Discrete Fourier Transform Pair

$$X(f_k) = X_k = \frac{1}{N}\sum_{n=0}^{N-1} x_n e^{2\pi i n k/\mathrm{N}} \qquad f_k = \frac{k}{N\Delta} = 2 f_c \frac{k}{N}, \qquad k = 0, 1, \ldots (N-1)$$

Nyquist Frequency $f_c = \dfrac{1}{2\Delta}$

$$x(t_n) = x_n = \sum_{k=0}^{N-1} X_k e^{-2\pi i n k/\mathrm{N}} \qquad t_n = n\Delta, \qquad n = 0, 1, 2, 3, \ldots (N-1)$$

Note: $X_0 = \dfrac{1}{N}\sum_{n=0}^{N-1} x_n$ $= \langle x \rangle$, the mean value of the signal x_n

Note: X_k and x_n have the same dimensions

Power Spectral Density

$$x_n = x(t_n), \qquad t_n = n\Delta, \qquad n = 0, 1, 2, 3, \ldots \text{ N-1}$$

$$X_k = X(f_k), \quad f_k = \frac{k}{N\Delta}, \qquad k = 0, 1, \ldots \frac{N}{2}$$

Variance of signal

$$\boxed{\sigma_x^2 = 2\sum_{k=1}^{\frac{N}{2}-1} \left|X_k\right|^2 + \left|X_{N/2}\right|^2}$$

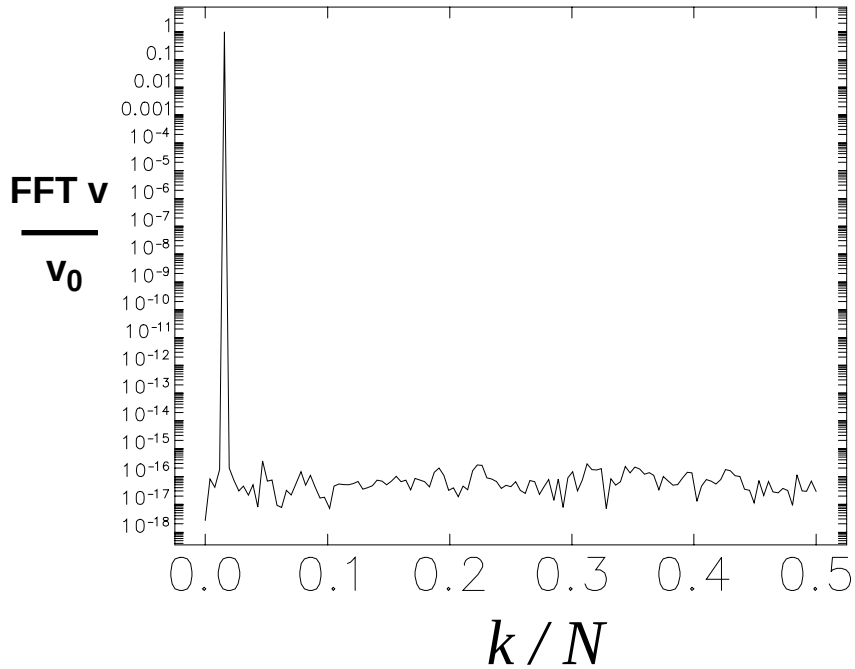
The amount of mean square signal per unit of frequency

in the frequency span from $f_k - \frac{1}{2N\Delta}$ to $f_k + \frac{1}{2N\Delta}$ is approximately

$$P(f_k) = 2\left|X_k\right|^2/(\Delta f) = 2N\Delta\left|X_k\right|^2, \qquad k = 1, 2, \ldots \frac{N}{2} - 1$$

$$= N\Delta\left|X_{N/2}\right|^2, \qquad k = \frac{N}{2}$$

Note: The power spectral density P is measured in units = (dimension of x)^2 / frequency



$$f_k = \frac{k}{N\Delta}$$

$$V_k = -\frac{\delta_{k,4}}{2i} \qquad 0 \le k \le \frac{N}{2}$$

$$\sigma_v^2 = 2\sum_{k=1}^{\frac{N}{2}-1} |V_k|^2 + |V_{N/2}|^2 = \frac{1}{2}$$

$$\sqrt{2}|V_k| = \frac{1}{\sqrt{2}}\delta_{k,4} = \text{RMS Signal}$$

Finally $\dfrac{\text{FFT v}}{\text{v}_0} = \sqrt{2} * \text{RMS Signal} = 2|V_k| = \delta_{k,4} = \text{Peak Signal}$