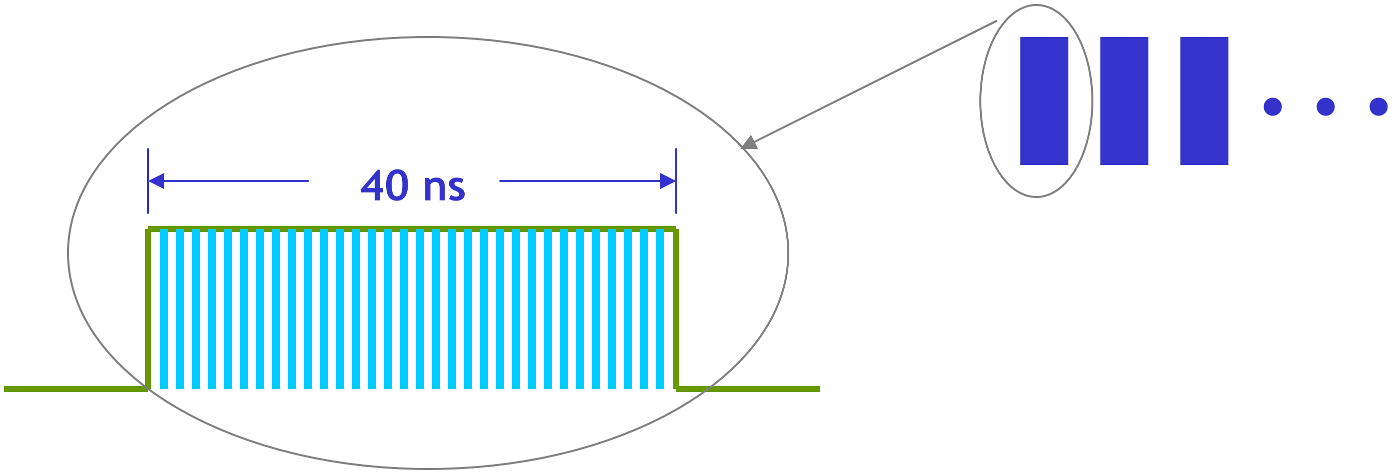
Linac Bunch Injection Sequence:

00 01 02 03 04 05 06 07 08 … 24 25 26 27 28 29

Linac macro-pulse length is determined by the linac rf thermionic gun(s) macro-pulse length

Thermionic gun 1: Nominal pulse length 40 ns

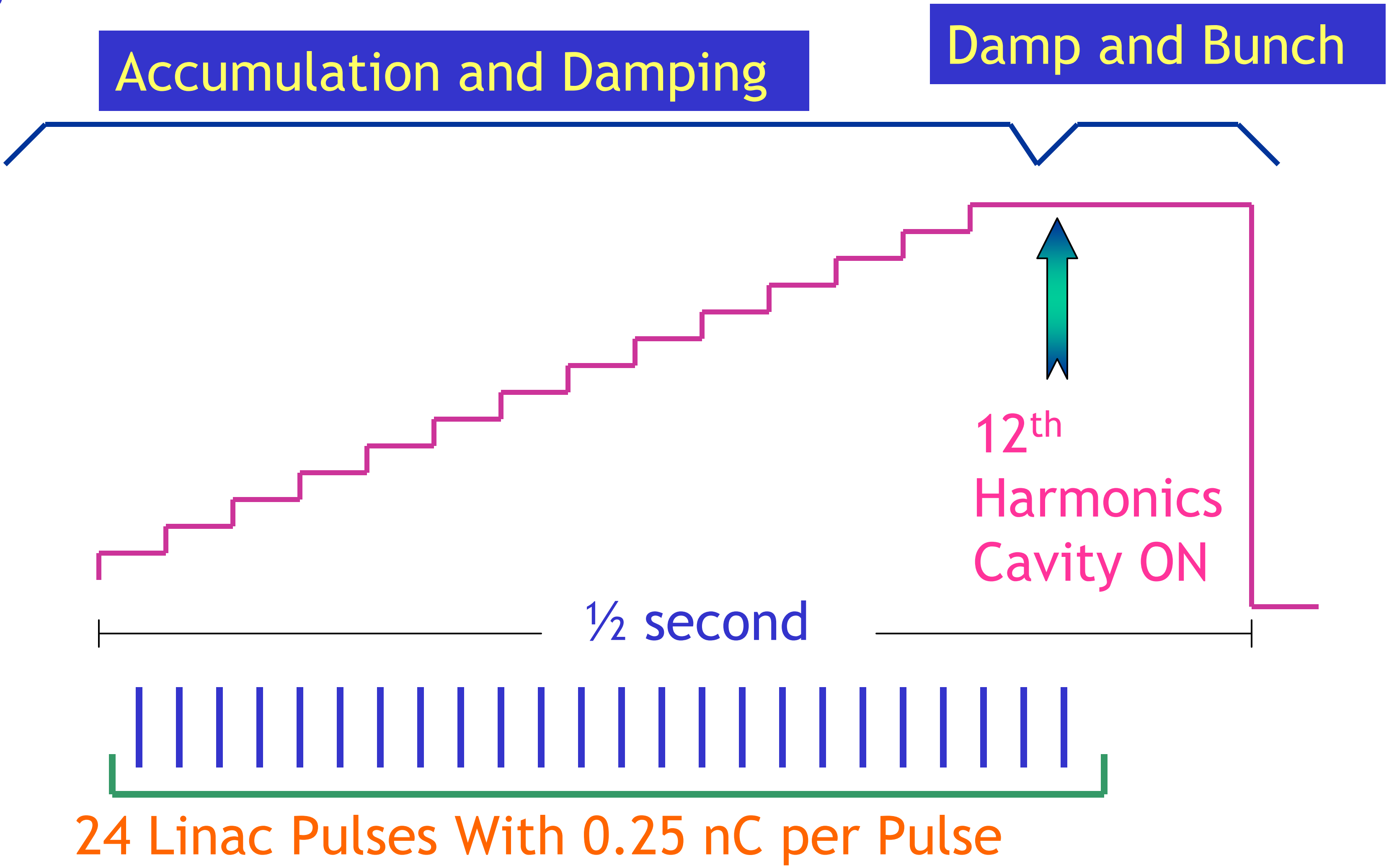
Thermionic gun 2: Nominal pulse length 10 ns



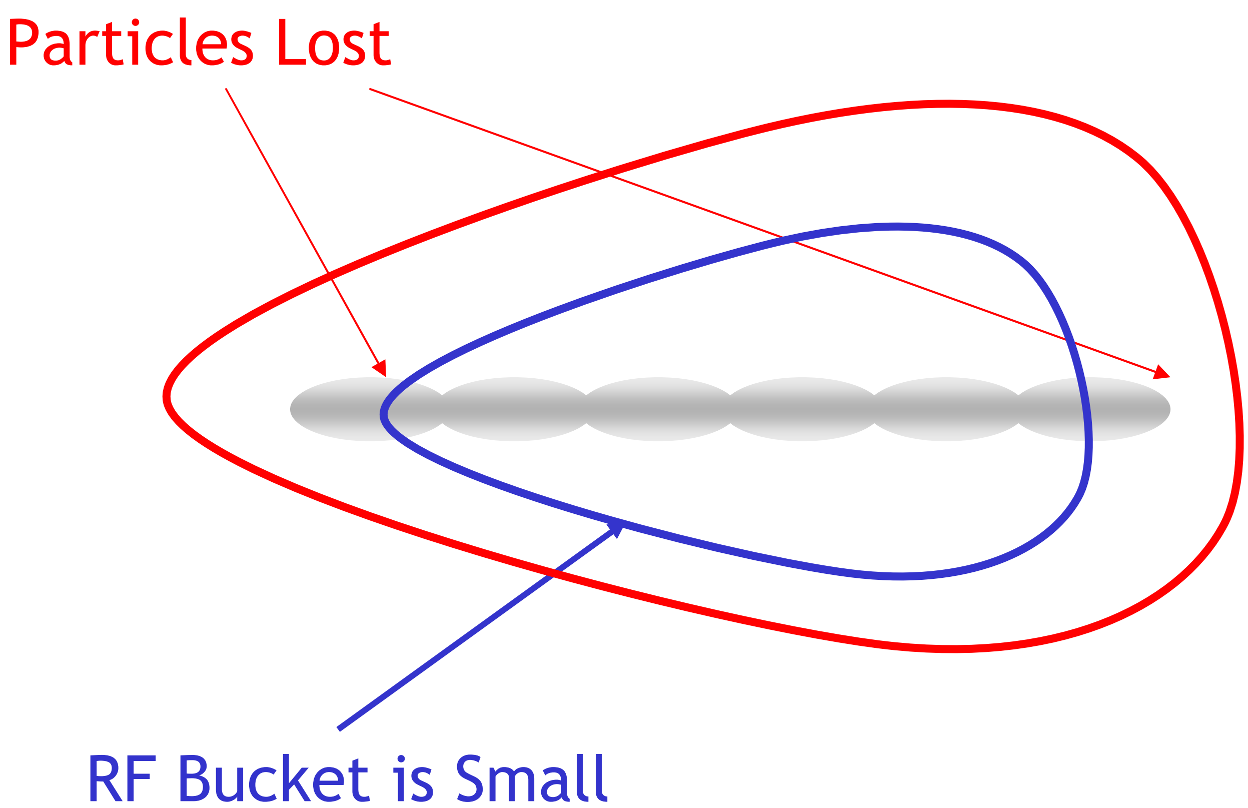
One Linac Micro-Pulse

PAR Operation Cycle

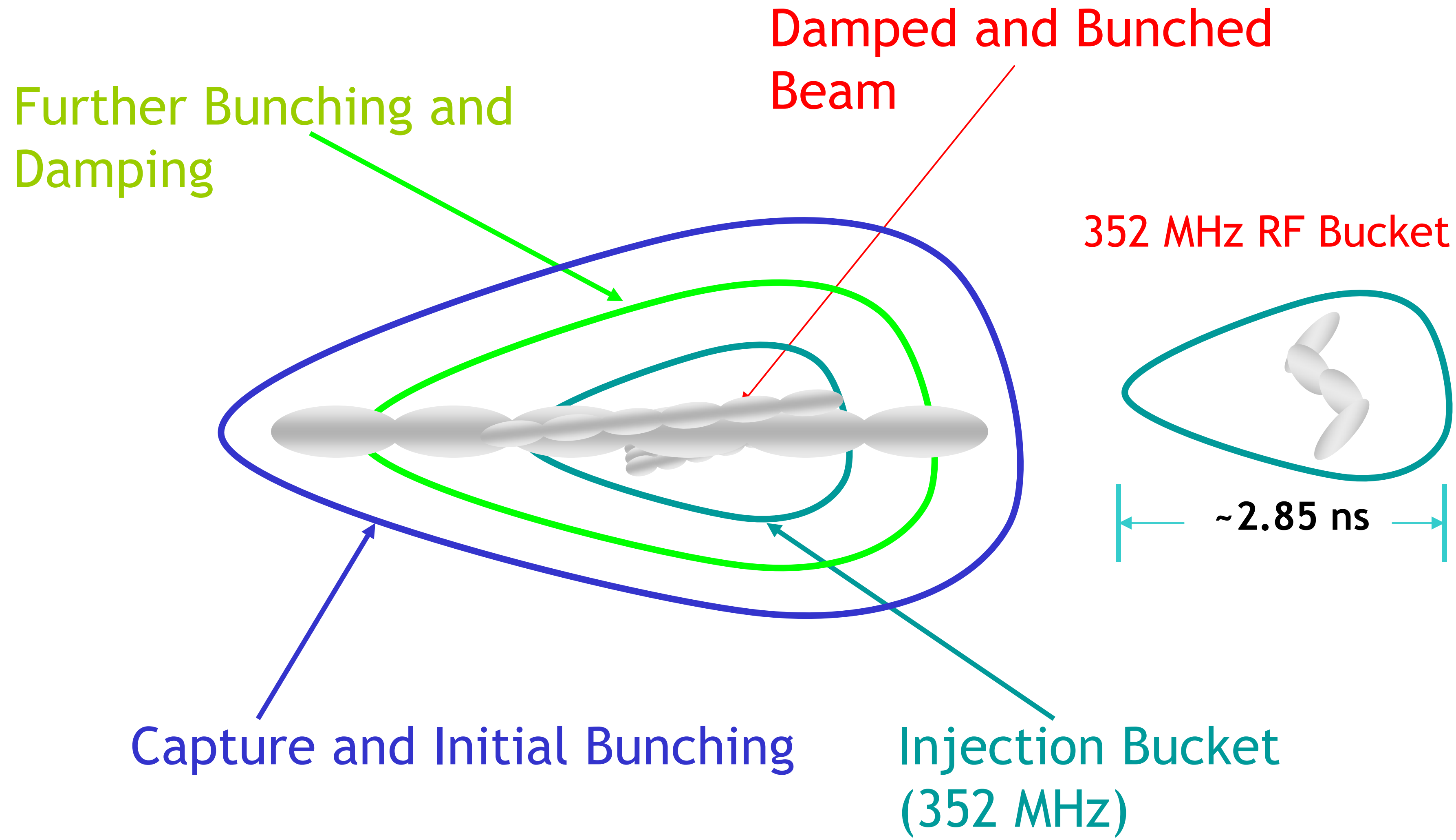
Injection Scenario:



Injection RF Bucket



Injection RF Bucket



PAR Operation Cycle

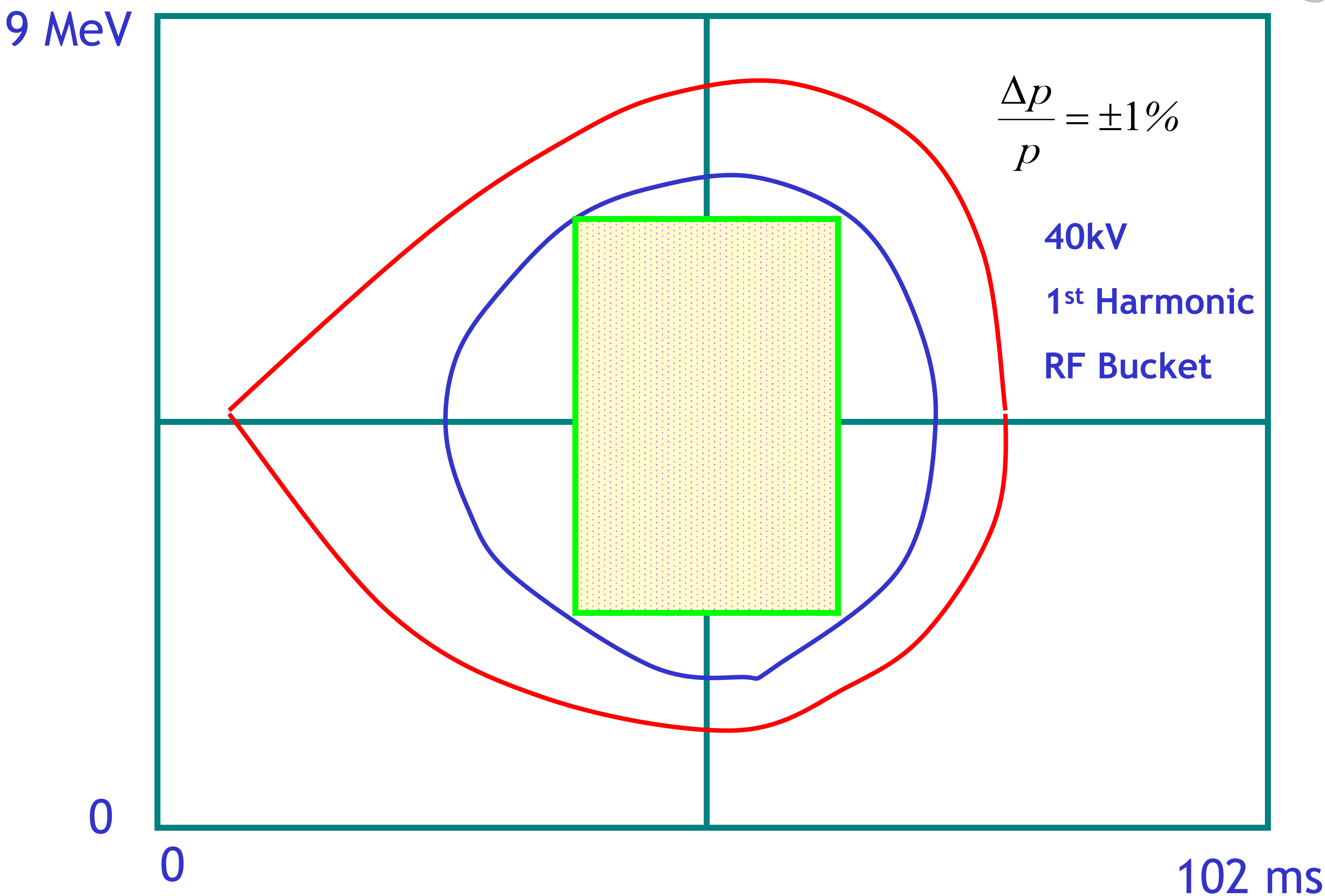
PAR cycle last 500 ms. At the beginning of the cycle, there in no stored beam, and only the first harmonic rf system (Fundamental Cavity) is on.

During the next 400 ms, 24 electron pulses (for example) from the linac are injected via LTP into the PAR.

After the last bunch train pulse is injected, the 12th harmonic rf system is turned on. During about 100 msec pf the cycle, the beam is damped by synchrotron radiation and compressed by the 12th harmonic cavity.

At the end of the cycle, the beam is ejected into the PTB for transport into the injector synchrotron.

PAR



PAR

The longitudinal phase space damping can be determined by

$$\frac{\sigma_E}{E(t)} = \sqrt{\left(\frac{\sigma_E}{E(\infty)}\right)^2 + \left(\frac{\sigma_E}{E(0)}\right)^2 - \left(\frac{\sigma_E}{E(\infty)}\right)^2 exp\left(-2t/\tau_E\right)}$$

is the natural (completely damped) energy spread in the ring.

During the 100 ms allowed for final damping, σ_E/E is reduced to the natural value of 0.4×10^{-3}. If the RF voltage remains at 40 kV, the final bunch length is σ_τ =0.92 ns, which is 1/3 of the 252 MHz injector synchrotron bucket.

I.S bucket = $\pm1.55\,\sigma_\tau$

PAR

The bunch length damping provided by the 1st harmonic system is not adequate for efficient capture in the injector synchrotron. Therefore, a 12th harmonic 118 MHz, 30 kV system is needed for the final damping. This system is turned on after the last pulse.

The first harmonic system damps the beam to

$$\sigma_E / E = 0.44 \times 10^{-3}$$
$$\sigma_\tau = 0.99\,ns$$

Final Bunch length

$$\sigma_E / E = 0.4 \times 10^{-3}$$
$$\sigma_\tau = 0.30\,ns$$

PAR Operation Cycle

What do RF Systems do?

Fundamental (1st Harmonic) RF System

Restore the energy that electrons lose to synchrotron radiation

Perform most of the bunch length compression

12th Harmonic RF System

Complete compression of the bunch length

Parameters for PAR RF Systems

System I

Frequency, f	9.77584	MHz
Harmonic number, h	1	
Peak Voltage ***V***	40	kV
Synchrotron frequency f_s	19.0	kHz
Natural bunch length (damped)	0.92	ns

Parameters for PAR RF Systems

System II

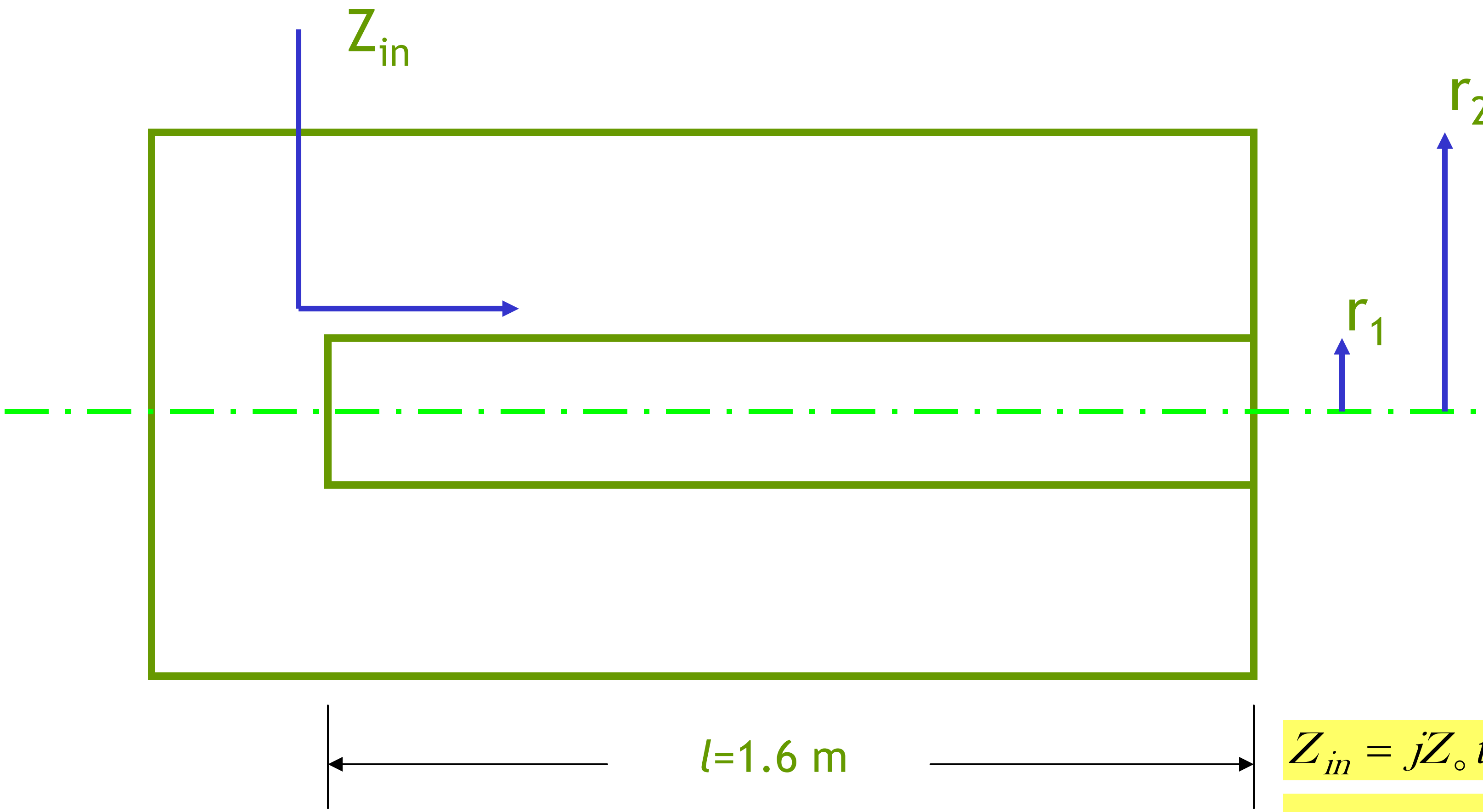
Frequency, f	117.3101	MHz
Harmonic number, h	12	
Peak Voltage *V*	30	kV
Synchrotron frequency f_s	60.2	kHz
Natural bunch length (damped)	0.30	ns

Cavities RF Parameters

Parameter	1st Harmonic	12th Harmonic
Frequency, f (MHz)	9.77584	117. 3101
V (kV)	40	30
Type	$\lambda/4$, loaded	$\lambda/2$
Z_0(Ω) 50	50	50
Power (kW)	4.7	0.222
R_s (kΩ)	170	2020
Q	7630	25300

PAR RF Cavities

1st Harmonic (9.77584 MHz)

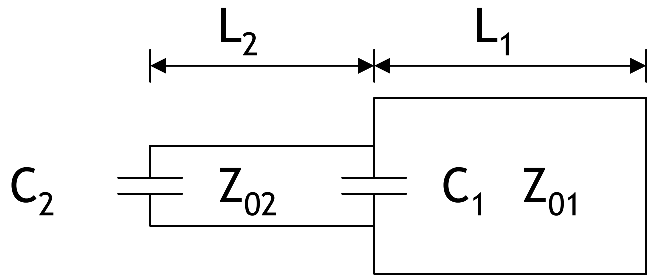


Diameter=1.2 m

$$Z_{in} = jZ_{\circ}\tan(\beta\ell)$$

$$Z_{\circ} = 60\,ln\frac{r_2}{r_1}$$

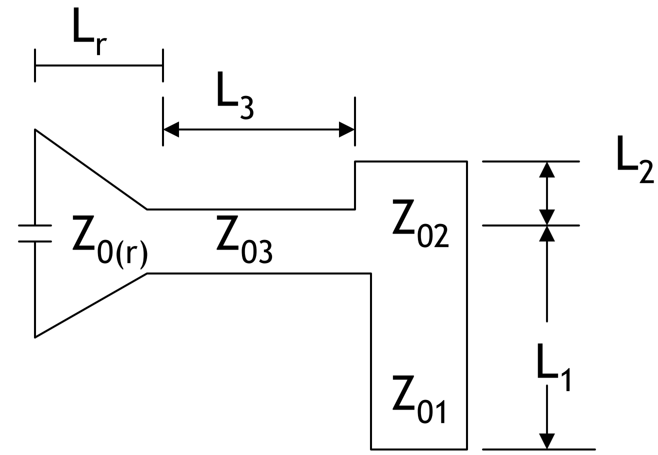
1^{st} Harmonic (9.77584 MHz)

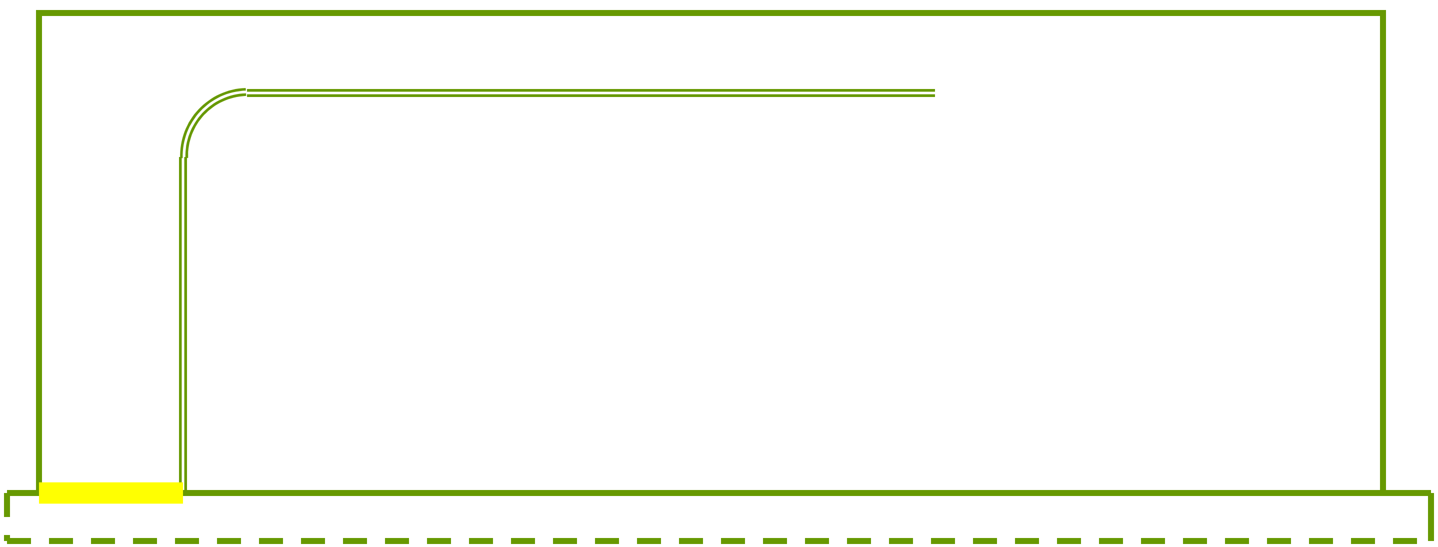


Upper half of the folded coaxial cavity

PAR RF Cavities

1st Harmonic (9.77584 MHz)





Radial Transmission Line Loaded Cavity

Radial Transmission Line

For the dominant TEM to r mode of a parallel plate radial transmission line structure, the fields with inward and outward traveling waves are

$$
\begin{aligned}
E_z &= AH_0^{(1)}(kr) + BH_0^{(2)}(kr) \\
&= \left[J_0^2(kr) + N_0^2(kr)\right]\left[Ae^{j\vartheta(kr)} + Be^{-j\vartheta(kr)}\right]
\end{aligned}
$$

$$
\begin{aligned}
H_\phi &= \frac{j}{\eta}\left[AH_1^{(1)}(kr) + BH_1^{(2)}(kr)\right] \\
&= \frac{\sqrt{J_0^2(kr) + N_0^2(kr)}}{Z_0^W(kr)}\left[Ae^{j\Psi(kr)} - Be^{-j\Psi(kr)}\right]
\end{aligned}
$$

Where A and B are magnitude of the incident and reflected waves, $H^{(1)}$ and $H^{(2)}$ are the Hankel functions of the first and second kind respectively.

$$
\left.
\begin{aligned}
H_n^{(1)} &= J_n + jN_n \\
H_n^{(2)} &= J_n - jN_n
\end{aligned}
\right]
$$

J_n is the n-th order Bessel function of the first kind and N_n is the n-th order Bessel function of the second kind.

Radial Transmission Line

$Z_o(kr)$ is the characteristic wave impedance of the radial transmission line:

$$Z_0^W(kr) = \eta\sqrt{\frac{J_0^2(kr)+N_0^2(kr)}{J_1^2(kr)+N_1^2(kr)}}$$

The phase functions are

$$\theta(v) = tan^{-1}\left[\frac{N_0(v)}{J_0(v)}\right]$$

$$\psi(v) = tan^{-1}\left[\frac{N_1(v)}{J_1(v)}\right]$$

Radial Transmission Line

The input impedance at a point $r=r_i$ with a load impedance Z_L at r_L is

$$Z^b = Z_0(r_i)\frac{Z_L\cos\{\theta(kr_i)-\psi(kr_L)\}+jZ_0(r_L)\sin\{\theta(kr_i)-\theta(kr_L)\}}{Z_0(r_L)\cos\{\psi(kr_i)-\theta(kr_L)\}+jZ_L\sin\{\psi(kr_i)-\psi(kr_L)\}}$$

Where the characteristic impedance is give as

$$Z_0(r) = Z_0^W \frac{h}{2\pi r}$$

h line is the height of the radial transmission.

The electric field or voltage reflection coefficient at $r=r_i$ in the radial transmission line section is

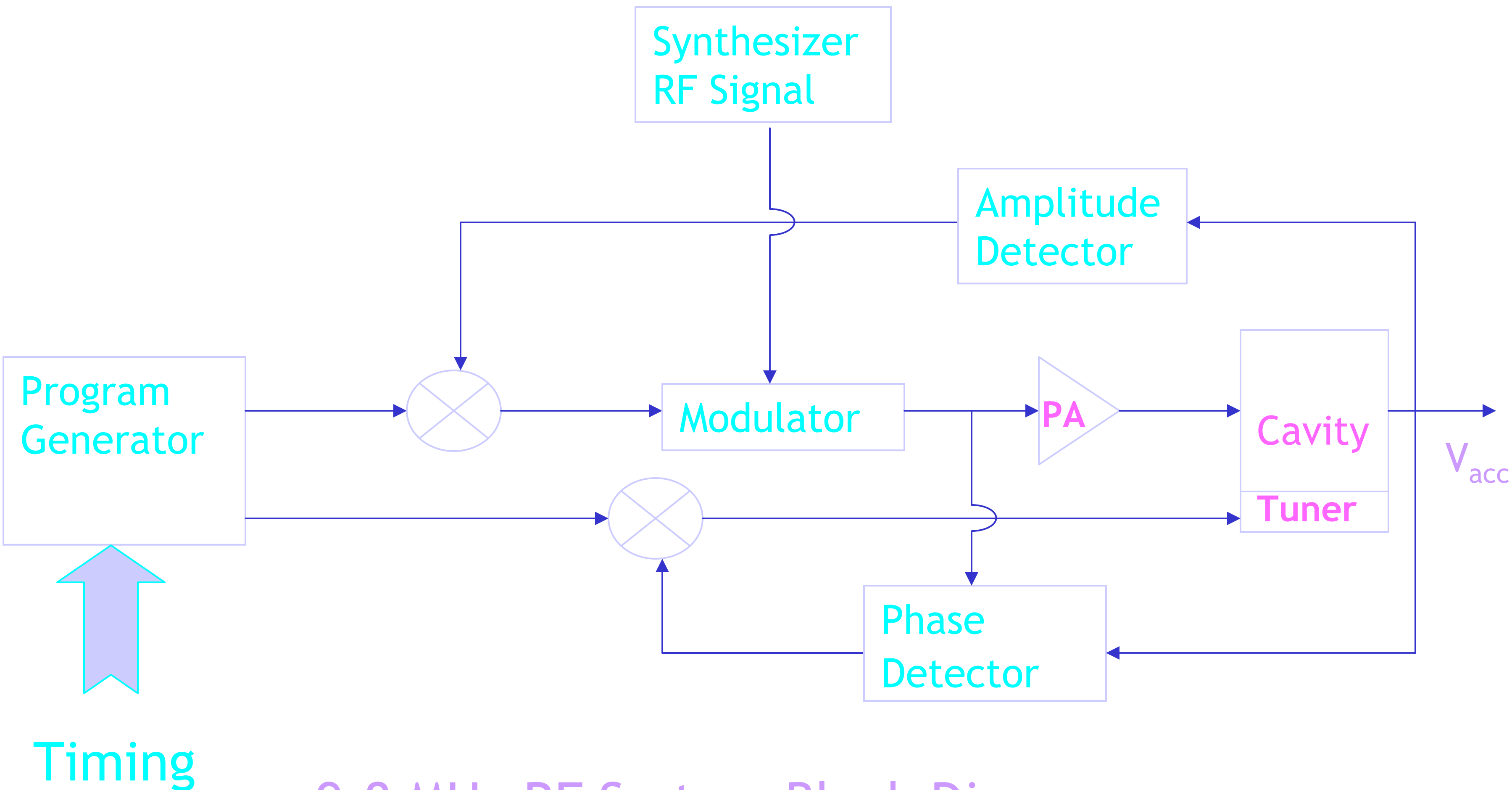
$$\Gamma(r_i) = \Gamma(r_L)e^{2j\{\theta(kr_i)-\theta(kr_L)\}}$$

The magnetic field or current reflection coefficient at $r=r_i$ in the radial transmission line section is

$$\Gamma(r_i) = \Gamma(r_L)e^{2j\{\psi(kr_i)-\psi(kr_L)\}}$$

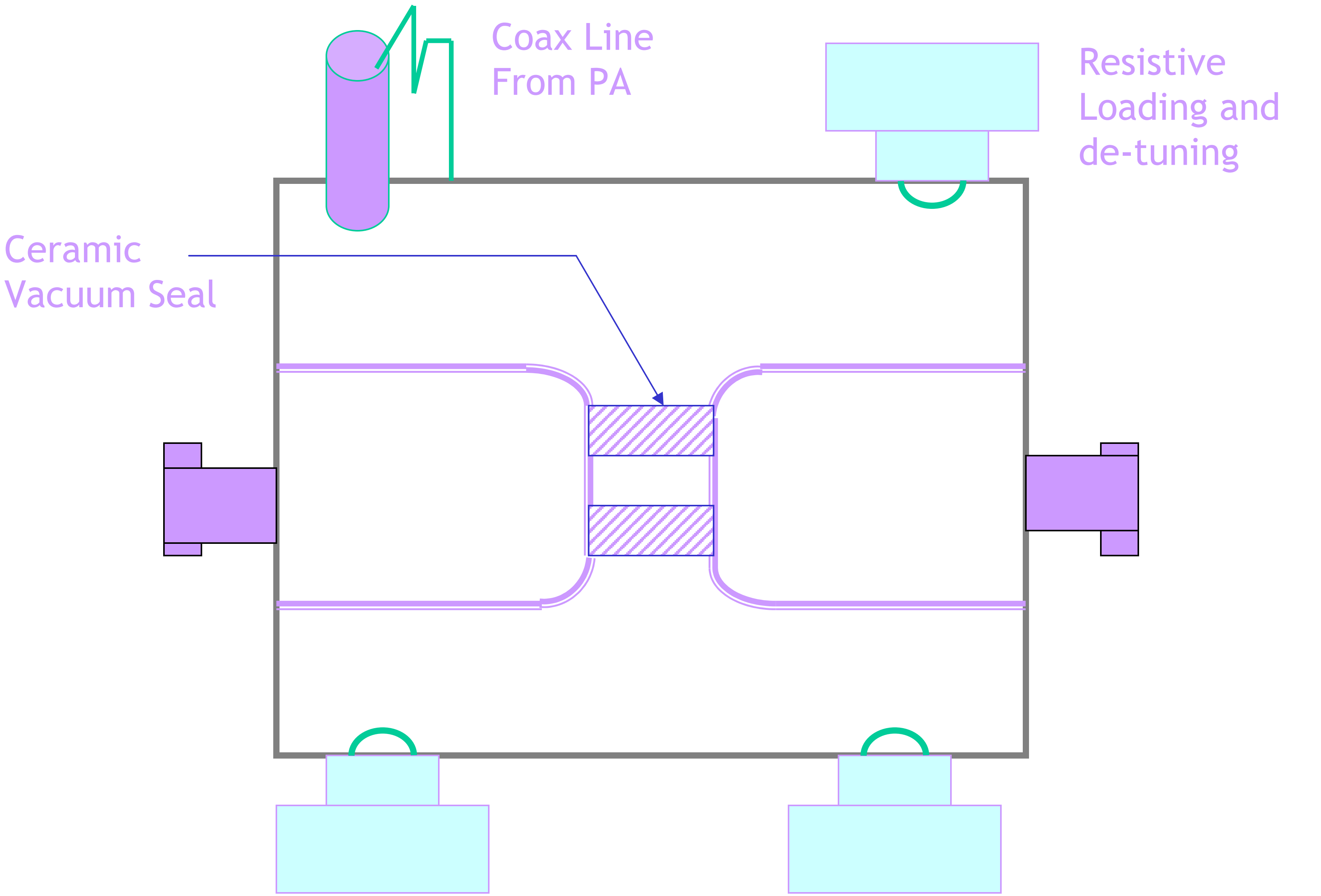
$$\Gamma(r_L) = \frac{Z_L - Z_0(r_L)}{Z_L + Z_0(r_L)}$$

PAR RF

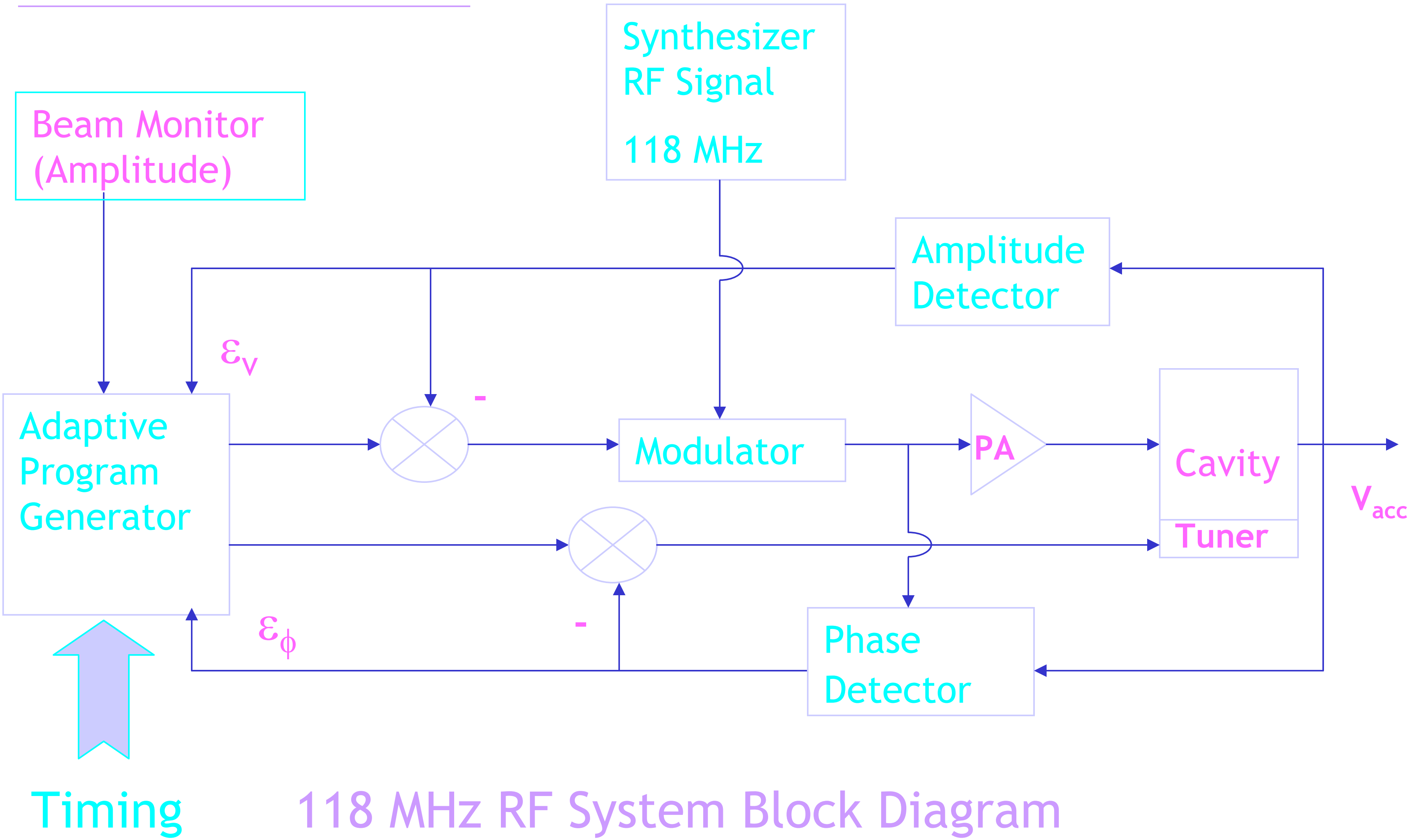


9.8 MHz RF System Block Diagram

PAR 12th Harmonic RF Cavity

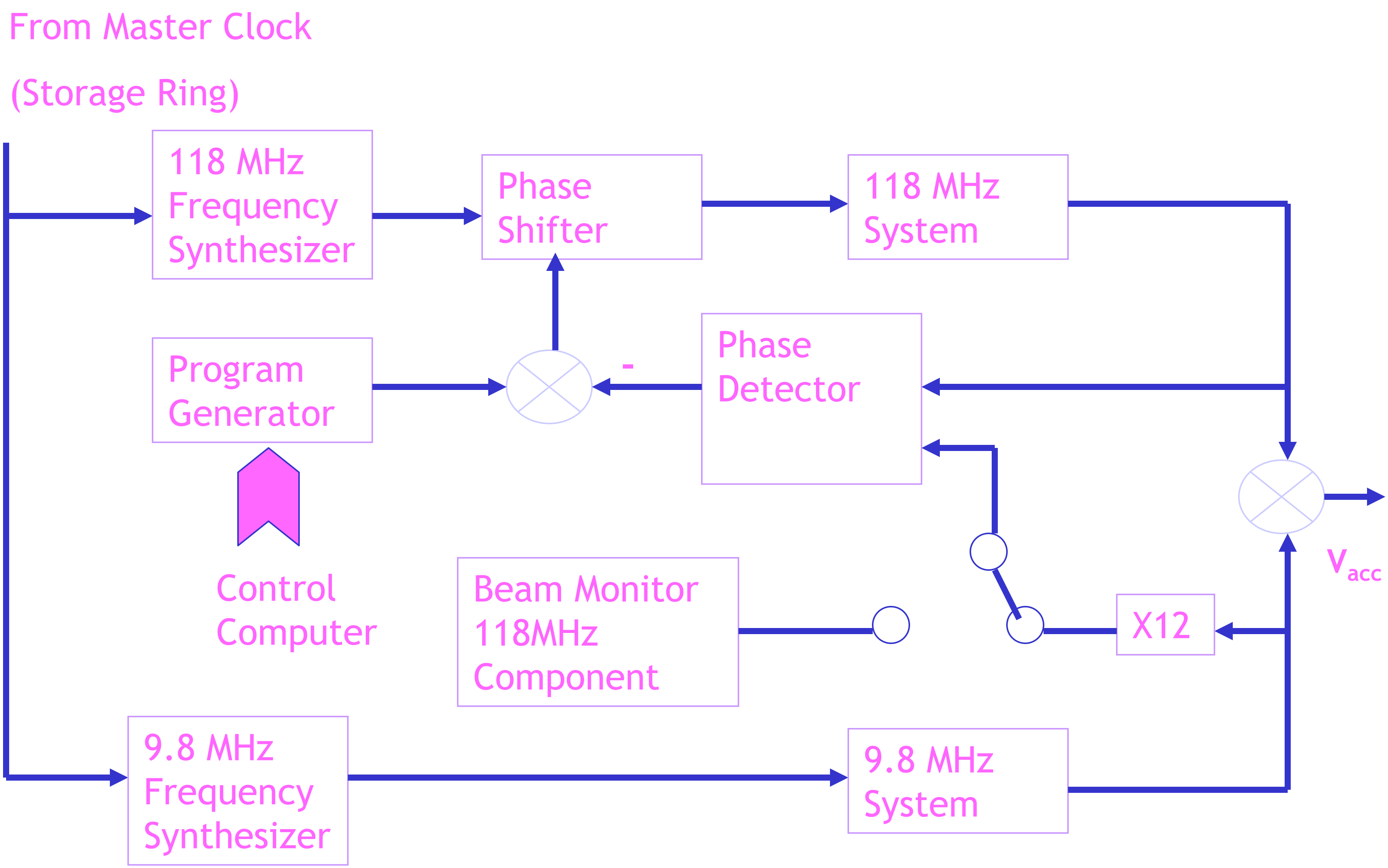


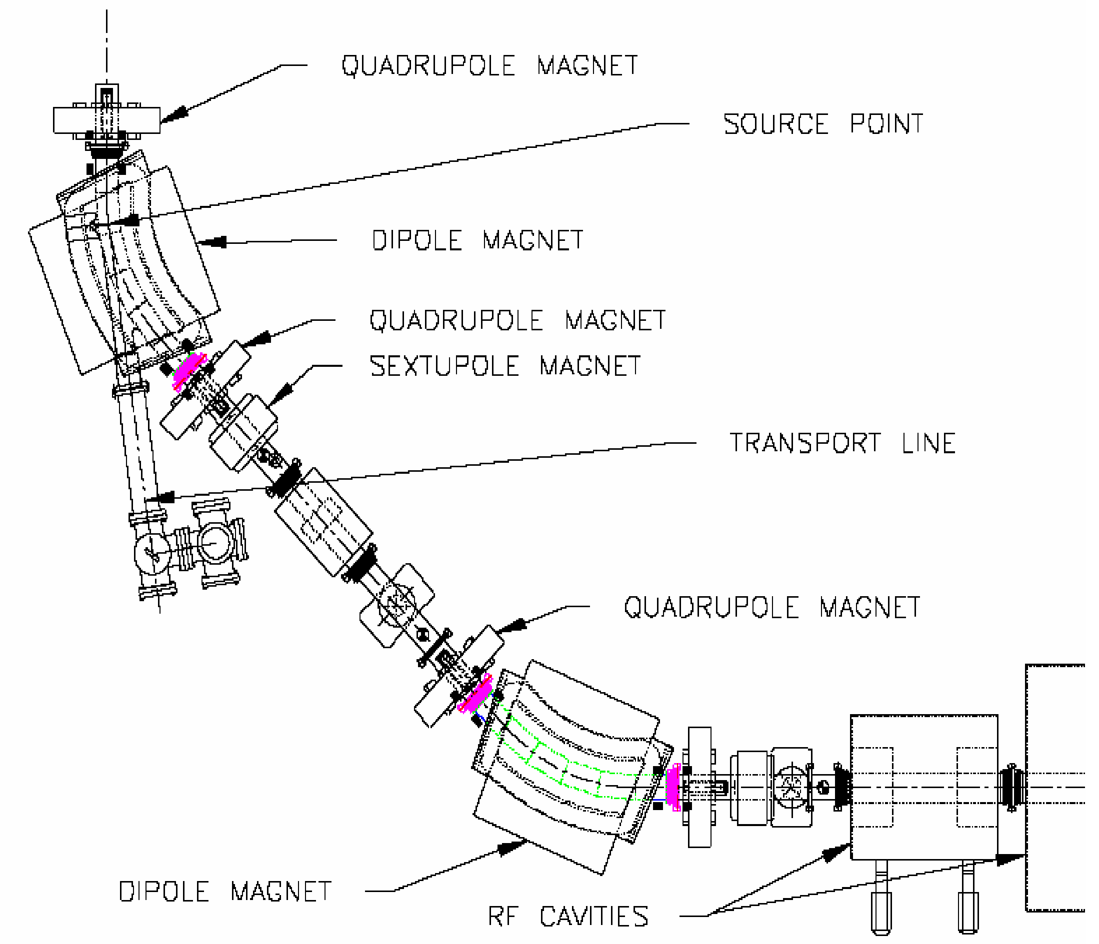
PAR 12th Harmonic



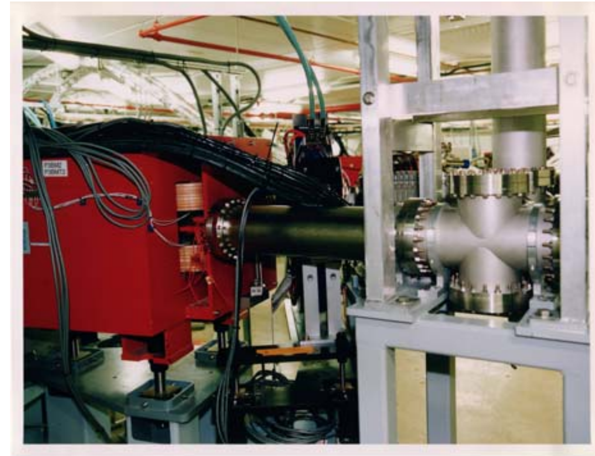
118 MHz RF System Block Diagram

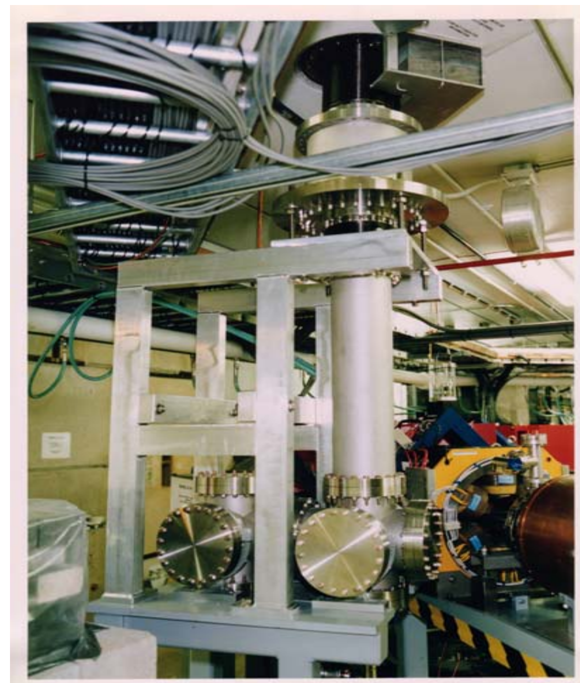
PAR RF Systems Synchronization

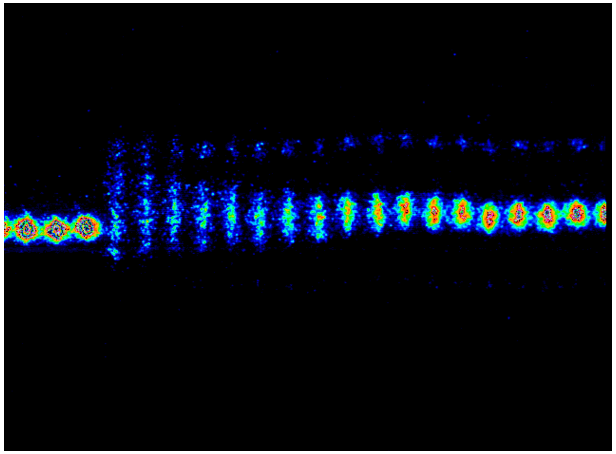


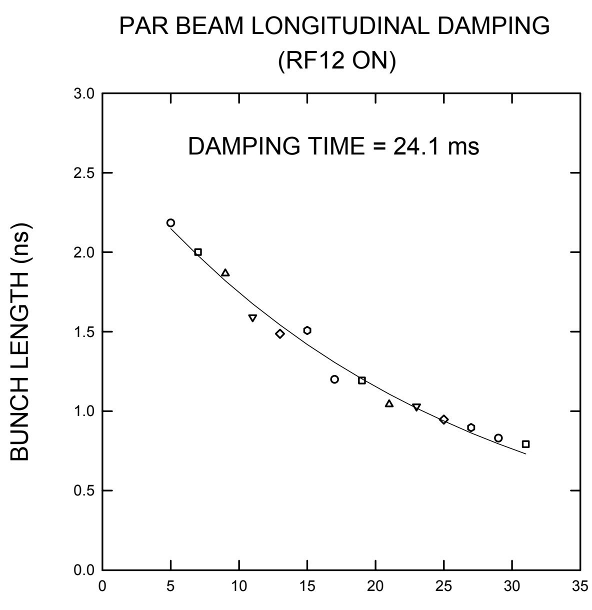

QUADRUPOLE MAGNET
SOURCE POINT
DIPOLE MAGNET
QUADRUPOLE MAGNET
SEXTUPOLE MAGNET
TRANSPORT LINE
QUADRUPOLE MAGNET
DIPOLE MAGNET
RF CAVITIES

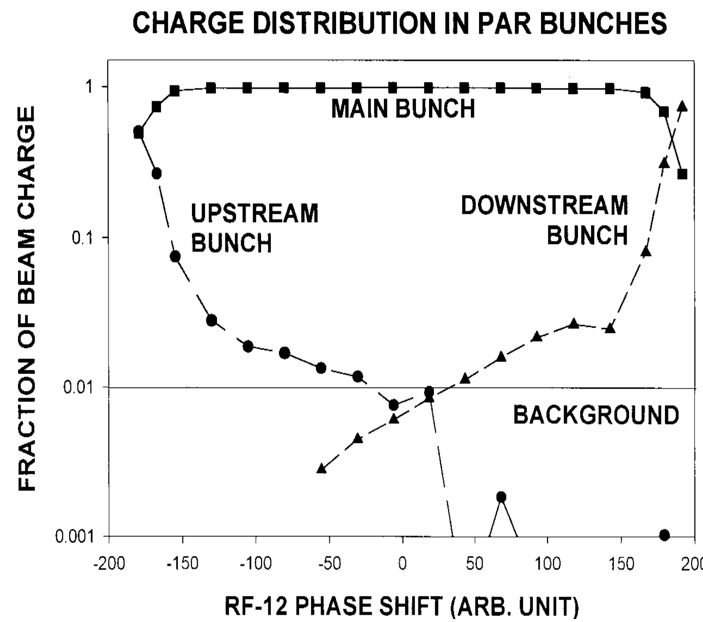
B. Yang











Intensity of bunches as a function of RF-12 phase. By varying the phase, one can minimize the satellite bunch intensity to a level acceptable to users.

B. Yang