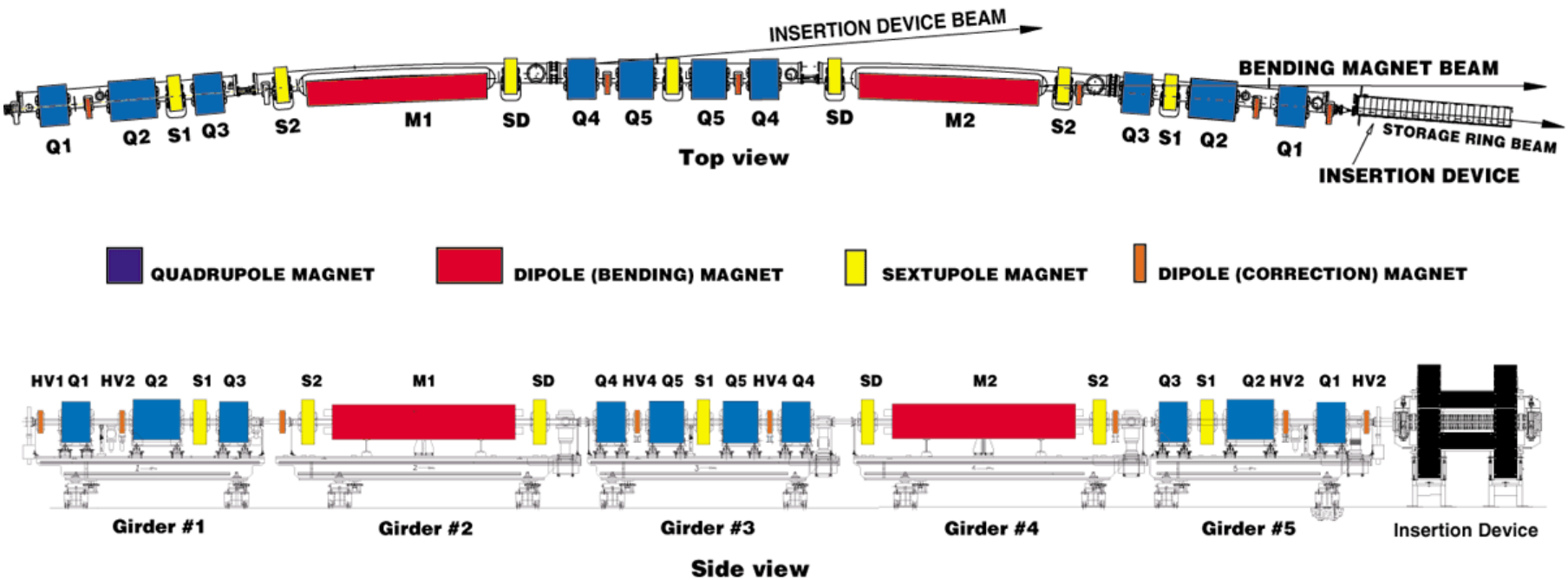
Essentials of Accelerator Physics



One of the 40 Sectors of the Advanced Photon Source

Essentials of Accelerator Physics

Single particle horizontal displacement relative to “the” equilibrium closed orbit:

$$x(s) = [W_x\, \beta_x(s)]^{1/2} \cos[\psi_x(s) - \psi_{0x}]$$

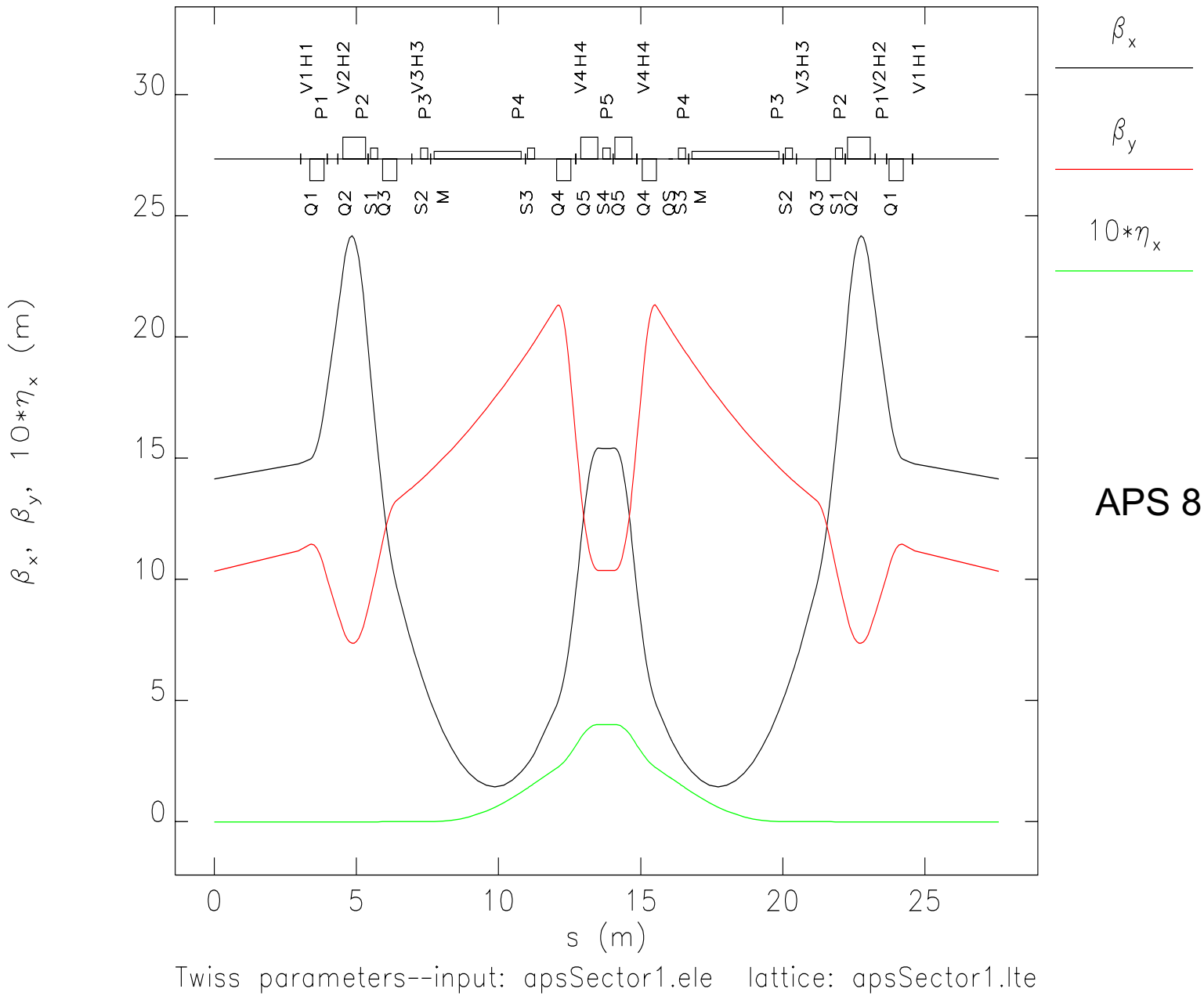
Single particle vertical displacement relative to “the” equilibrium closed orbit:

$$y(s) = [W_y\, \beta_y(s)]^{1/2} \cos[\psi_y(s) - \psi_{0y}]$$

Displacement of the equilibrium closed orbit for an off-energy particle:

$$\Delta x_E(s) = \eta(s)\, \frac{\Delta E}{E_0} = \eta(s)\, \delta$$

Double Bend Achromat Lattice - also known as Chasman-Green Lattice



APS 8.2 nm-rad

Essentials of Accelerator Physics

Twiss Parameters $\alpha(s)$, $\beta(s)$, $\gamma(s)$

$$\alpha(s) = - \beta'(s) / 2 \qquad \gamma(s) = \frac{1 + \alpha^2(s)}{\beta(s)}$$

All Twiss parameters are periodic functions:

$$\beta(s+L) = \beta(s)$$
$$\alpha(s+L) = \alpha(s)$$
$$\gamma(s+L) = \gamma(s)$$

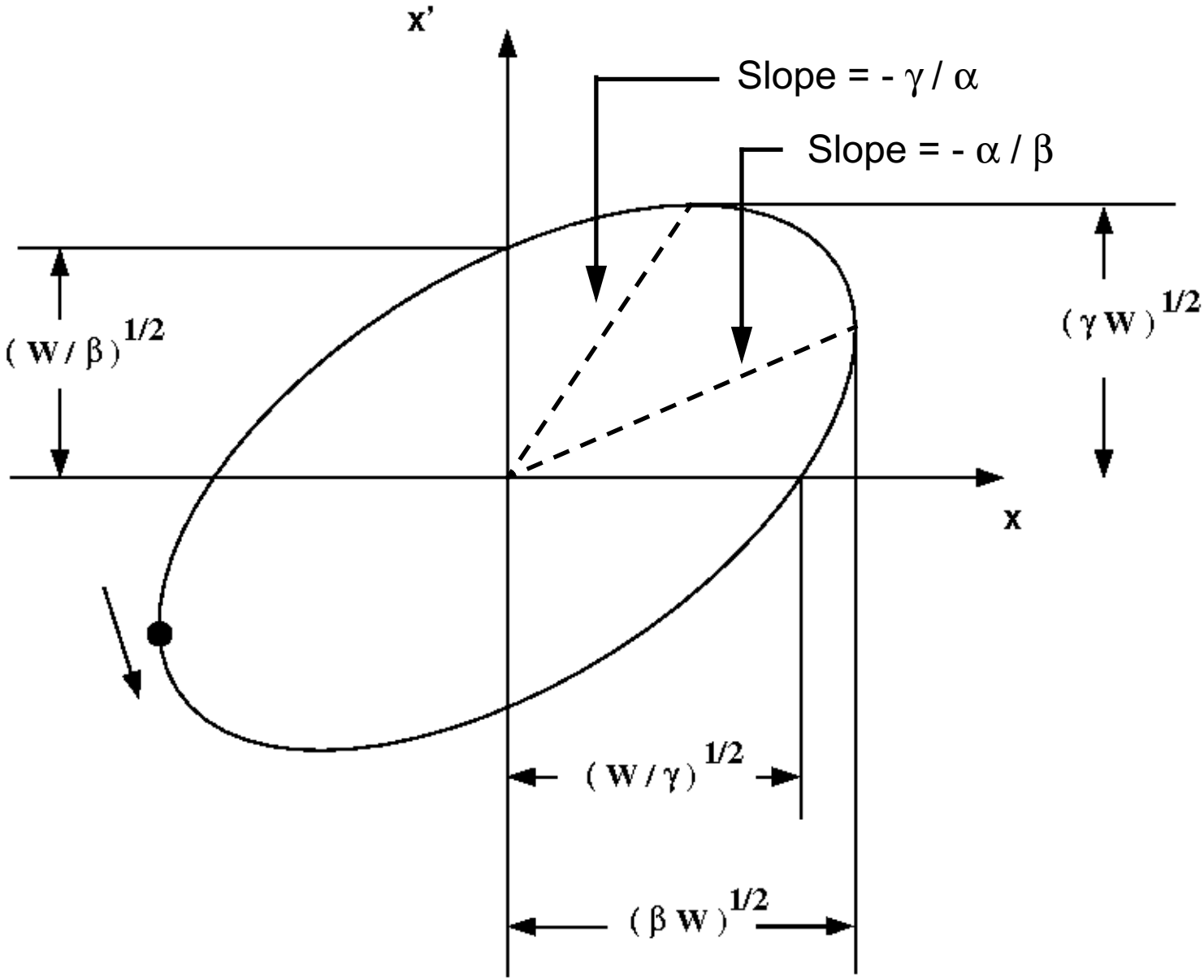
L = Ring "Circumference" = c T_{rev}

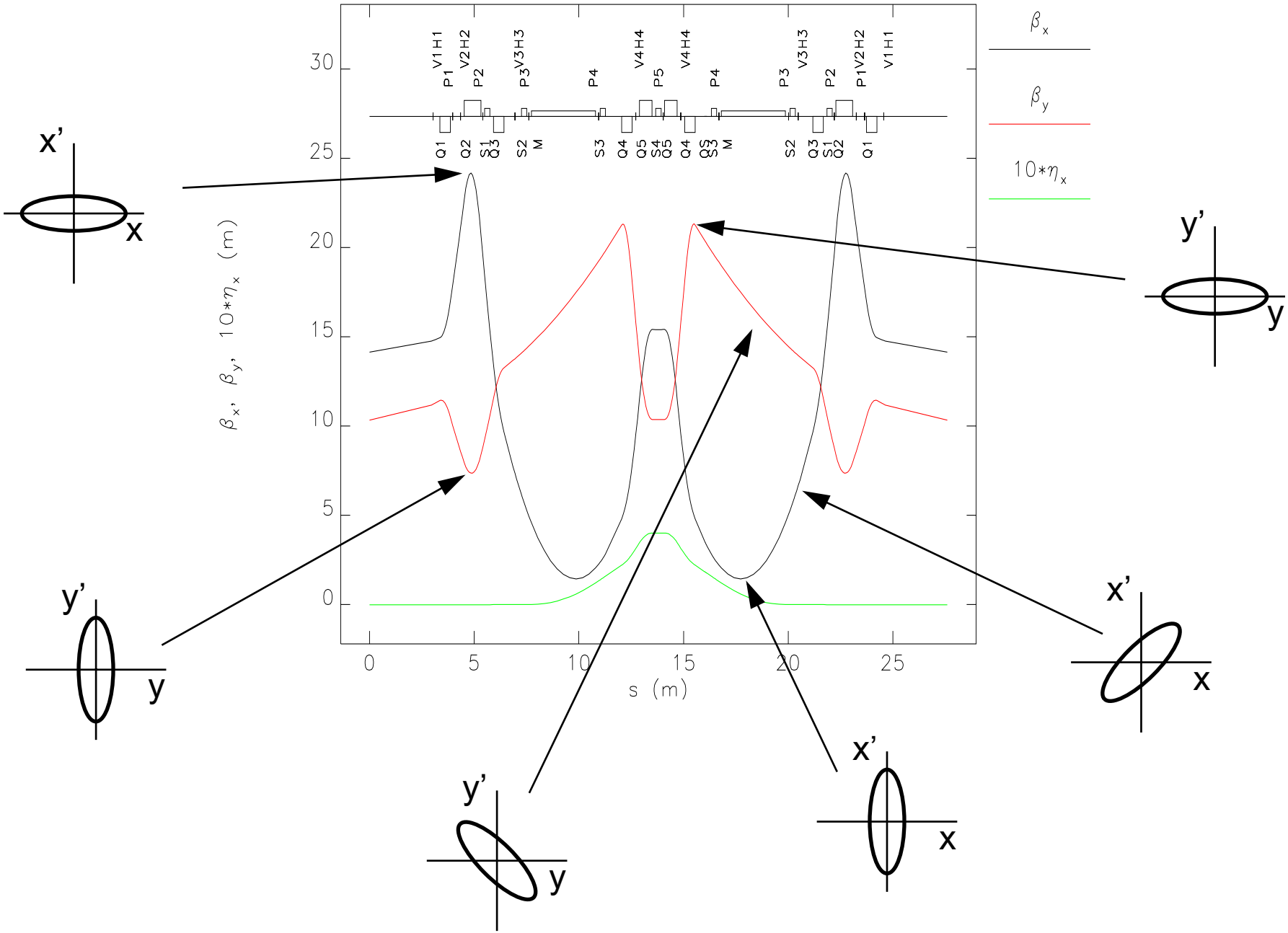
Courant-Snyder Invariant* $W_x = \gamma_x(s)\, x^2(s) + 2\, \alpha_x(s)\, x(s)\, x'(s) + \beta_x(s)\, x'(s)^2$

$$= [\, x^2 + (\, \alpha_x\, x + \beta_x\, x')^2 \,] / \beta_x$$

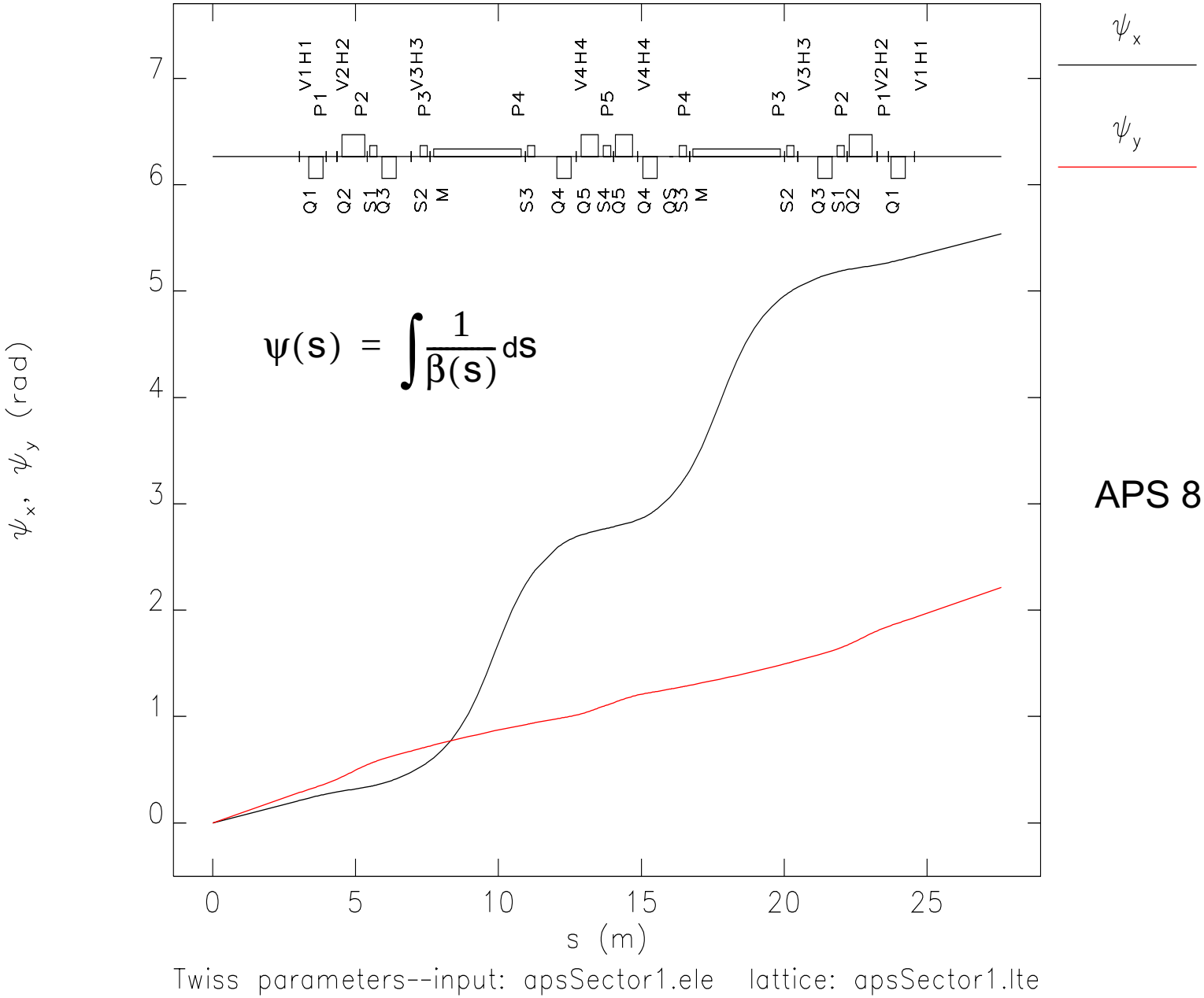
* E.Courant, H.S. Snyder, Annals of Physics 3, 1-48 (1958)

Particle Beam Phase Space




V1H1
V2H2
P1
P2
P3V3H3
P4
V4H4
P5
V4H4
P4
P3
V3H3
P2
P1V2H2
V1H1
Q1
Q2
S1
Q3
S2
M
S3
Q4
Q5
S4
Q5
Q4
Q3
S3
M
S2
Q3
S1
Q2
Q1
β_x
β_y
$10*\eta_x$
β_x, β_y, $10*\eta_x$ (m)
s (m)
0
5
10
15
20
25
30
x'
x
y'
y

Horizontal, Vertical Betatron Phases



Twiss parameters--input: apsSector1.ele lattice: apsSector1.lte

APS 8.2 nm-rad

Betatron Tunes

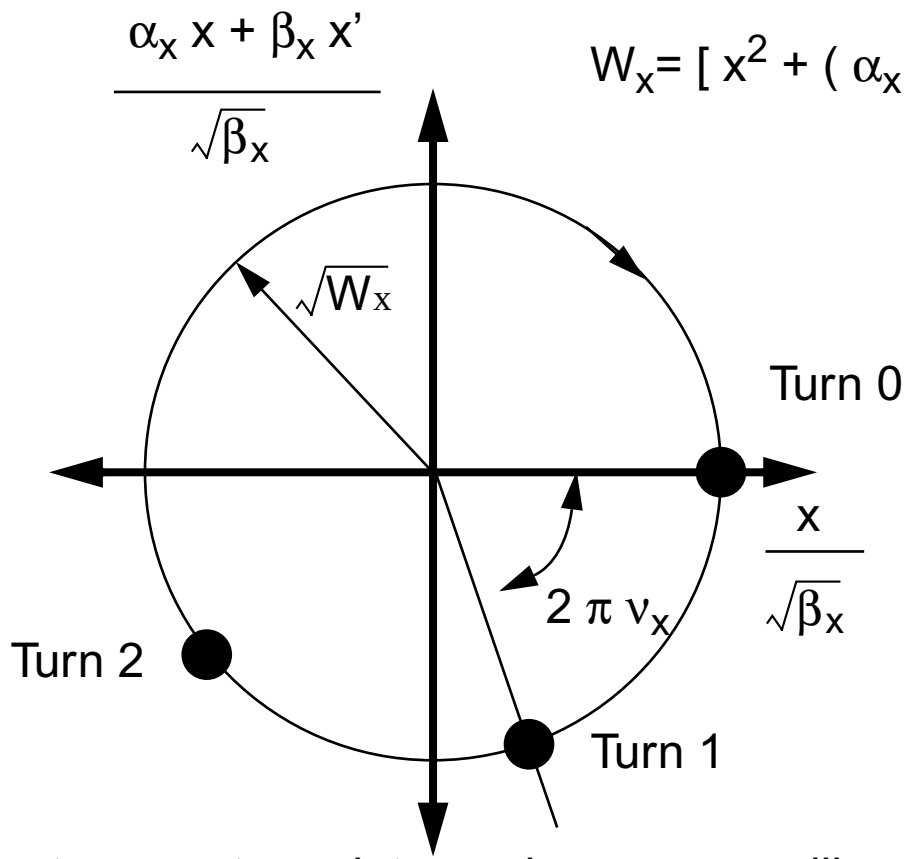
The betatron tune ν is the betatron phase advance in one circuit around the machine, divided by 2π, i.e. it is the number of betatron wavelengths once around the machine:

$$2\pi\nu_x = \psi_x(s+L) - \psi_x(s) = \int_s^{s+L} \frac{1}{\beta_x(s)} ds$$

$$2\pi\nu_y = \psi_y(s+L) - \psi_y(s) = \int_s^{s+L} \frac{1}{\beta_y(s)} ds$$

Normalized Electron Phase Space at Fixed Location s

$$W_x = [\, x^2 + (\alpha_x x + \beta_x x')^2 \,] / \beta_x$$

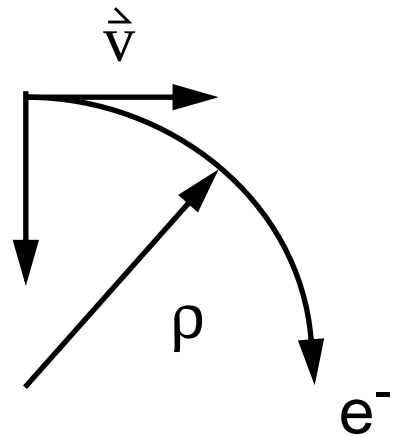


$\frac{d\, \beta_x(s)}{d\, s} = -2\, \alpha_x = 0$ at symmetry points -> phase space ellipse is upright, so

$$\frac{\alpha_x x + \beta_x x'}{\sqrt{\beta_x}} \longrightarrow \sqrt{\beta_x}\, x'$$

Charged Particle Moving in a Magnetic Field B

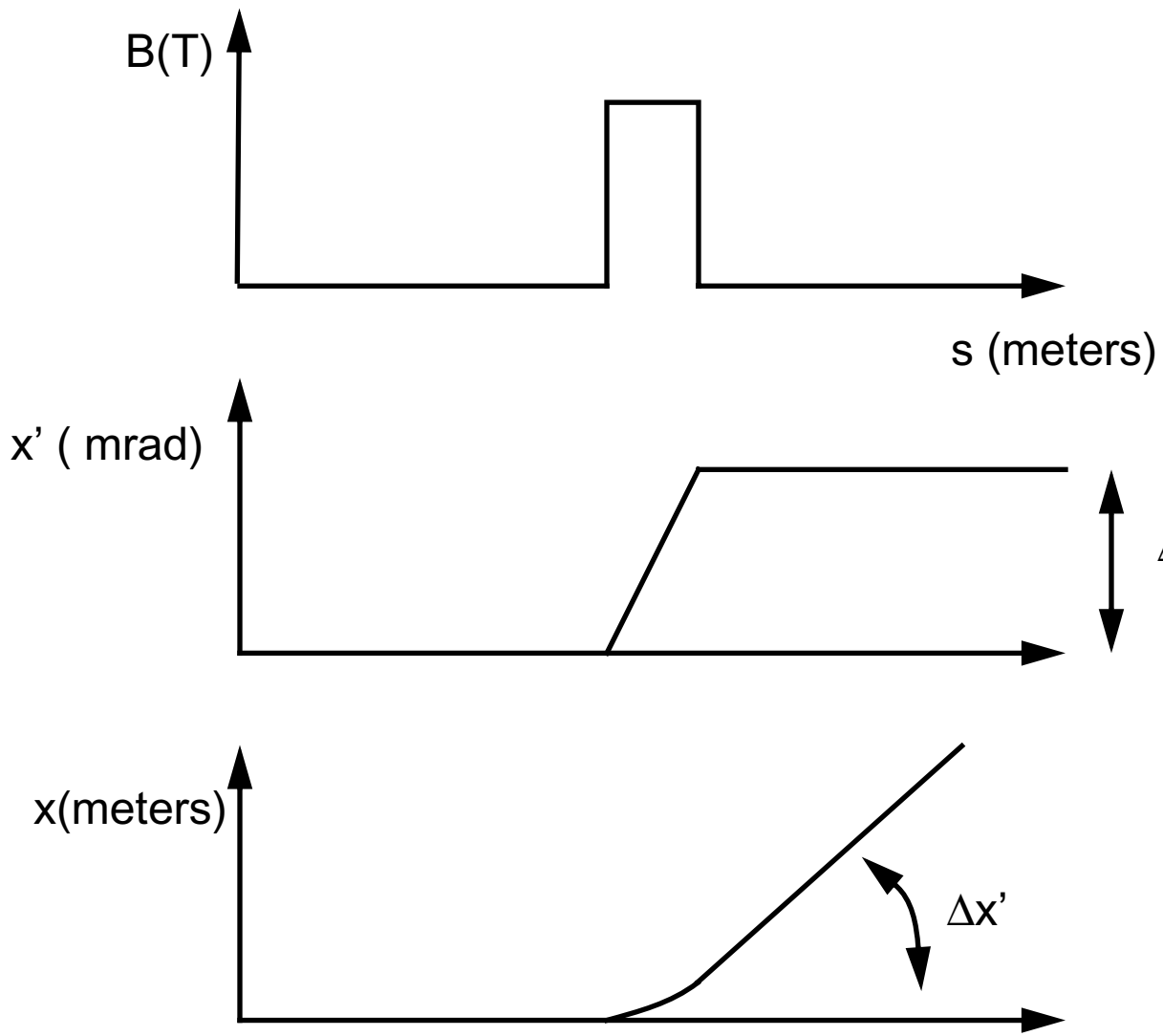
$$\vec{F} = -e\vec{v} \times \vec{B}$$



$$B(T)\ \rho\ (m) = 3.335641\ E\ (GeV)$$

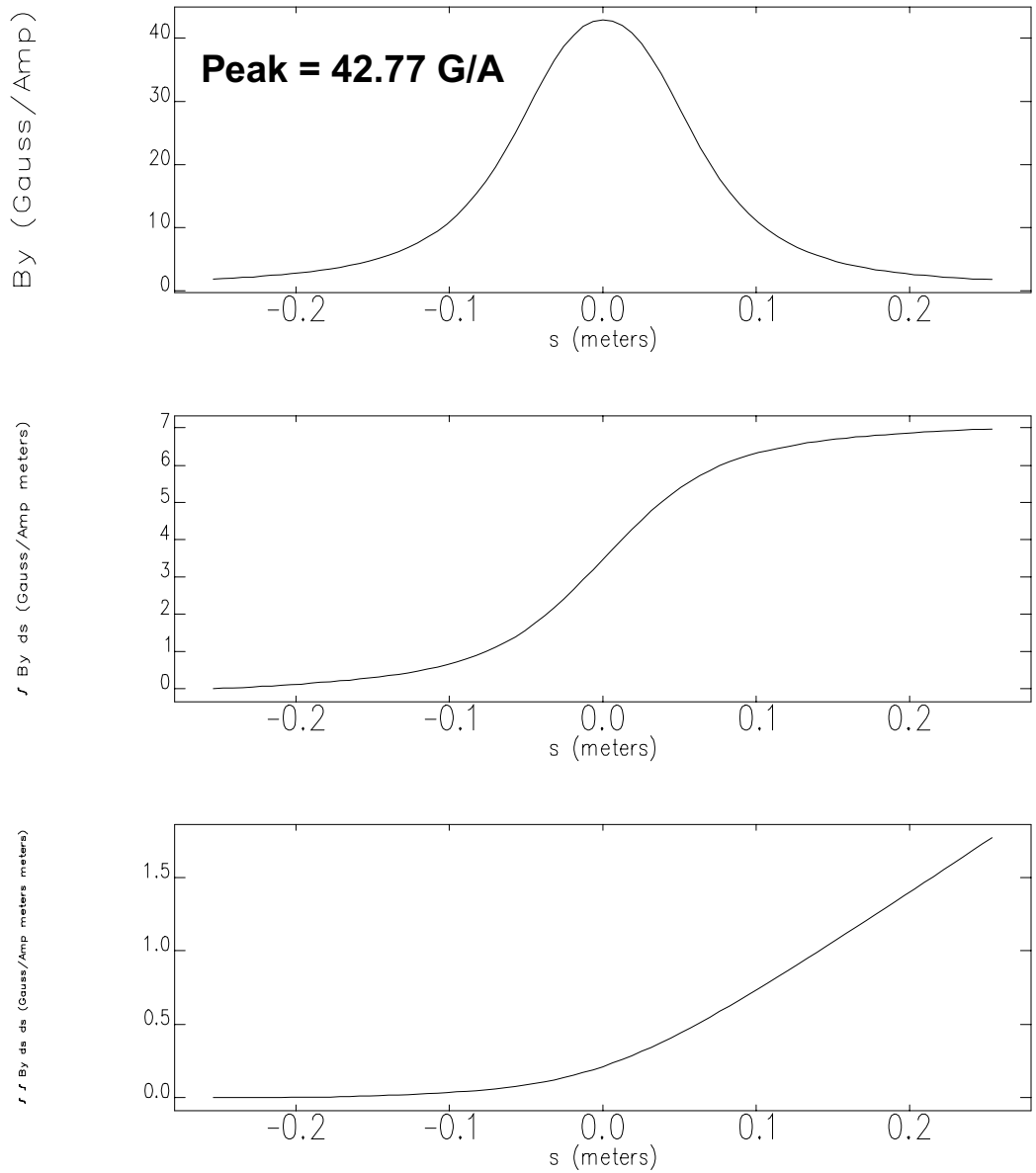
(magnetic field pointing into page)

Hard-edge dipole steering corrector
First and Second Magnetic Field Integrals, in Sensible Units



$$\Delta x' = \frac{1}{B\rho}\int B(s)\,ds$$

Actual Magnet Measurement Data, APS Linac Steering Corrector



Gauss / Amp

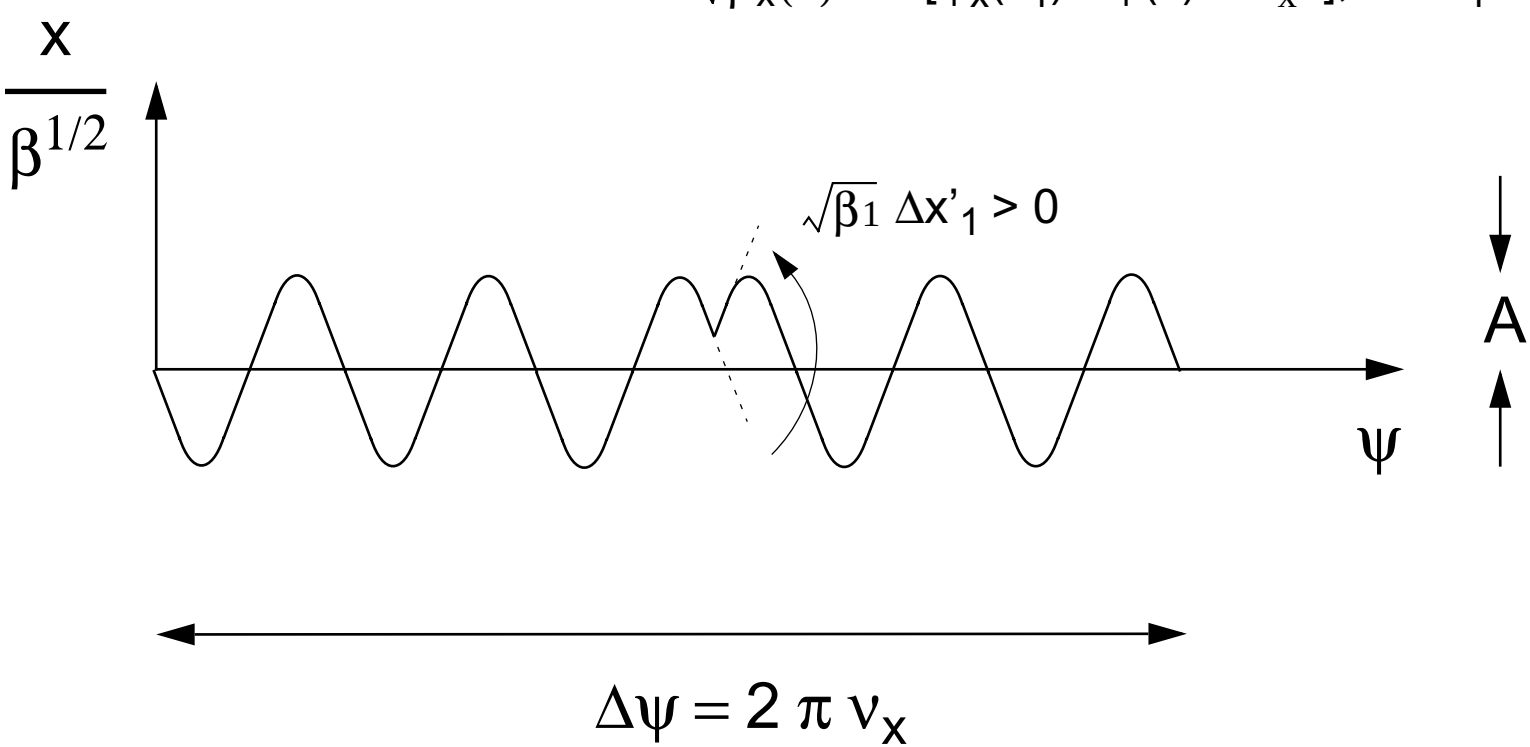
Gauss-meters / Amp

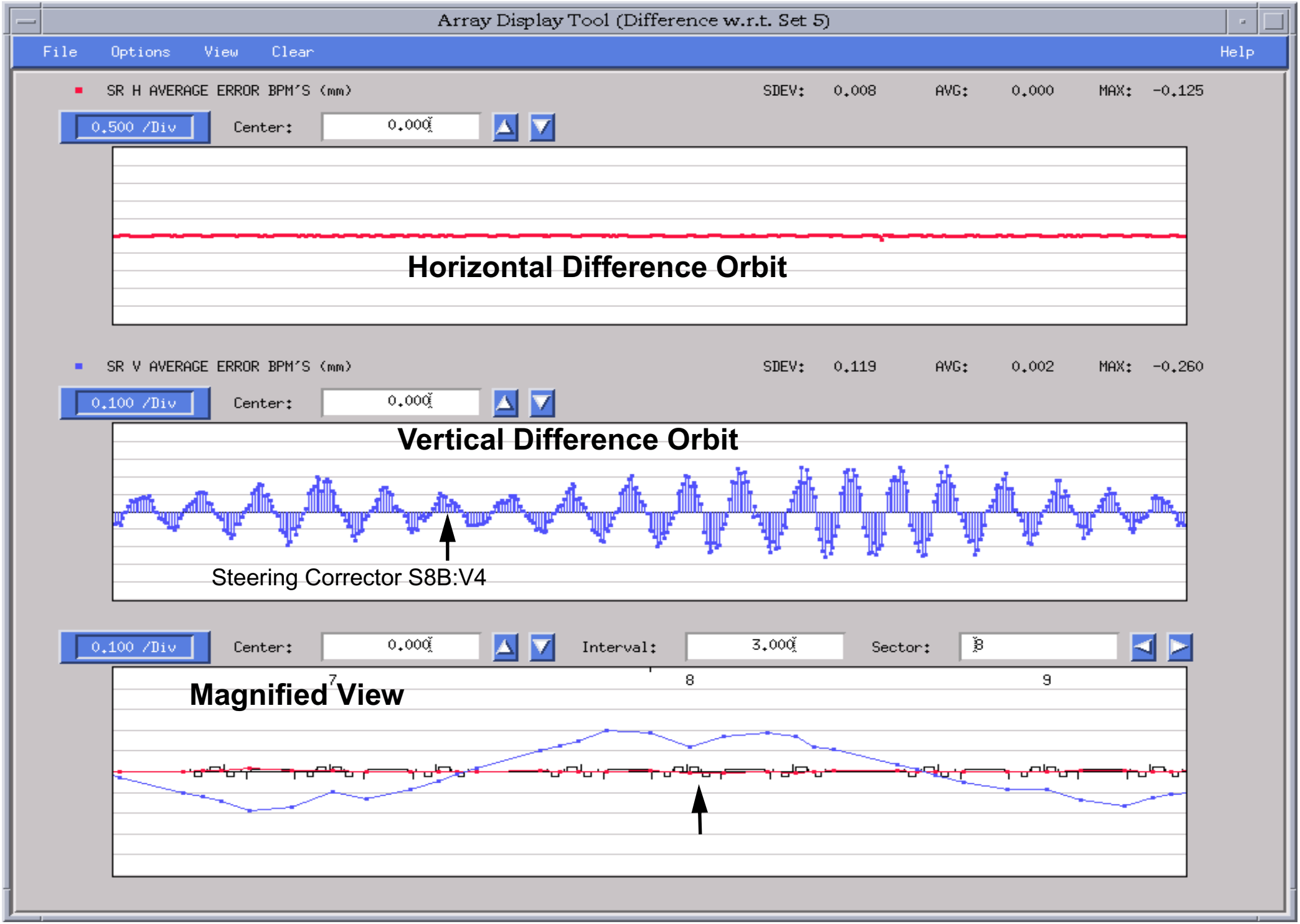
Gauss-meters^2 / Amp

Closed Orbit Distortion Resulting from Single Corrector Change $\Delta x'_1$

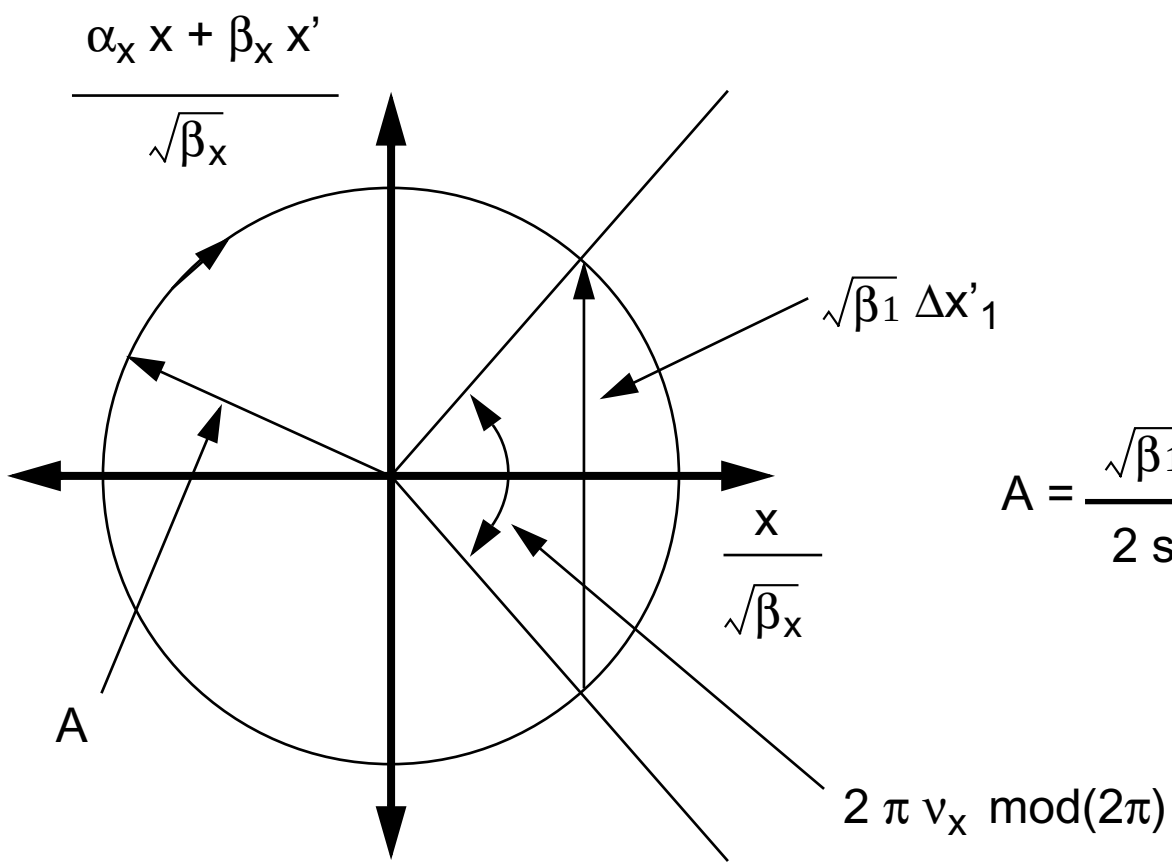
$$x(s) = A\sqrt{\beta_x(s)}\cos[\psi_x(s) - \psi(s_1) + \nu_x\pi]\ ;\ s < s_1$$

$$= A\sqrt{\beta_x(s)}\cos[\psi_x(s_1) - \psi(s) + \nu_x\pi];\ s > s_1$$




Array Display Tool (Difference w.r.t. Set 5)
File
Options
View
Clear
Help
SR H AVERAGE ERROR BPM'S (mm)
SDEV: 0.008
AVG: 0.000
MAX: -0.125
0.500 /Div
Center: 0.000
Horizontal Difference Orbit
SR V AVERAGE ERROR BPM'S (mm)
SDEV: 0.119
AVG: 0.002
MAX: -0.260
0.100 /Div
Center: 0.000
Vertical Difference Orbit
Steering Corrector S8B:V4
0.100 /Div
Center: 0.000
Interval: 3.000
Sector: 8
7
8
9
Magnified View

Closed Orbit Distortion Resulting from Single Corrector Change $\Delta x'_1$ Phase Space Representation



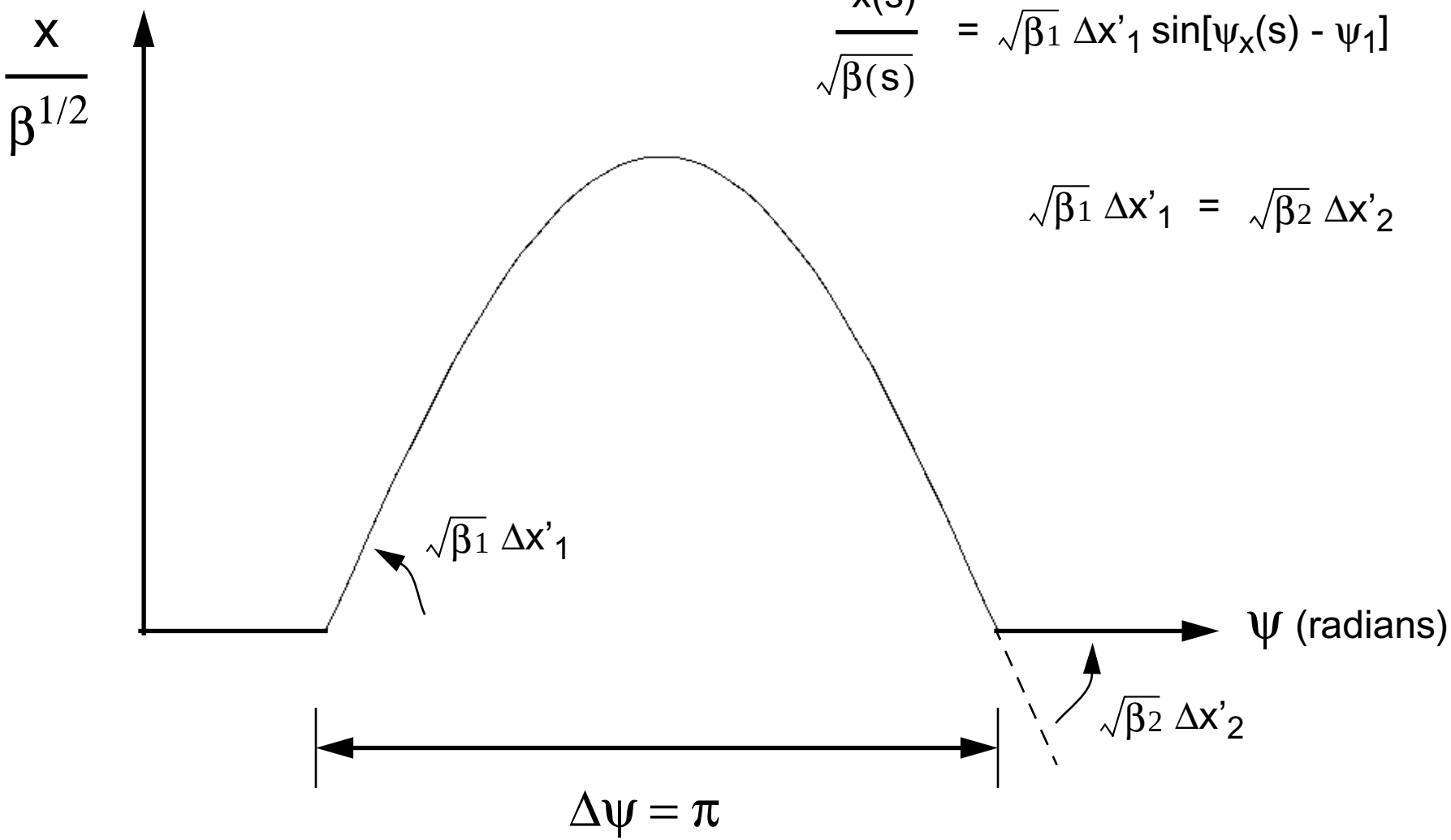
$$A = \frac{\sqrt{\beta_1}\,\Delta x'_1}{2\sin\pi\nu_x}$$

$$x(s) = \frac{\sqrt{\beta_x(s)\beta_1}\,\Delta x_1'}{2\sin\pi\nu_x}\cos[\psi_x(s) - \psi_x(s_1) + \nu_x\pi] \quad s < s_1 \text{ etc.}$$

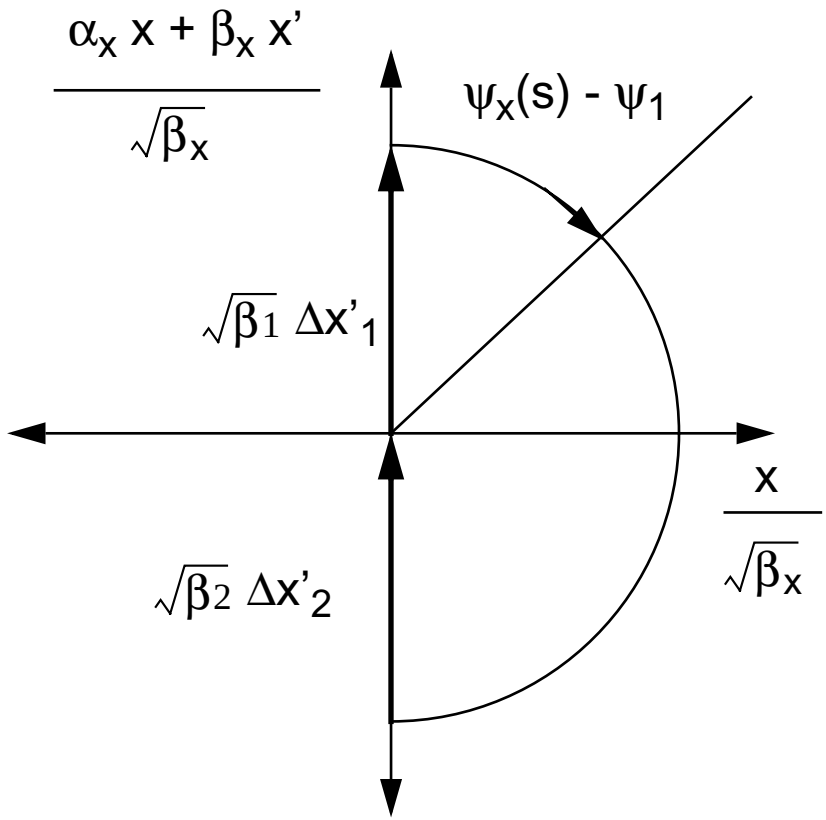
Closed Orbit Half-Wave Bump - Two Correctors

$$\frac{x(s)}{\sqrt{\beta(s)}} = \sqrt{\beta_1}\,\Delta x'_1 \sin[\psi_x(s) - \psi_1]$$

$$\sqrt{\beta_1}\,\Delta x'_1 = \sqrt{\beta_2}\,\Delta x'_2$$

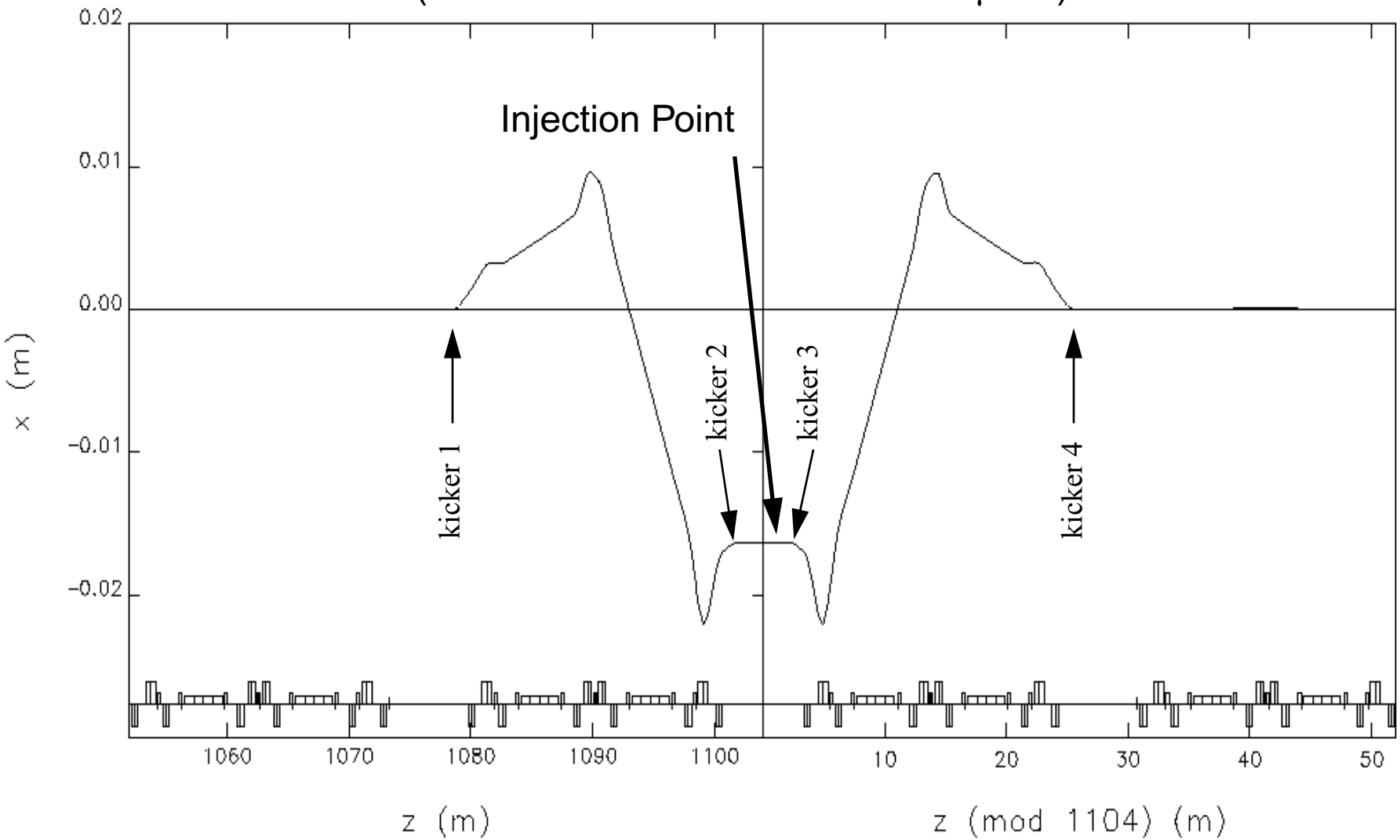


Phase Space Representation of Two-Corrector Half-Wave Bump



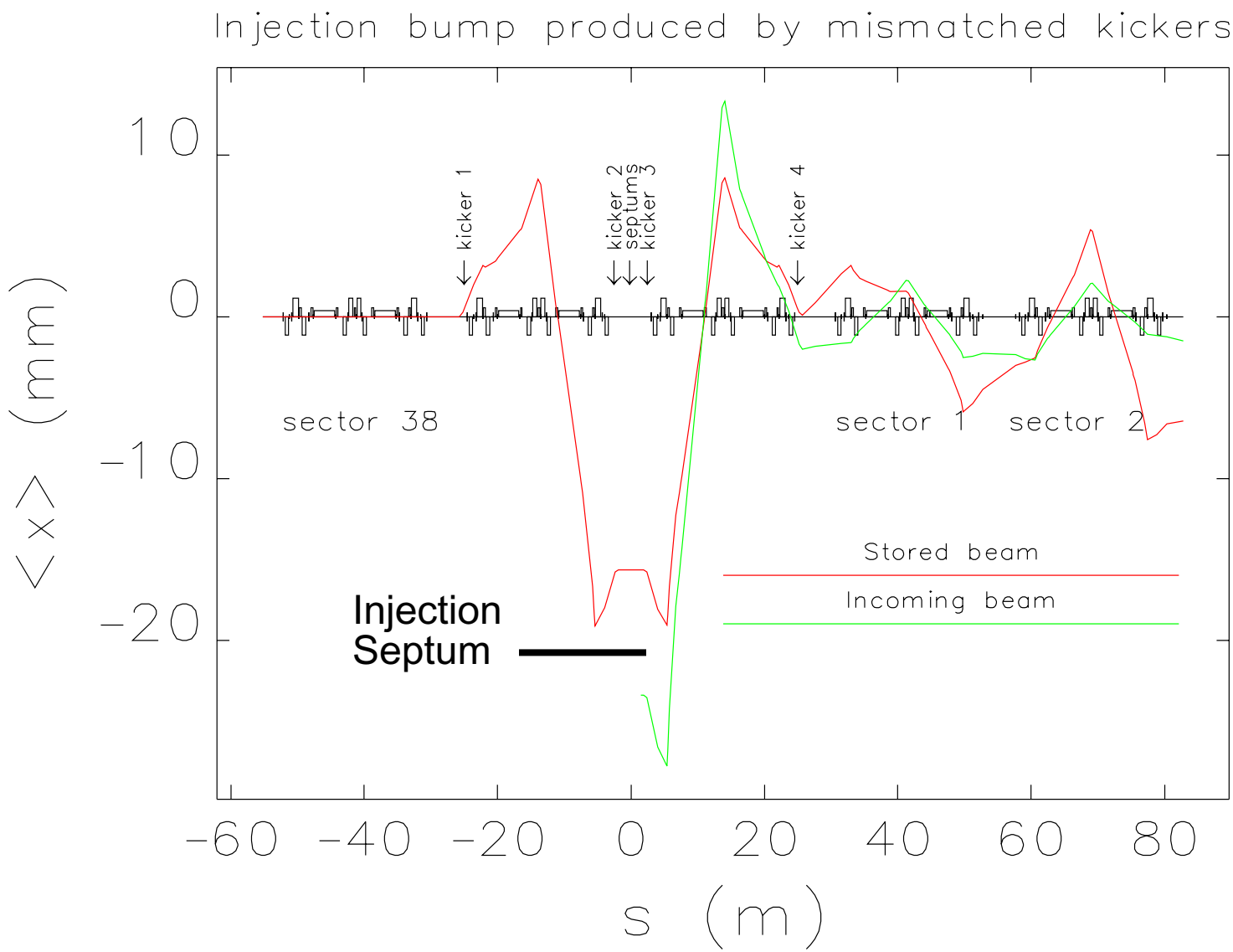
$$x_{max} = \sqrt{\beta_0 \beta_1}\,\Delta x'_1$$

APS Injection Closed Orbit Bump as Designed
(Pulsed - Full Width < 2 turns = 7 μsec)



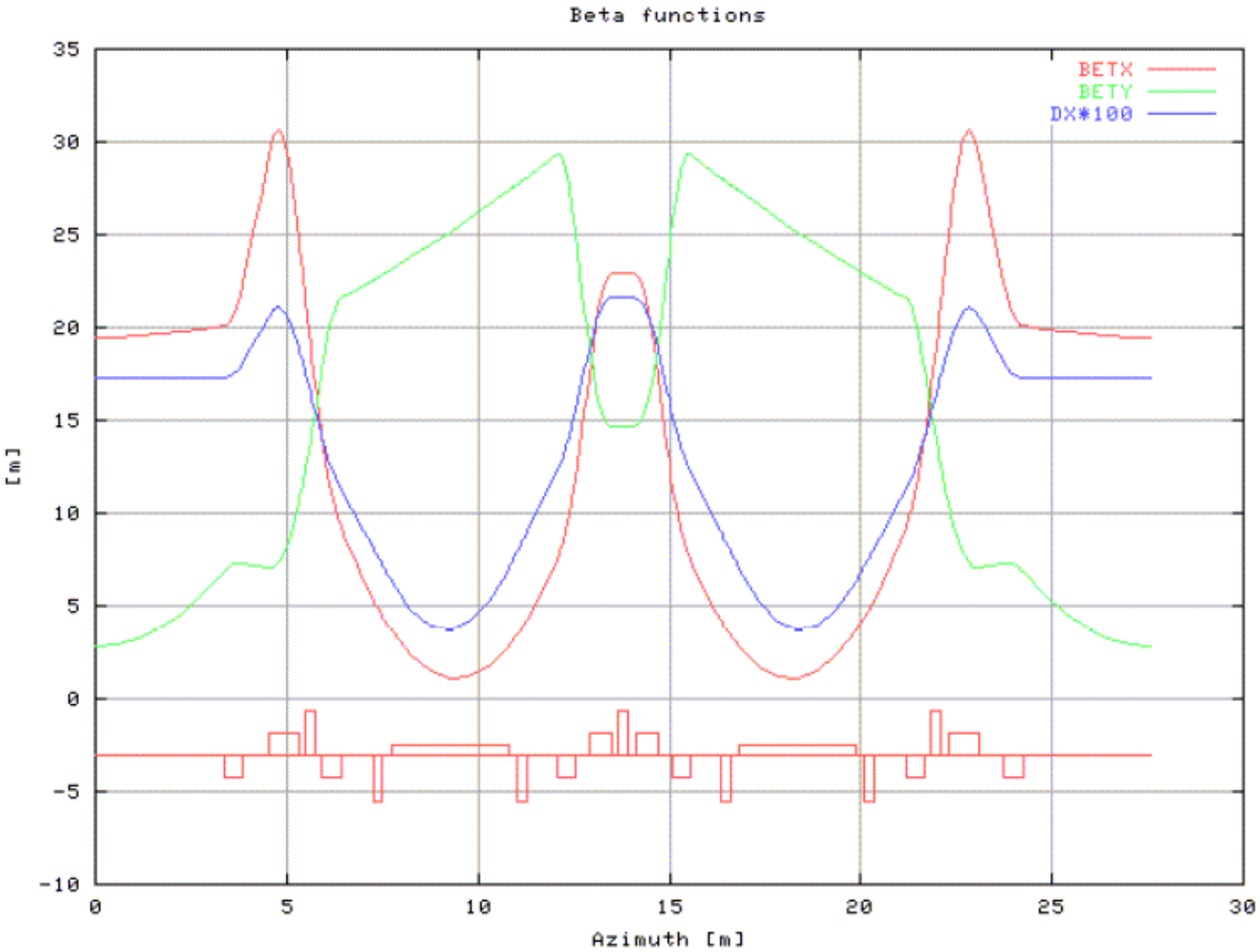
Maximum orbit bump during injection

APS Injection Orbit Bump in Practice (not closed)



APS lattice functions

2.4 nm×rad lattice, one sector



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