Problem Set 2 Tuesday June 17, 2003

Problem 1:

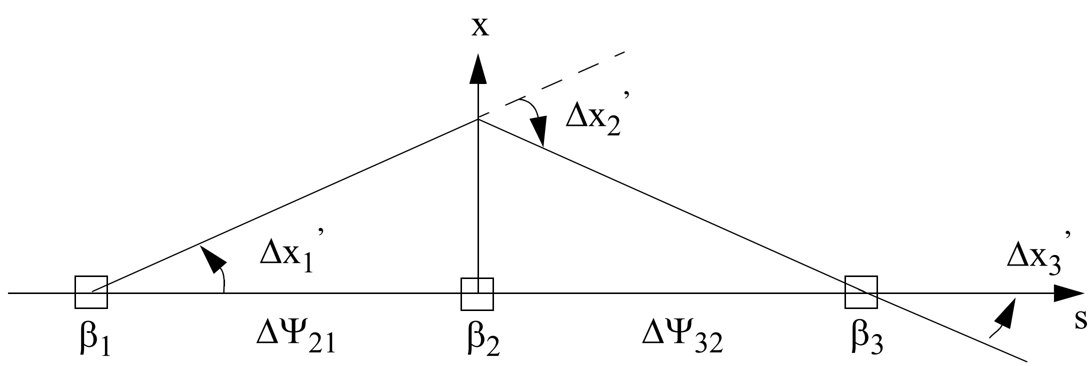


Figure for problem 1

$$\Delta\Psi_{21} = \Psi_2 - \Psi_1 = -\Delta\Psi_{12}\,,\ \Delta\Psi_{32} = \Psi_3 - \Psi_2 = -\Delta\Psi_{23}\,,\ \Delta\Psi_{31} = \Psi_3 - \Psi_1 = -\Delta\Psi_{13}.$$

1a) The figure shows a 3 corrector bump. Draw the beam trajectory shown in the figure in the phase space coordinate plane x, x' assuming the phase advance between correctors is $< \pi / 2$ and $> \pi / 2$. Label the axis and the relevant phase advances.

1b) Derive the bump height x_{2-} at the center of the corrector 2 in terms of the beta functions β_1, β_2 associated with correctors 1 and 2, the phase advance $\Delta\Psi_{21}$, and the corrector strength $\Delta x_1'$.

1c) Derive the bump height x_{2+} at the center of corrector 2 in terms of the beta functions β_2 , β_3 associated with correctors 2 and 3, the phase advance $\Delta\Psi_{32}$, and the corrector strength Δx_3'.

1d) After equating the results obtained in 1b) and 1c) to obtain the bump ratio $\Delta \mathbf{x_3}'$ / $\Delta \mathbf{x}_1$' , derive a formula for the required kick Δx_2' for the middle corrector to close the bump in terms of the kicks $\Delta \mathbf{x}_1$' and $\Delta \mathbf{x_3}$' and beta functions shown in the figure. Simplify your formula as much as possible.

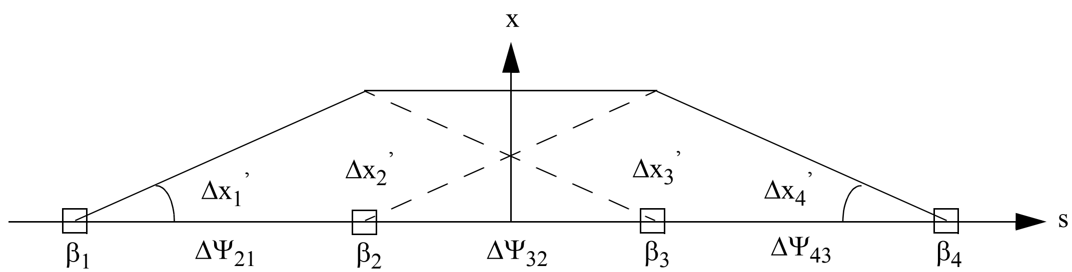
1e) Use your answers to 1b through 1d to derive the bump ratios $\Delta \mathbf{x}_1$'/ $\Delta \mathbf{x}_2$' , $\Delta \mathbf{x}_2$'/ $\Delta \mathbf{x}_3$' , $\Delta \mathbf{x}_1$'/ $\Delta \mathbf{x}_3$' relating the required corrector kicks to each other to keep the bump closed. Simplify your formula as much as possible.

1f) Numerical example: Assume $\beta_1 = \beta_2 = \beta_3$, and $\Delta\Psi_{12} = \Delta\Psi_{23} = \pi / 4$, derive the bump ratios using the result from 1d.

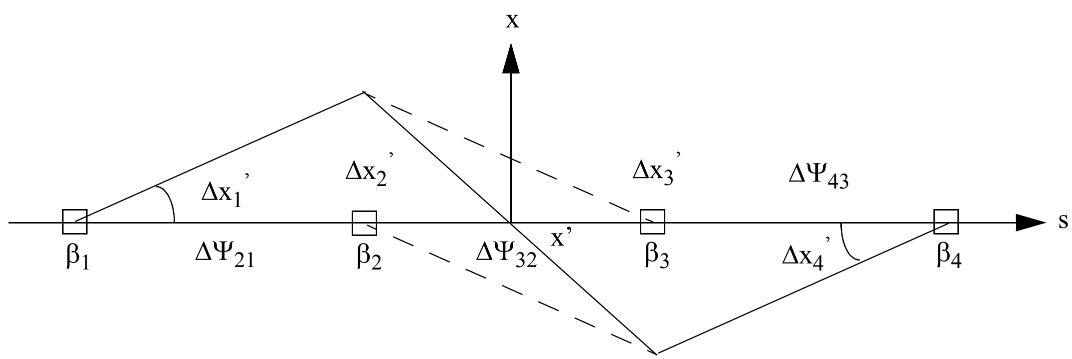
1g) Numerical example: Assume $\beta_1 = \beta_2 = \beta_3$, $\Delta\Psi_{12} = \Delta\Psi_{23} / 10$ and $\Delta\Psi_{23} = 5\pi / 11$, derive the bump ratios using the result from 1d.

1h) Which of the 3 corrector bumps (1f or 1g) is more difficult to realize in practice and why?

Problem 2



Symmetric 4 corrector bump



Antisymmetric 4 corrector bump

Figure for problem 2

2a) The figure shows a 4 corrector bump constructed of two overlapping 3 corrector bumps. In this problem we want to construct a symmetric bump as shown in the figure and an antisymmetric bump. For a symmetric bump, the corrector constraints are $\Delta x_1' = \Delta x_4'$ and $\Delta x_2' = \Delta x_3'$. For the antisymmetric bump, $\Delta x_1' = -\Delta x_4'$ and $\Delta x_2' = -\Delta x_3'$. Assume the twiss parameter constraints are $\beta_1 = \beta_4$, $\beta_2 = \beta_3$ and $\Delta\Psi_{21} = \Delta\Psi_{43}$ and the space between correctors 2 and 3 is a drift space. Derive the bump amplitude x at the center of the bump for the symmetric bump and the angle x' at the center of the bump for the antisymmetric bump in terms of the kick $\Delta x_1'$ beta functions β_1, β_2, and relevant phase advances.

2b) Numerical example. Compare original APS (symmetric) injection bump with small phase advance to actual implementation. For the original APS injection bump assume $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 20$ m, $\Delta\Psi_{12} = \Delta\Psi_{34} = 0.1$ radians. The existing injection bump has $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 20$ m, $\Delta\Psi_{12} = \Delta\Psi_{34} = 5$ radians. Derive the ratio of corrector strengths between the two cases for the same bump amplitude x.