Problem Set 1 Monday June 16, 2003

Problem 1:

1a) Calculate the total charge stored and maximum stored energy of a 7 GeV, 100 mA beam in a storage ring with a 3.68 μs revolution period (The APS ring nominal operating condition).

Answer: $Q = (0.1A)(3.68 \times 10^{-6}s) = 3.68 \times 10^{-7}$ Coul = 368 nC.

$E = (7 \times 10^{9}\ eV)(1.6 \times 10^{-19} J/eV)(0.1\ A)/(1.6 \times 10^{-19} Coul/e^{-})(3.68 \times 10^{-6}s) = 2576\ J$.

1b) Calculate the average power of the beam delivers in (1a) assuming it is dumped in one revolution period.

Answer: $P = (2576\ J) / (3.68 \times 10\text{-}6s) = 700 \times 10^{6}$ Watts = 700 MW.

1c) Calculate the average power the beam delivers in (1a) assuming it is slowly scraped in 100000 revolution periods.

Answer: P = (0.007 MW = 7 kW).

1d) Calculate the average power of a 7 GeV, 100 mA Energy Recover Linac (ERL) beam.

Answer: P = (700 MW).

1e) Which beam is more damaging to accelerator components if it is missteered?

Answer: The ERL beam since it delivers 700 MW of beam power to the components CW.

Problem 2:

2a) Calculate the total photon power (in kW) for a 7 GeV, 100 mA beam passing through a storage ring bending magnet source with a 30 m bend radius.

Answer: $P_{tot} = (C_\gamma E^4 I) / \rho = (8.8575 \times 10^{-5}$ **m /GeV^3 x 7 GeV^4 x 0.1 A) / 30 m = 0.709 MW = 709 kW.**

2b) Assuming the dipole is L = 3 m long calculate the power per unit bend angle for the dipole source in 2a.

Answer: $\sin(\theta/2) = L/2\rho = (3\ \mathrm{m} / 2 \times 30\ \mathrm{m}) \Rightarrow \theta = 0.1$ **radian.** $dP/d\theta = P_{tot} / \theta =$ **7090 kW/rad = 7 kW/mrad.**

2c) Calculate the critical energy for the bending magnet source in 2a.

Answer: $\varepsilon_c = 2.218\ E^3/\rho =$ **25.36 keV.**

2d) Estimate the spectral function $S(\omega/\omega_c)$ at the critical frequency using the low and high frequency approximations to $S(\omega/\omega_c)$.

Answer: $S(1) \sim (1.333 + 0.777/e) / 2 = 0.809$

2e) Estimate the power per unit frequency for the bending magnet source in 2a at the critical frequency ($\hbar = 6.582 \times 10^{-19}$ keV s).

Answer: $dP/d\omega = (709\ \mathrm{kW}) / (25.36\ \mathrm{keV} / 6.582 \times 10^{-19}\ \mathrm{keV\ s})\ S(1) = 1.489 \times 10^{-17}$ **kW/Hz**

Problem 3:

3a) Calculate B_{max} and K for the APS undulator A which has λ_{ID}= 3.3 cm and g = 5 mm.

Answer: B_{max} **= 33.3 exp[-(g /**λ_{ID}**)(5.47-1.8(g /**λ_{ID}**))] = 15.15 kG**

K = 0.934 B_{max} λ_{ID} **= (0.934) (1.515 T) (3.3 cm) = 4.67**

3b) Calculate the total photon power for a 7 GeV, 100 mA beam passing through undulator A (3a) assuming N = 70.

Answer: P_{tot} **= (7.26) (7** GeV^2**) (0.1 A) (70) (**4.67^2**) / (3.3 cm) = 1.646 kW.**

3c) Calculate the undulator rms angular divergence for the first harmonic for undulator A (assume 7 GeV, 100 mA and K = 0.1).

Answer: $\sigma' = (1 + 0.1^2/2)^{1/2}/((7e6 \text{ keV} / 0.511 \text{ keV})(70)^{1/2}) = 8.747 \times 10^{-6}$ **radians.**

3d) Calculate the power in the first harmonic per unit solid angle for the parameters listed in 3c.

.**Answer:** $dP/d\Omega = (7.26)\ (7\ GeV^2)\ (0.1\ A)\ (70)\ (0.1^2) / ((3.3\ cm)\ (2\ \pi)\ (8.747 \times 10^{-3} mrad)^2 = 1.57 \times 10^4$ **kW/ mrad²**.

3e) Calculate the first harmonic power per unit area using the answer in 3c and 3d at 50 m.

Answer: $dP/dA = (dP/dW) / (50 \times 10^3\ mm)^2 = 6.28 \times 10^{-6}\ kW/mm^2 = 6.28\ kW/m^2$.

Problem 4:

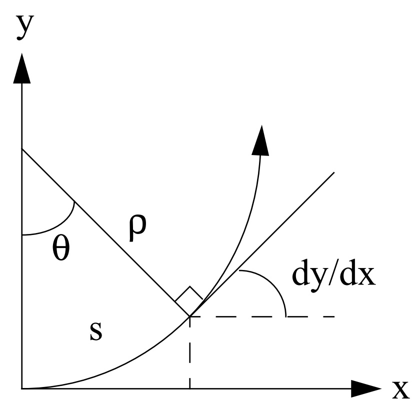

Figure problem 4

4a) Consider a charged particle moving in a constant magnetic field perpendicular to this page (see the figure). Write an expression for the bend angle θ in terms of the arc length parameter s and the radius of curvature ρ.

Answer: $\theta = s / \rho$.

4b) Write the rectangular coordinates of the point (s,ρ) in terms of s and ρ.

Answer: $x = \rho \sin(s/\rho)$, $y = \rho\,(1 - \cos(s/\rho))$.

4c) Derive an expression for the slope of the tangent to the charged particle path dy/dx in terms of s and ρ.

Answer: $dy/dx = (dy/ds)\,(ds/dx)$ **(chain rule)** $= \sin(s/\rho) / \cos(s/\rho) = \tan(s/\rho)$.

4d) Write to 3rd order in (s / ρ) the difference between dy/dx and θ.

Answer: $dy/dx - \theta = (s / \rho)^3 / 3$.

4e) What is the maximum angle θ such that approximating the angle of the particle trajectory with respect to the x axis $\Delta x' \sim dy/dx \sim \theta$ is good to 1 %?

Answer: $(dy/dx - \theta) / \theta = (s / \rho)^2 / 3 = \theta^2 / 3 = 0.01 \Rightarrow \theta = 0.173$ **radians = 173 mrad.**

Problem 5:

Figure for problem 5

5a) The figure shows a particle traveling a distance s through a simple drift space. Derive an expression for the minimum beta function β^* and the point s^* where the minimum occurs in terms of the initial twiss parameters. Which twiss parameter is constant in the drift space?

Answer: $d\beta/ds = 0 = -2\alpha_o + 2s^*\gamma_o \Rightarrow s^* = \alpha_o / \gamma_o$.

$\beta^* = \beta_o - 2s^*\alpha_o + s^{*2}\gamma_o = 1 / \gamma_o = 1 / \gamma$.

5b) Write the final twiss parameters α, β, γ and Ψ in terms of s, s^* and β^*.

Answer: $\alpha = \alpha_o - s\gamma_o = s^*\gamma_o - s\gamma_o = -\gamma_o(s - s^*) = -(s - s^*) / \beta^*$.

$\beta = \beta_o - 2s\alpha_o + s^2\gamma_o = \beta_o - 2ss^*\gamma_o + s^2\gamma_o = \beta_o + \gamma_o(s - s^*)^2 - s^{*2}\gamma_o =$

$(s - s^*)^2 / \beta^* + \beta_o - \alpha_o^2 / \gamma_o = \beta^* + (s - s^*)^2 / \beta^*.$

$\gamma = \gamma_o = 1 / \beta^*$

5c) Assuming $\alpha_o = 0$ (ie. $s^* = 0$ and $\beta_o = \beta^*$), derive an expression for the phase advance at s in terms of β^*.

Answer: $\Psi = \int_0^s \frac{ds'}{\beta(s')} = \tan^{-1}(s / \beta^*)$

5d) Assuming $\alpha_o = 0$, derive an expression for x at s (ie. x(s)) in terms of x' using the transfer matrix parameterized in terms of twiss functions ($M_{0\text{->}s}$).

Answer: $x = (\beta/\beta_o)^{1/2}\cos\Psi + (\beta_o\beta)^{1/2}\sin\Psi =$

$((\beta^* + s^2 / \beta^*) / \beta^*)^{1/2}(\beta^*/(\beta^{*2} + s^2)^{1/2})x_o +$

$(\beta^*(\beta^* + s^2/\beta^*))^{1/2}(s / (\beta^{*2} + s^2)^{1/2})x' = x_o + s\, x'$

(ie. the hard way to derive the equation of a straight line!)