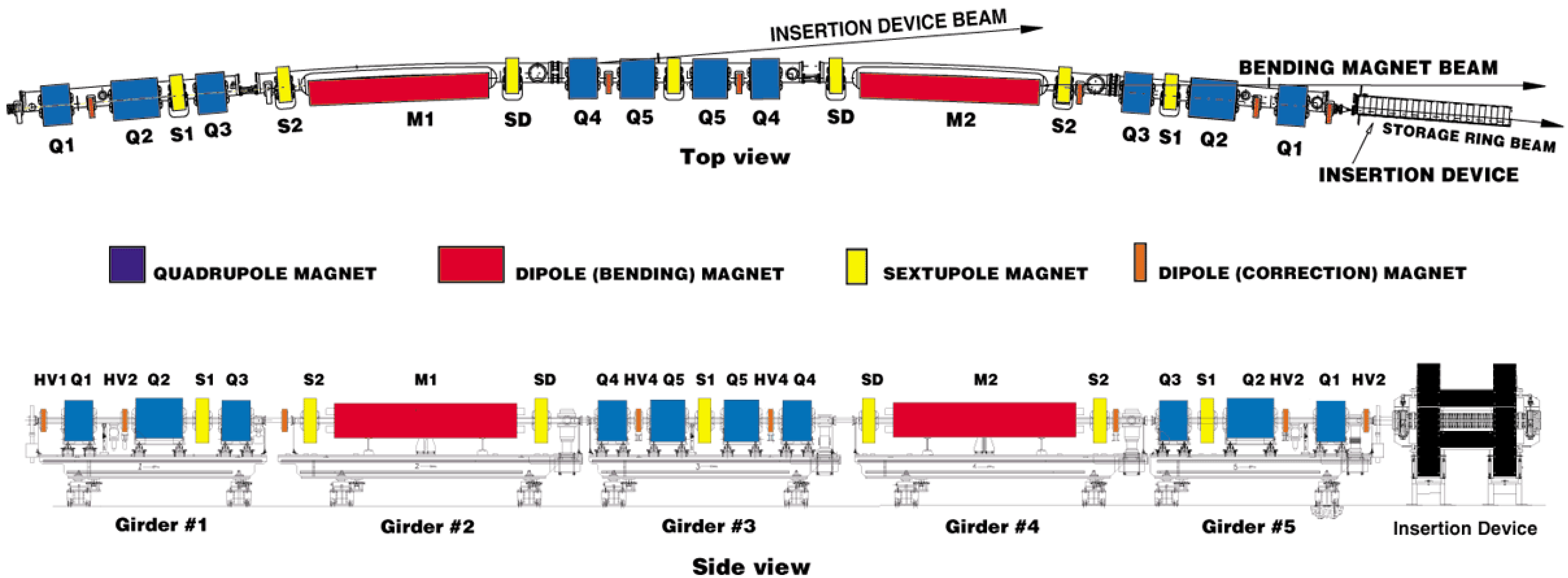
# Betatron and Synchrotron Oscillations in Space and Time




One of the 40 Sectors of the Advanced Photon Source

## Betatron Oscillations

Observations

$$x(s) = [W_x \beta_x(s)]^{1/2} \cos[\psi_x(s) - \psi_{0x}]$$

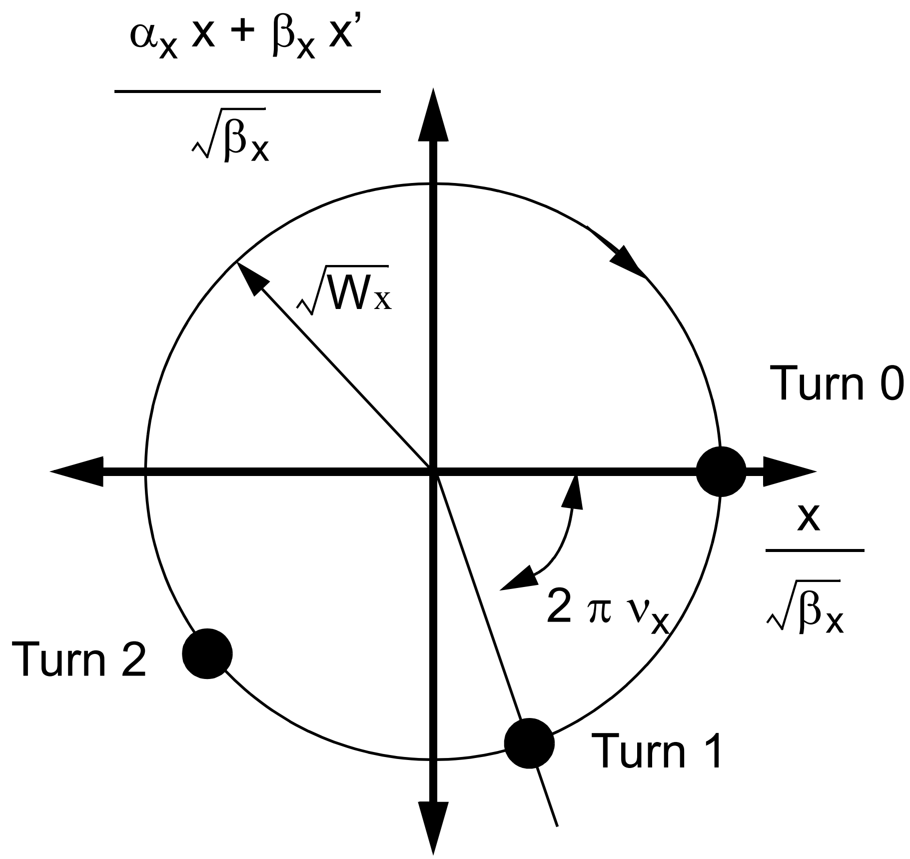
1) To first order, a bunch of electrons behaves like one gigantic super-electron satisfying the above relation.

2) The curvilinear coordinate s in the above can equally well be understood to represent time, according to the relation $s = vt = ct$

3) Since no human can be in all places at all times, measurements of $x(s)$ are observable only in a quantized fashion dictated by the bunch structure of the beam. For any particular bunch, any detector bolted to the vacuum chamber can observe the bunch's transverse position $x(s)$ only at times corresponding to $s = s_0+cT_0$, $s+2cT_0$,... etc, where $T_0$ = the revolution period, $cT_0$ = the ring circumference L.

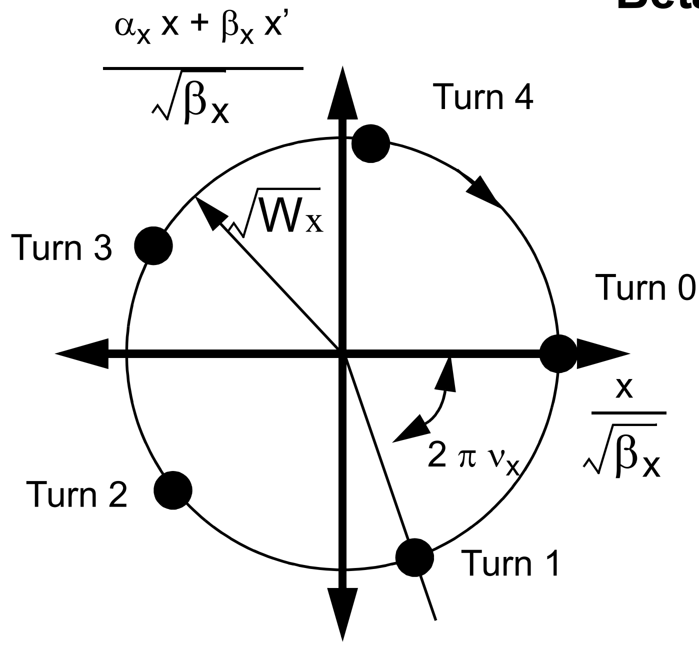
4) $\beta_x(s_0+L) = \beta_x(s_0) = \beta_x$, i.e. every detector gets its own unique $\beta_x$; and
$\psi_x(s_0+L) = \psi_x(s_0) + 2\pi\nu_x$,
i.e. the tune $\nu_x$ sets the time scale for betatron oscillations

## Normalized Electron Phase Space at Fixed Location s




Unlike the angle x', the position variable x is directly observable at a fixed location along the ring circumference

# Betatron Oscillations



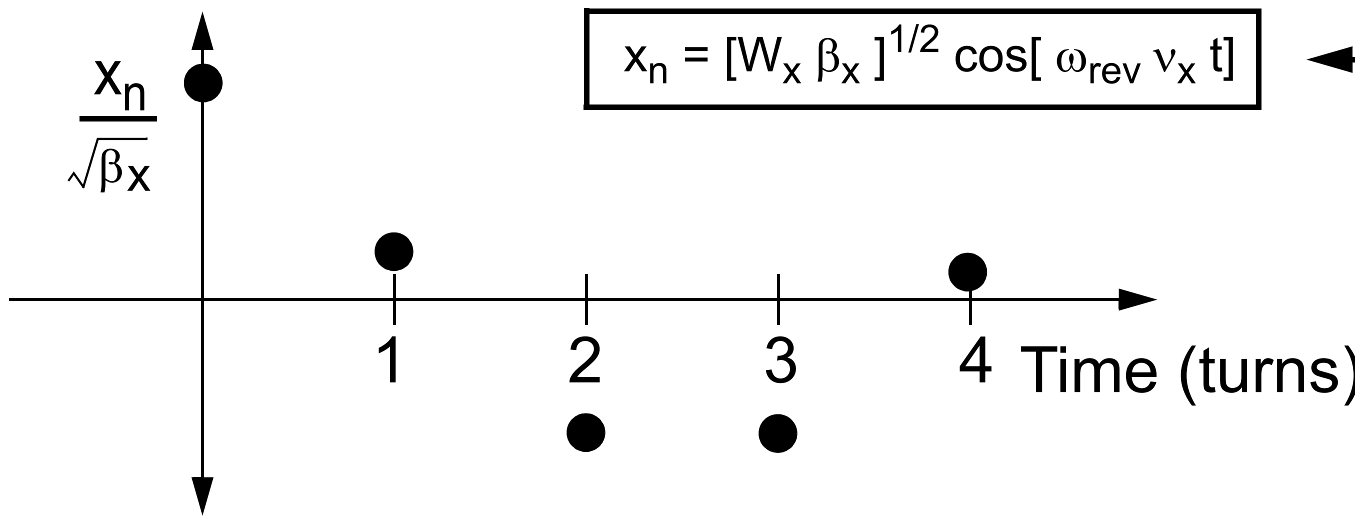

$$x(s) = [W_x \beta_x(s)]^{1/2} \cos[\psi_x(s) - \psi_{0x}]$$

$$x_n = [W_x \beta_x ]^{1/2} \cos[2 \pi \nu_x n]$$

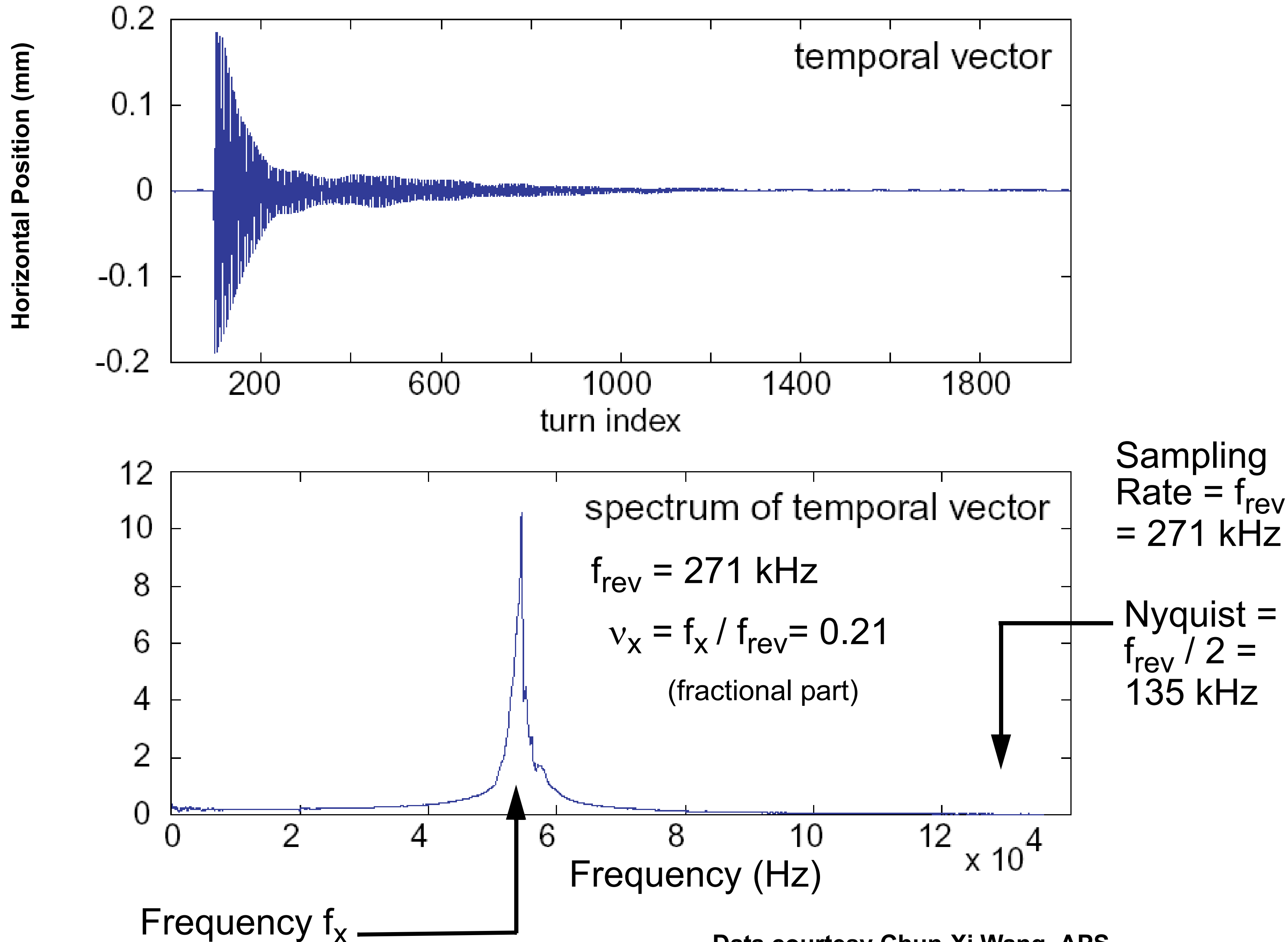
$$n = t / T_0 = f_{rev} t$$

$$x_n = [W_x \beta_x ]^{1/2} \cos[2 \pi f_{rev} \nu_x t]$$

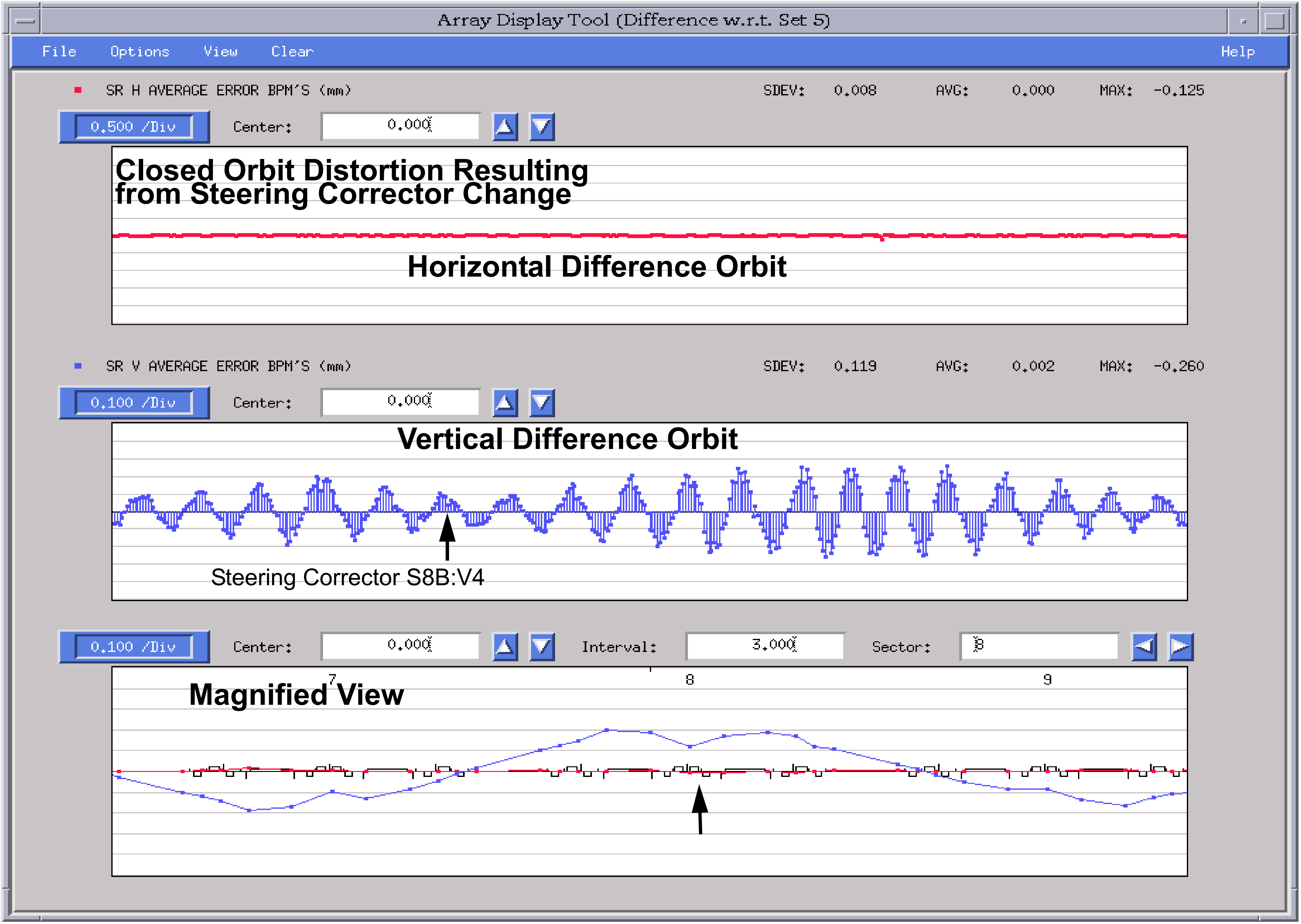
$$\boxed{x_n = [W_x \beta_x ]^{1/2} \cos[ \omega_{rev} \nu_x t]}$$



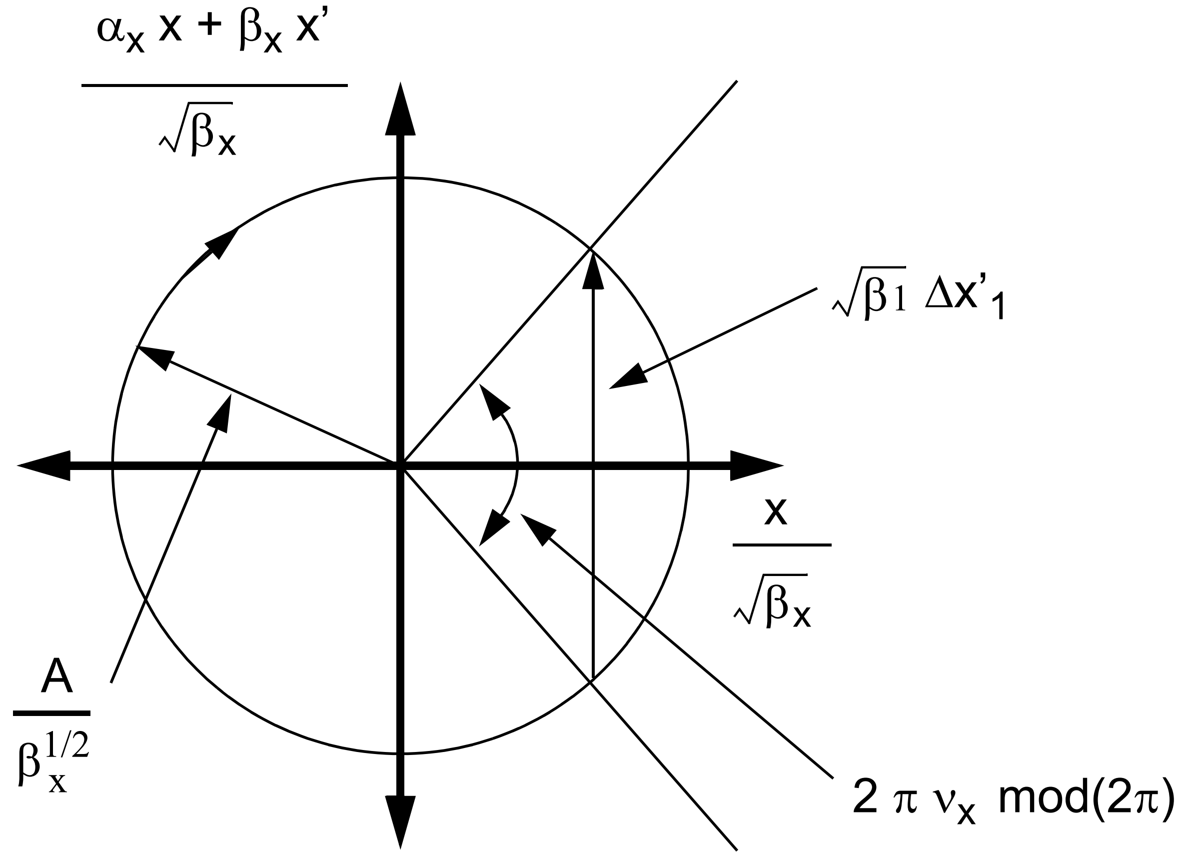
## Betatron oscillation resulting from firing injection kicker




**Data courtesy Chun-Xi Wang APS**


Array Display Tool (Difference w.r.t. Set 5)
File
Options
View
Clear
Help
SR H AVERAGE ERROR BPM'S (mm)
SDEV: 0.008
AVG: 0.000
MAX: -0.125
0.500 /Div
Center: 0.000
Closed Orbit Distortion Resulting from Steering Corrector Change
Horizontal Difference Orbit
SR V AVERAGE ERROR BPM'S (mm)
SDEV: 0.119
AVG: 0.002
MAX: -0.260
0.100 /Div
Center: 0.000
Vertical Difference Orbit
Steering Corrector S8B:V4
0.100 /Div
Center: 0.000
Interval: 3.000
Sector: 8
Magnified View
7
8
9

# Closed Orbit Distortion Resulting from Single Corrector Change $\Delta x'_1$ Phase Space Representation



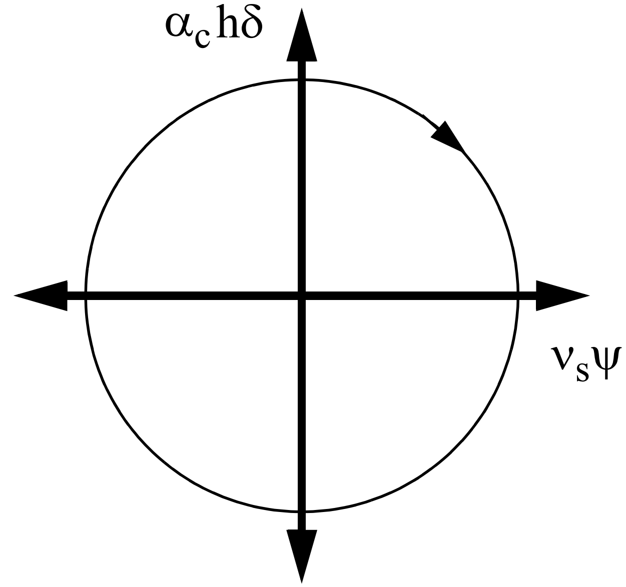
# Synchrotron Oscillations

$$\psi = \psi_0 \cos \Omega_s t$$

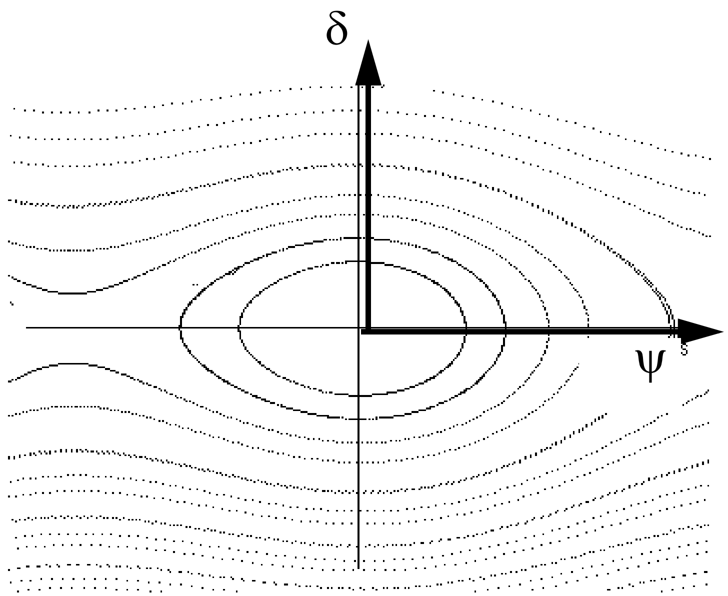
$$\Omega_s = \sqrt{\frac{\alpha_c \omega_{RF} e V_{RF}^0 \cos\phi_s}{E_0 T_0}}$$

$$\delta = \delta_{max} \sin \Omega_s t$$

Normalized Small-Amplitude Phase Space



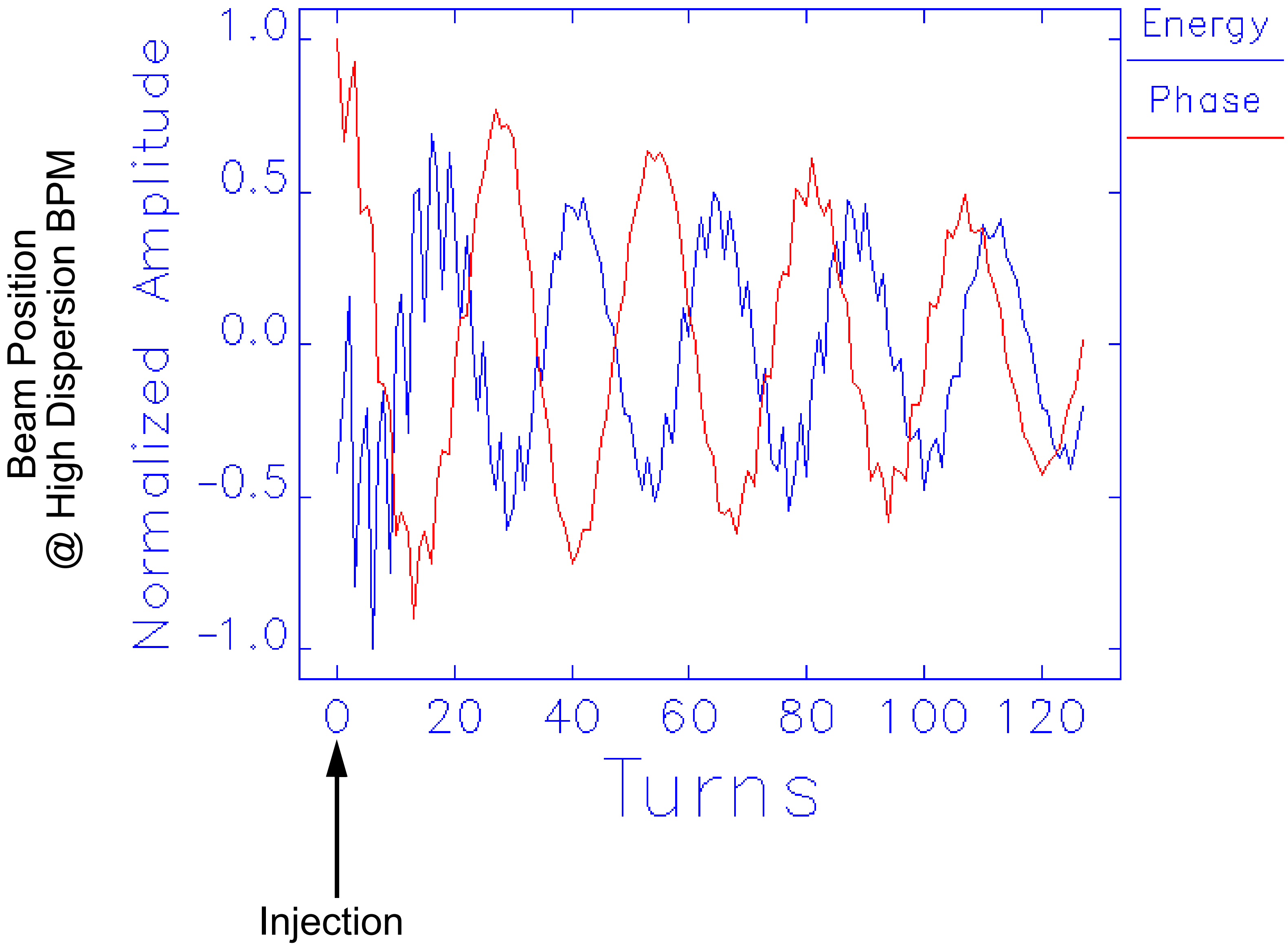

$$\left(\alpha_c{}^2 h^2\right)\delta^2 + \left(\nu_s^2\right)\psi^2 = \text{constant}$$



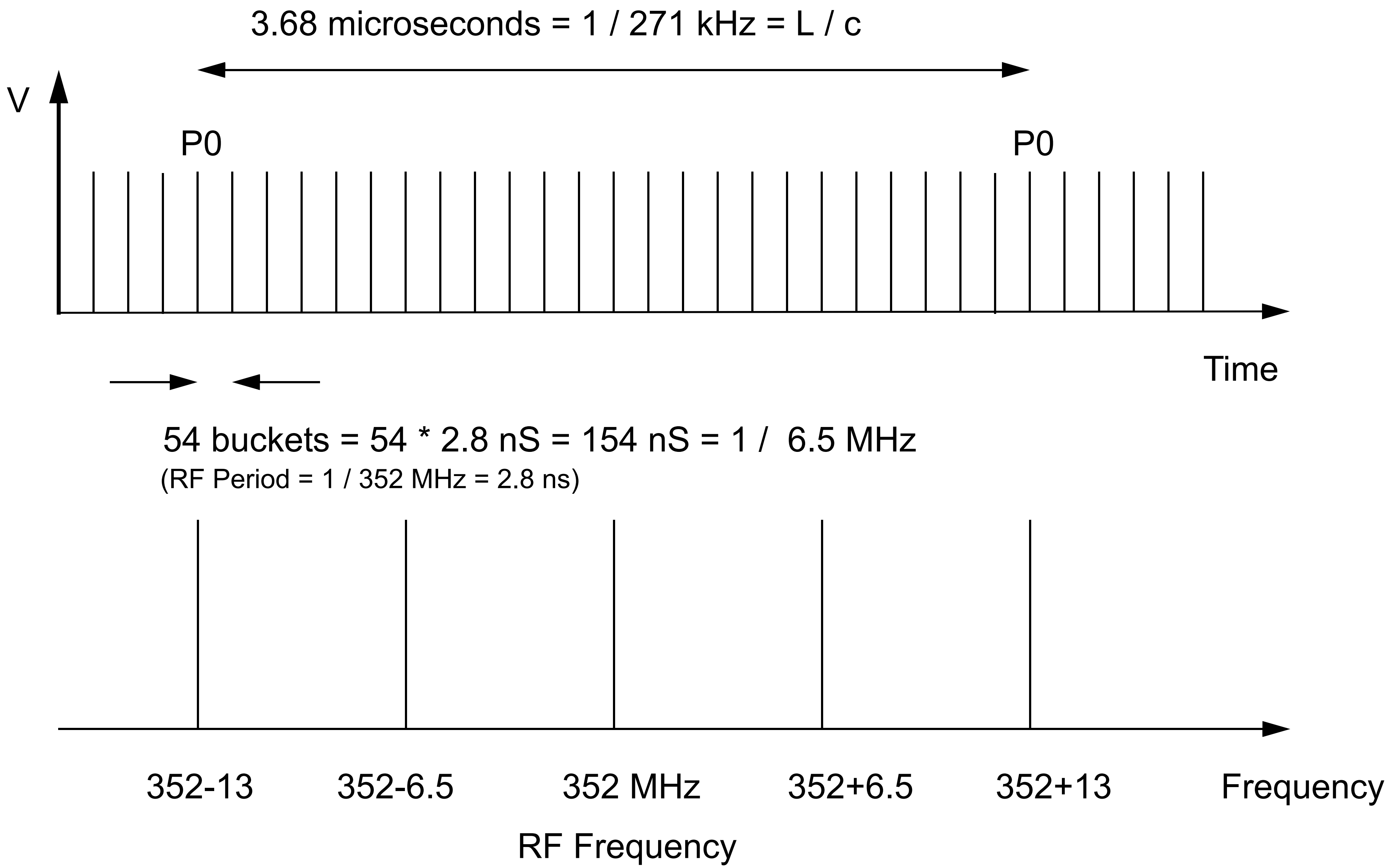

Phase Space - Separatrix and All

$$\Delta x_E(s) = \eta(s)\,\frac{\Delta E}{E_0} = \eta(s)\,\delta$$

Synchrotron Oscillations Resulting from Energy / Phase Mismatch at Injection APS Booster



# 24 Bunch Fill Pattern - APS



## Impact of Synchrotron Phase Oscillations on All Beam-derived Signals

$$V(t) = A \{ \mathrm{Cos}[\omega_{RF}\, t + \psi] + \mathrm{Cos}[ (\omega_{RF} + 2\,\pi * 6.5\ \mathrm{MHz})t + \psi ] + \ldots\}$$

But $\psi = \psi_0 \,\mathrm{Cos}\, \Omega_s t$

So $V(t) = A\, \mathrm{Cos}[\omega_{RF}\, t + \psi_0 \cos \Omega_s t\, ] + \ldots$ etc.

# Digression - Frequency / Phase Modulation

$$v(t) = \mathrm{Cos}(\omega_c t + \beta\,\mathrm{Sin}(\omega_m t))$$

$$v(t) \cong \mathrm{Cos}(\omega_c t) - \frac{\beta}{2}\{\mathrm{Cos}[(\omega_c + \omega_m)t] - \mathrm{Cos}[(\omega_c - \omega_m)t]\}$$

$\omega_c$ = 2 $\pi$ * Carrier Frequency

$\omega_m$ = 2 $\pi$ * Modulation Frequency

$\beta$ = Frequency Modulation Amplitude ( radians )
... usually << 1