Problem Set 2 Tuesday June 17, 2003

Problem 1:

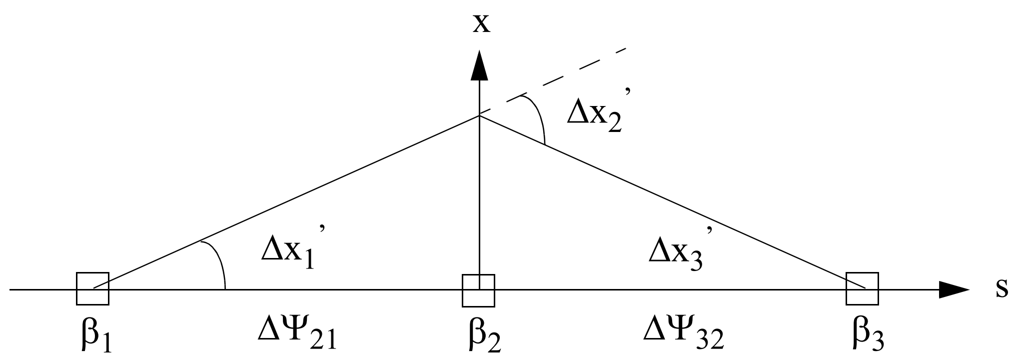
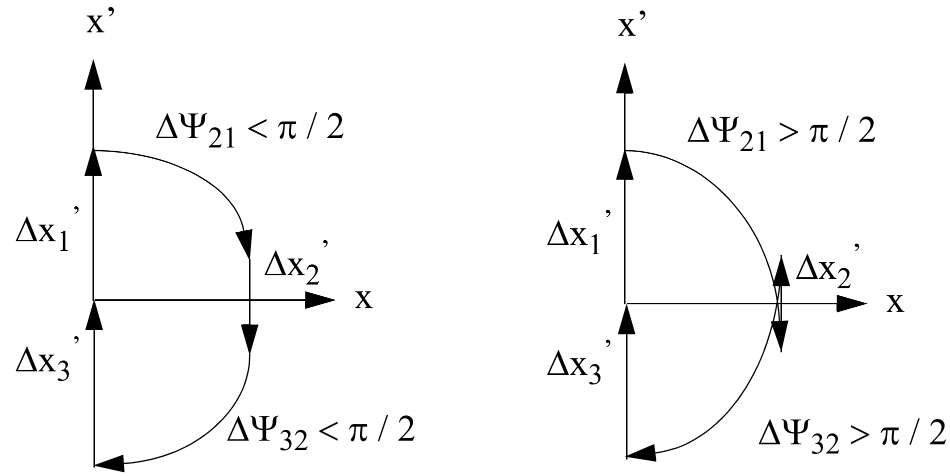


Figure for problem 1

1a) The figure shows a 3 corrector bump. Draw the beam trajectory shown in the figure in the phase space coordinate plane x, x' assuming the phase advance between correctors is $< \pi / 2$ and $> \pi / 2$. Label the axis and the relevant phase advances.



1b) Derive the bump height x_{2-} at the center of the corrector 2 in terms of the twiss parameters for correctors 1 and 2. $\Delta\Psi_{21} = \Psi_2 - \Psi_1 = -\Delta\Psi_{12}$, $\Delta\Psi_{32} = \Psi_3 - \Psi_2 = -\Delta\Psi_{23}$, $\Delta\Psi_{31} = \Psi_3 - \Psi_1 = -\Delta\Psi_{13}$.

Answer: $\mathbf{x_{2-} = \Delta x_1'(\beta_2\beta_1)^{1/2}\sin(\Delta\Psi_{21})}$.

1c) Derive the bump height x_{2+} at the center of corrector 2 in terms of the twiss parameters for correctors 2 and 3.

Answer: $\mathbf{x_{2+} = -\Delta x_3'(\beta_3\beta_2)^{1/2}\sin(-\Delta\Psi_{23}) = \Delta x_3'(\beta_3\beta_2)^{1/2}\sin(\Delta\Psi_{32})}$.

1d) Derive a formula for the required kick $\Delta x_2'$ for the middle corrector to close the bump in terms of the kicks $\mathbf{\Delta x_1'}$ and $\mathbf{\Delta x_3'}$ and twiss parameters shown in the figure. Simplify your formula as much as possible.

Answer: $\mathbf{\Delta x_2' = dx_{2-}/ds - dx_{2+}/ds \,|_{(s = s2)}}$

$\mathbf{dx_{2-}/ds} \mid_{(s = s2)} = \mathbf{(\Delta x_1'/ 2)\ (d\beta(s) / ds)\ (\beta_1/ \beta(s))^{1/2} sin(\Psi(s) - \Psi_1)} \mid_{(s = s2)} + \mathbf{\Delta x_1'(\beta_1\beta(s))^{1/2}\ cos(\Psi(s) - \Psi_1)\ (d\Psi(s) / ds)} \mid_{(s = s2)} =$

$-\alpha_2(\beta_1/ \beta_2)^{1/2}\mathbf{sin}(\Delta\Psi_{21})\ \Delta \mathbf{x_1}' + (\beta_1/ \beta_2)^{1/2}\mathbf{cos}(\Delta\Psi_{21})\Delta \mathbf{x_1}'$

$\mathbf{dx_{2+}/ds} \mid_{(s = s2)} = \mathbf{(\Delta x_3'/ 2)\ (d\beta(s) / ds)\ (\beta_3/ \beta(s))^{1/2} sin(\Psi_3 - \Psi(s))} \mid_{(s = s2)} + \mathbf{-\Delta x_3'(\beta_3\beta(s))^{1/2}\ cos(\Psi_3 - \Psi(s))\ (-d\Psi(s) / ds)} \mid_{(s = s2)} =$

$-\alpha_2(\beta_3/ \beta_2)^{1/2}\mathbf{sin}(\Delta\Psi_{32})\Delta \mathbf{x_3}' - (\beta_3/ \beta_2)^{1/2}\mathbf{cos}(\Delta\Psi_{32})\Delta \mathbf{x_3}'$

$\mathbf{x_{2+} = x_{2-} = x_2} \Rightarrow \Delta \mathbf{x_1}'\beta_1{}^{1/2}\mathbf{sin}(\Delta\Psi_{21}) = \Delta \mathbf{x_3}'\beta_3{}^{1/2}\mathbf{sin}(\Delta\Psi_{32})$ **(Use this to eliminate $\Delta x_3'$ in the previous equations and solve for $\Delta x_2'$. The terms with α_2 cancel.)**

$\Delta \mathbf{x_2}' = (\beta_1/ \beta_2)^{1/2}\mathbf{cos}(\Delta\Psi_{21})\Delta \mathbf{x_1}' + (\beta_3/ \beta_2)^{1/2}\mathbf{cos}(\Delta\Psi_{32})\Delta \mathbf{x_3}'$.

1e) Use your answers to 1b through 1d to derive equations relating the required corrector kicks to each other to keep the bump closed. Simplify your formula as much as possible.

Answer: Combine equations derived in 1a and 1b.

$\Delta \mathbf{x_1}'\beta_1{}^{1/2}\mathbf{sin}(\Delta\Psi_{21}) = \Delta \mathbf{x_3}'\beta_3{}^{1/2}\mathbf{sin}(\Delta\Psi_{32})$ **(Eliminate the kick $\Delta x_3'$)**

$\Delta \mathbf{x_2}' = \Delta \mathbf{x_1}'(\beta_1/\beta_2)^{1/2}\mathbf{cos}(\Delta\Psi_{21}) + \Delta \mathbf{x_3}'(\beta_3/\beta_2)^{1/2}\mathbf{cos}(\Delta\Psi_{32}) = \Delta \mathbf{x_1}'(\beta_1/\beta_2)^{1/2}\mathbf{cos}(\Delta\Psi_{21}) + (\Delta \mathbf{x_1}'\beta_1{}^{1/2}\mathbf{sin}(\Delta\Psi_{21})\ \mathbf{cos}(\Delta\Psi_{32})) / (\beta_2{}^{1/2}\mathbf{sin}(\Delta\Psi_{32}))$.

$\sin(\Delta\Psi_{32}) \cos(\Delta\Psi_{21}) + \cos(\Delta\Psi_{32}) \sin(\Delta\Psi_{21}) = \sin(\Delta\Psi_{31}) = \sin(\Psi_3 - \Psi_1)$.

Rearrange and simplify:

$\Delta x_1' \beta_1^{1/2} / \sin(\Delta\Psi_{32}) = \Delta x_2' \beta_2^{1/2} / \sin(\Delta\Psi_{31})$.

Similarly, relating $\Delta x_2'$ to $\Delta x_3'$:

$\Delta x_2' \beta_2^{1/2} / \sin(\Delta\Psi_{31}) = \Delta x_3' \beta_3^{1/2} / \sin(\Delta\Psi_{21})$.

1f) Numerical example: Assume $\beta_1 = \beta_2 = \beta_3$, and $\Delta\Psi_{12} = \Delta\Psi_{23} = \pi / 4$, derive the corrector kicks using the result from 1d.

Answer: $\Delta x_1' = \Delta x_3'$, $\Delta x_1' = \Delta x_2' / (2 \cos(\Delta\Psi_{23})) = \Delta x_2' / (2)^{1/2} = 0.707\ \Delta x_2'$.

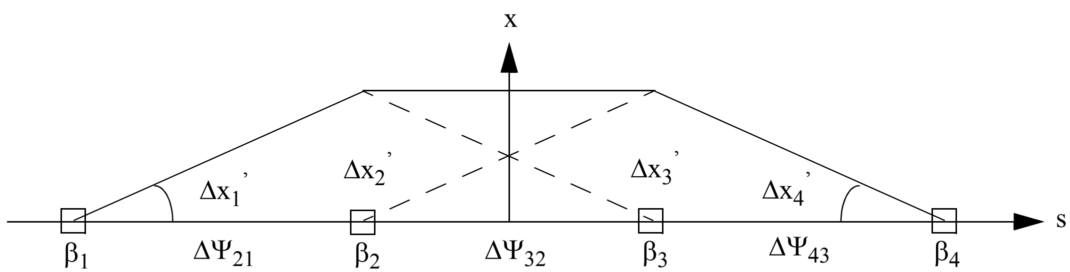
1g) Numerical example: Assume $\beta_1 = \beta_2 = \beta_3$, $\Delta\Psi_{12} = \Delta\Psi_{23} / 10$ and $\Delta\Psi_{23} = 5\pi / 11$, derive the corrector kicks using the result from 1d.

Answer: $\Delta x_1' = \Delta x_2' \sin(\Delta\Psi_{23}) = 0.990\ \Delta x_2'$, $\Delta x_1' = \Delta x_3' \sin(\Delta\Psi_{23}) / \sin(\Delta\Psi_{12}) = 6.96\ \Delta x_3'$.

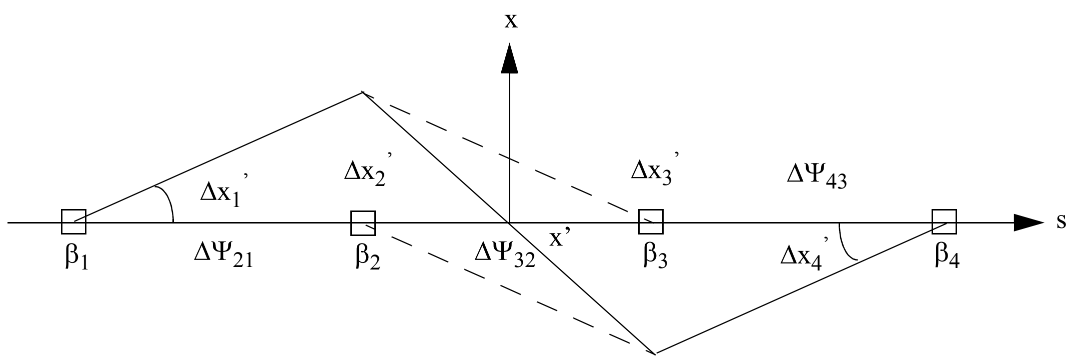
1h) Which of the 3 corrector bumps (1f or 1g) is more difficult to realize in practice and why?

Answer: 1g. Correctors 1 and 2 in 1g need to be 7 times as strong for a given corrector 3 kick. Corrector 3 also needs to be regulated an order of magnitude better than correctors 1 and 2 so excessive AC orbit noise is not generated from bump closure errors.

Problem 2:



Symmetric 4 corrector bump



Antisymmetric 4 corrector bump

Figure for problem 2

2a) The figure shows a 4 corrector bump constructed of two overlapping 3 corrector bumps. In this problem we want to construct a symmetric bump as shown in the figure and an antisymmetric bump. For a symmetric bump, the corrector constraints are $\Delta x_1' = \Delta x_4'$ and $\Delta x_2' = \Delta x_3'$. For the antisymmetric bump, $\Delta x_1' = -\Delta x_4'$ and $\Delta x_2' = -\Delta x_3'$. Assume the twiss parameter constraints are $\beta_1 = \beta_4$, $\beta_2 = \beta_3$ and $\Delta\Psi_{21} = \Delta\Psi_{43}$ and the space between correctors 2 and 3 is a drift. Derive the bump height x at the center of the bump for the symmetric bump and the angle x' at the center of the bump for the antisymmetric bump in terms of the kicks and twiss parameters.

Answer: Symmetric bump height x at bump midpoint (from 3 corrector bump problem).

$$\mathbf{x = x_2 = \Delta x_1'}(\beta_2\beta_1)^{1/2}\mathbf{sin}(\Delta\Psi_{12}) = \mathbf{x_3 = \Delta x_4'}(\beta_3\beta_4)^{1/2}\mathbf{sin}(\Delta\Psi_{34}).$$

Antisymmetric bump angle $\Delta x'$ at bump midpoint.

$$\mathbf{x'} = -2\Delta\mathbf{x_{31}'} = -2\Delta\mathbf{x_1'}\beta_1^{1/2}\mathbf{sin}(\Delta\Psi_{21}) / (\beta_3^{1/2}\ \mathbf{sin}(\Delta\Psi_{32}))$$

$$\mathbf{x'} = 2\Delta\mathbf{x_{22}} = 2\Delta\mathbf{x_4'}\beta_4^{1/2}\mathbf{sin}(\Delta\Psi_{43}) / (\beta_2^{1/2}\ \mathbf{sin}(\Delta\Psi_{32})) = \mathbf{-2\Delta x_{31}'}$$

2b) Numerical example. Compare original APS (symmetric) injection bump with small phase advance to actual implementation. For the original APS injection bump assume $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 20$ m, $\Delta\Psi_{12} = \Delta\Psi_{34} = 0.1$ radians. The existing injection bump has $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 20$ m, $\Delta\Psi_{12} = \Delta\Psi_{34} = 5$ radians. Derive the ratio of corrector strengths between the two cases for the same bump height x.

Answer: $(\Delta \mathbf{x}_{\mathbf{original}}' / \Delta\mathbf{x}_{\mathbf{actual}}')_{\mathbf{1,2,3,4}}$ **= sin(5) / sin(0.1) = -9.60.**

So, the original idea with small phase advance would require kickers about 10 times stronger than required for the existing bump.