Digital Receivers for BPM Processing

USPAS

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1 Outline

- Desired functions of BPM processor
- Beam signal spectrum
- Digital receiver
  - Principles of operation
  - Numerical accuracy
- SPEAR 3 implementation design
- FNAL proposed implementation

2 BPM Processor Functions

- Accurately and quickly measure beam orbit
  - $\approx 1\,\mu\mathrm{m}$ out of $1\,\mathrm{cm} \Rightarrow 80\,\mathrm{dB}$ resolution
  - $\approx 200\,\mathrm{Hz}$ feedback bandwidth $\Rightarrow \approx 1\,\mathrm{kHz}$ acquistion rate
- Measure dynamics of beam
  - "First-turn" injection monitoring
  - Orbits of bunches in different sections of the bucket train
  - Machine tunes
    - ν_s (longitudinal synchrotron tune)
    - ν_x, ν_y (transverse betatron tunes)

3 Beam Signal Spectrum of Storage Ring

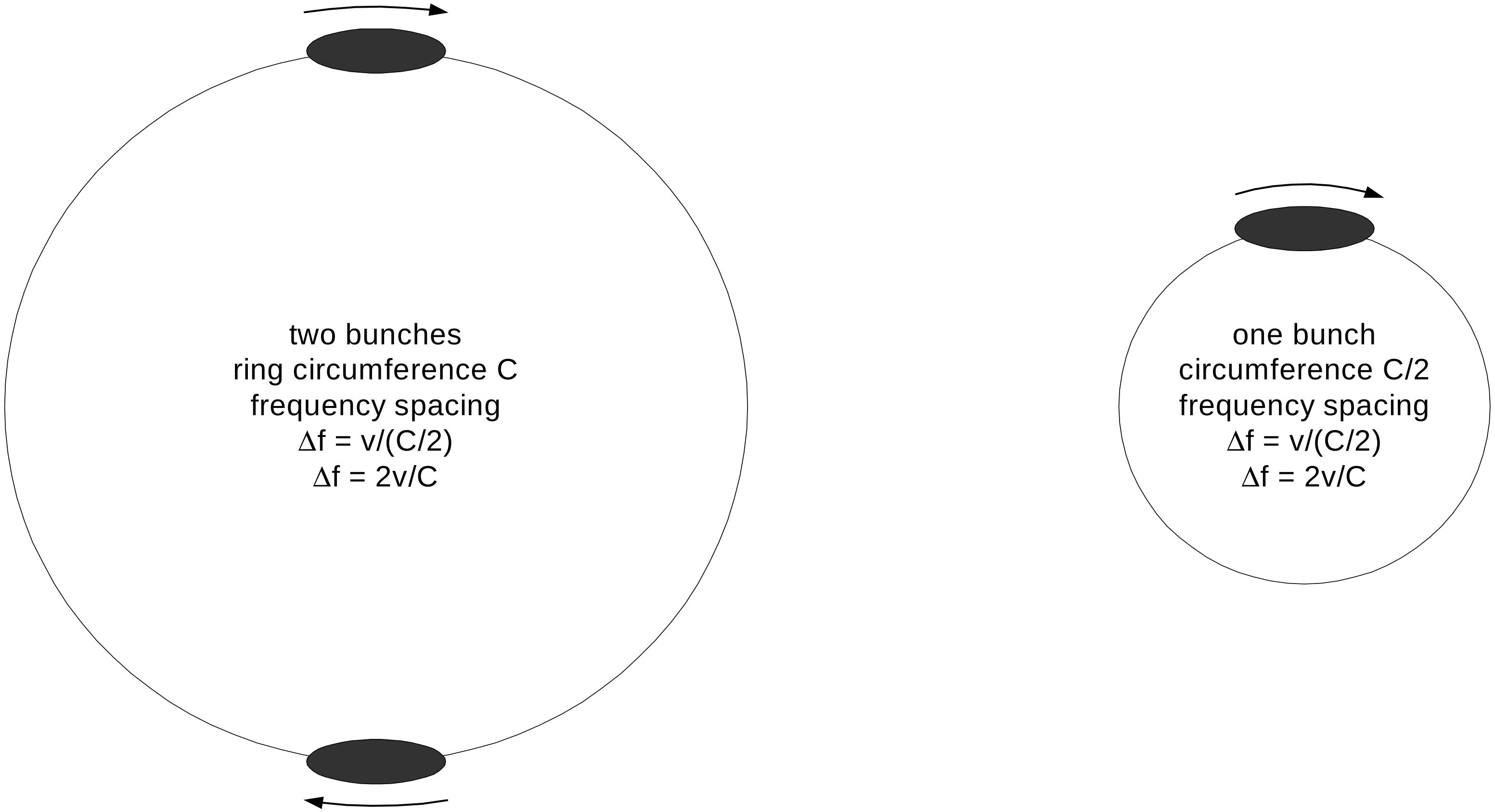
- Bunches driven by RF frequency source at f_{RF}
- Bunches circle the ring
  - Revolution period T
  - Revolution frequency $f_{REV} = 1/T$
- From Fourier theory, a periodic signal can be represented by a spectrum of the harmonics of its fundamental frequency

$$\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} X_k e^{i\frac{2\pi k t}{T}} \\ &= \sum_{k=-\infty}^{\infty} X_k e^{i2\pi k f_{REV} t} \end{aligned}$$

  The spectrum is a series of delta functions in the frequency domain separated by f_{REV}

- In a storage ring, with harmonic number h
  - f_{REV} is harmonically related to f_{RF}
$$f_{RF} = hf_{REV}$$
  - There are only h possible bunches, so
$$I_{k+h} = I_k$$
  - Potential bunches are spaced $T/h = 1/f_{RF}$ apart
  - The Fourier representation is now a finite sum
$$\begin{aligned} i\left(nT/h\right) \; = \; i\left[n\right] &= \sum_{k=0}^{h-1} I\left[k\right] e^{i\frac{2\pi kn}{T}\frac{T}{h}} \\ &= \sum_{k=0}^{h-1} I\left[k\right] e^{i\frac{2\pi kn}{h}} \end{aligned}$$
  - The inverse transform is
$$I\left[k\right] = \frac{1}{h}\sum_{n=0}^{h-1} i\left[n\right] e^{-i\frac{2\pi kn}{h}}$$

- The amplitude of each harmonic is determined by the fill pattern
  - If only bunch n_0 is filled, then
$$I[k] = \frac{1}{h} i[n_0] e^{-i\frac{2\pi k n_0}{h}}$$
    * Each harmonic has the same amplitude
    * But each can have a different phase depending on n_0.
  - Different phases of bunches can lead to nulling of harmonics
    * If two equally spaced bunches are equally filled, then the frequency spectrum is composed of harmonics spaced at $2f_{REV}$

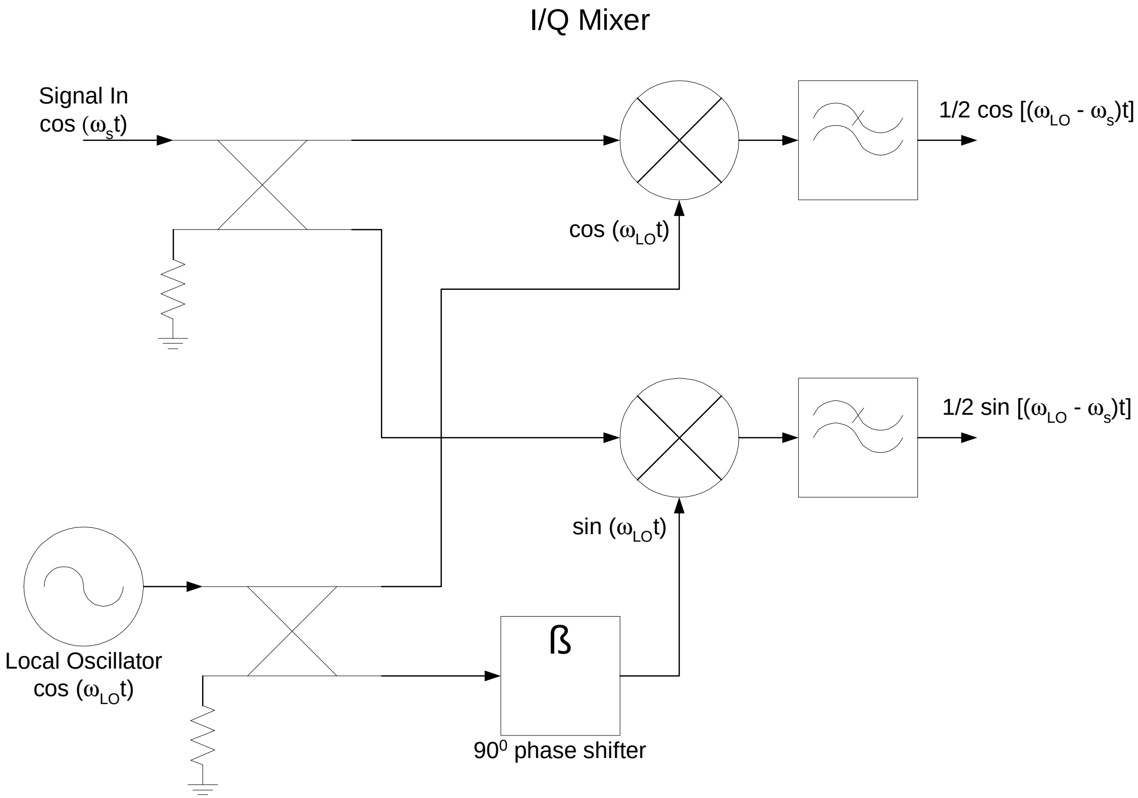

two bunches
ring circumference C
frequency spacing
Δf = v/(C/2)
Δf = 2v/C
one bunch
circumference C/2
frequency spacing
Δf = v/(C/2)
Δf = 2v/C

- In any case, all bunches have the same phase at multiples of $hf_{REV} = f_{RF}$
  - Spectral pattern repeats every h harmonics
  - f_{RF} is the image of the DC current in the machine
- Complete amplitude spectrum of the harmonics is not just determined by fill pattern
  - Finite structure of each bunch rolls off the spectrum at high frequencies
  - Typical light sources roll off in the few GHz range

4 Elements of BPM Processor Design

- Usually process signal at f_{RF}
  - Want a signal independent of fill pattern
  - Want to stay below cutoff in vacuum chambers
- Use narrowband processor
  - Want to minimize noise
  - Beam signal confined to revolution harmonics
  - Place a narrowband filter around f_{RF}
- Usually mix f_{RF} down to a lower, intermediate frequency, f_{IF}
  - Easier to make narrowband filters at lower frequencies
  - Digital conversion easier at lower frequencies
  - People are starting to look at direct digitization of f_{RF}
- Amplitude and phase information are required to know complete information about signal
  - Obtain position from amplitude
  - Betatron motion produces amplitude oscillations
  - Synchrotron motion produces phase oscillations

5 Analog I/Q Mixer



- Requires stable oscillator
- Requires analog components, splitters, phase shifter, mixers, filters
- Would like $80\,\mathrm{dB}$ stability from $30\,\mathrm{dB}$ to $40\,\mathrm{dB}$ components

6 Digital Receiver

6.1 Digital I/Q Mixer

- Requires stable clock
- Remainder of mixer is digital
- Accuracy is limited by internal arithmetic precision, typically $90\,\mathrm{dB}$

6.2 Data Bandwidth

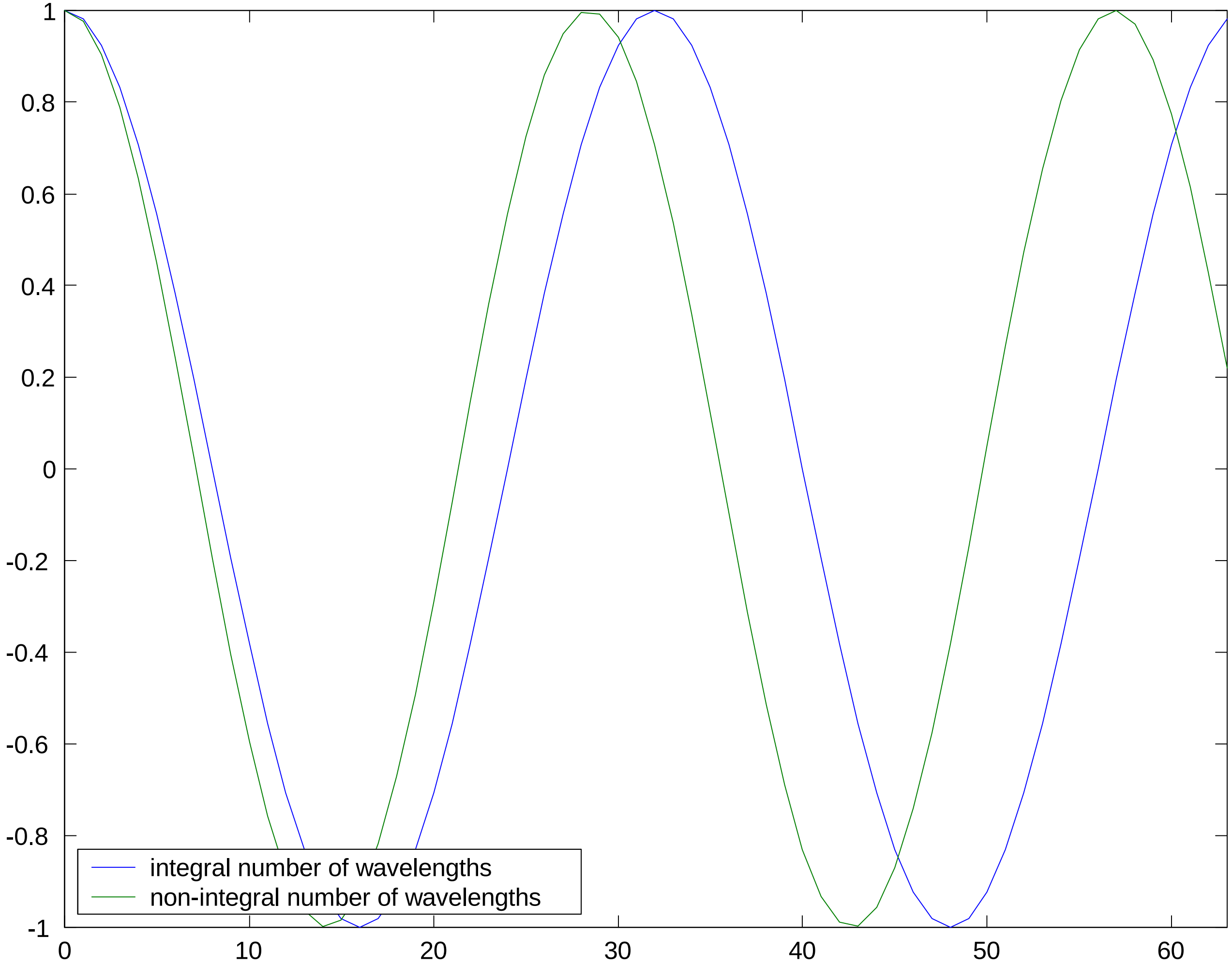
- Digitizing a fast stream of data at a fast rate
- Must process the data to reduce bandwidth
- Want to find good algorithm
- Use filter digital receiver to select out harmonic of interest

6.3 Theory of Operation

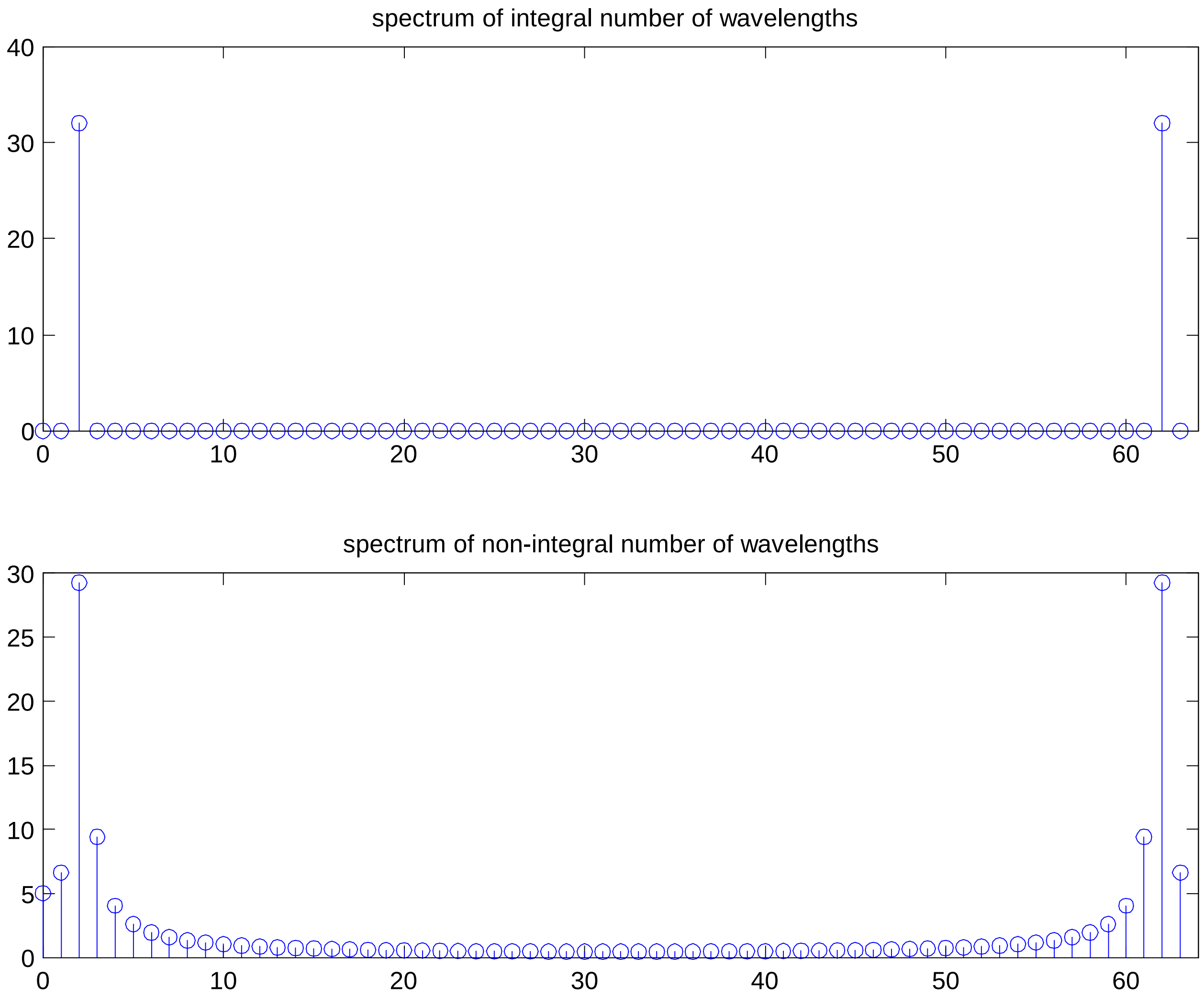
- ADC converts input analog signal
  - We use 65MS/s 14 bit digitizer, speeds now up over 100MS/s
  - "Leading edge" product aimed for commercial mass market
- 14 bit digital word $x[n]$ input into digital receiver
- Receiver clocked with digitizer clock
- Each clock increments, by a programmed amount, an internal phase accumulator, $\phi[n]$
- Receiver generates $\cos\phi[n]$, $\sin\phi[n]$
- Generates two real (one complex) digital paths
  - In-phase — $x[n]\cos\phi[n]$
  - Quadrature — $x[n]\sin\phi[n]$
- Digital filter further processes this complex signal

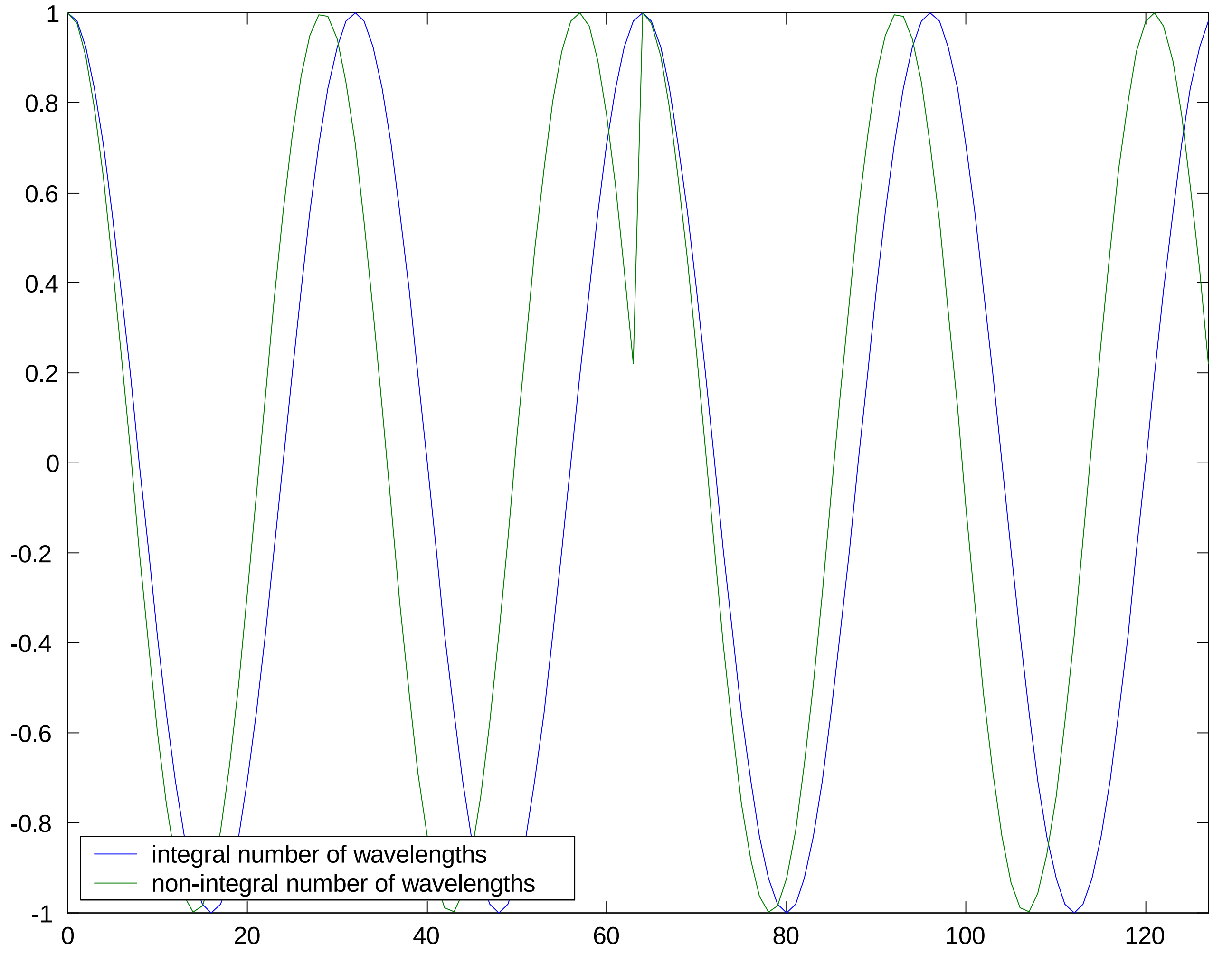
6.4 Digital Filter Design

- Utilize advantages of periodic signals to create simple, orthogonal filter



Two waveforms to be sampled


spectrum of integral number of wavelengths
40
30
20
10
0
0
10
20
30
40
50
60
spectrum of non-integral number of wavelengths
30
25
20
15
10
5
0
0
10
20
30
40
50
60



Waveform represented by DFT

- We have a periodic signal, and have access to f_{RF}, the system clock
- Make everything synchronous to f_{RF}; make everything a multiple of f_{REV}
  - Process signal at f_{RF}
  - Set local oscillator frequency, f_{LO}, at a multiple of f_{REV}
  - Then intermediate frequency, f_{IF}, is a multiple of f_{REV}
  - Set sampling clock to another multiple of f_{REV}
- Result is a digital system sampling an integral number of wavelengths each revolution

- Example: 8 samples gives 8 independent basis vectors

$\boldsymbol{\phi}$	$\cos \mathbf{0}$	$\cos \boldsymbol{\phi}$	$\sin \boldsymbol{\phi}$	$\cos \mathbf{2}\boldsymbol{\phi}$	$\sin \mathbf{2}\boldsymbol{\phi}$	$\cos \mathbf{3}\boldsymbol{\phi}$	$\sin \mathbf{3}\boldsymbol{\phi}$	$\cos \mathbf{4}\boldsymbol{\phi}$
$\mathbf{0}$	1	1	0	1	0	1	0	1
$\boldsymbol{\pi}/\mathbf{4}$	1	$\sqrt{2}/2$	$\sqrt{2}/2$	0	1	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1
$\boldsymbol{\pi}/\mathbf{2}$	1	0	1	-1	0	0	-1	1
$\mathbf{3}\boldsymbol{\pi}/\mathbf{4}$	1	$-\sqrt{2}/2$	$\sqrt{2}/2$	0	-1	$\sqrt{2}/2$	$\sqrt{2}/2$	-1
$\boldsymbol{\pi}$	1	-1	0	1	0	-1	0	1
$\mathbf{5}\boldsymbol{\pi}/\mathbf{4}$	1	$-\sqrt{2}/2$	$-\sqrt{2}/2$	0	1	$\sqrt{2}/2$	$-\sqrt{2}/2$	-1
$\mathbf{3}\boldsymbol{\pi}/\mathbf{2}$	1	0	-1	-1	0	0	1	1
$\mathbf{7}\boldsymbol{\pi}/\mathbf{4}$	1	$\sqrt{2}/2$	$-\sqrt{2}/2$	0	-1	$-\sqrt{2}/2$	$-\sqrt{2}/2$	-1

- To project out $\cos\phi$, multiply all columns by $\cos\phi$ and sum

$\boldsymbol{\phi}$	$\cos \mathbf{0}$	$\cos \boldsymbol{\phi}$	$\sin \boldsymbol{\phi}$	$\cos \mathbf{2}\boldsymbol{\phi}$	$\sin \mathbf{2}\boldsymbol{\phi}$	$\cos \mathbf{3}\boldsymbol{\phi}$	$\sin \mathbf{3}\boldsymbol{\phi}$	$\cos \mathbf{4}\boldsymbol{\phi}$
$\mathbf{0}$	1	1	0	1	0	1	0	1
$\boldsymbol{\pi}/\mathbf{4}$	$\sqrt{2}/2$	$1/2$	$1/2$	0	$\sqrt{2}/2$	$-1/2$	$1/2$	$-\sqrt{2}/2$
$\boldsymbol{\pi}/\mathbf{2}$	0	0	0	0	0	0	0	0
$\mathbf{3}\boldsymbol{\pi}/\mathbf{4}$	$-\sqrt{2}/2$	$1/2$	$-1/2$	0	$\sqrt{2}/2$	$-1/2$	$-1/2$	$\sqrt{2}/2$
$\boldsymbol{\pi}$	-1	1	0	-1	0	1	0	1
$\mathbf{5}\boldsymbol{\pi}/\mathbf{4}$	$-\sqrt{2}/2$	$1/2$	$1/2$	0	$-\sqrt{2}/2$	$-1/2$	$1/2$	$\sqrt{2}/2$
$\mathbf{3}\boldsymbol{\pi}/\mathbf{2}$	0	0	0	0	0	0	0	0
$\mathbf{7}\boldsymbol{\pi}/\mathbf{4}$	$\sqrt{2}/2$	$1/2$	$-1/2$	0	$-\sqrt{2}/2$	$-1/2$	$-1/2$	$-\sqrt{2}/2$
$\boldsymbol{\Sigma}$	0	4	0	0	0	0	0	0

- Only the $\cos\phi$ component remains, since these are the orthogonal basis vectors of the Fourier representation

- This can be demonstrated algebraically. If the digitized beam signal $i\left[n\right] = e^{i\frac{2\pi n k_0}{N}}$

$$\begin{aligned} I\left[k\right] &= \frac{1}{N}\sum_{n=0}^{N-1} i\left[n\right] e^{-i\frac{2\pi k n}{h}} \\ &= \frac{1}{N}\sum_{n=0}^{N-1} e^{i\frac{2\pi n k_0}{N}} e^{-i\frac{2\pi n k}{N}} \\ &= \frac{1}{N} e^{i\pi\left(k_0 - k\right)\left(1-\frac{1}{N}\right)} \frac{\sin\pi\left(k_0 - k\right)}{\sin\pi\left(\frac{k_0-k}{N}\right)} \\ &= \delta_{kk_0} \end{aligned}$$

  All undesired signals, the other harmonics, are orthogonal to the signal of interest.

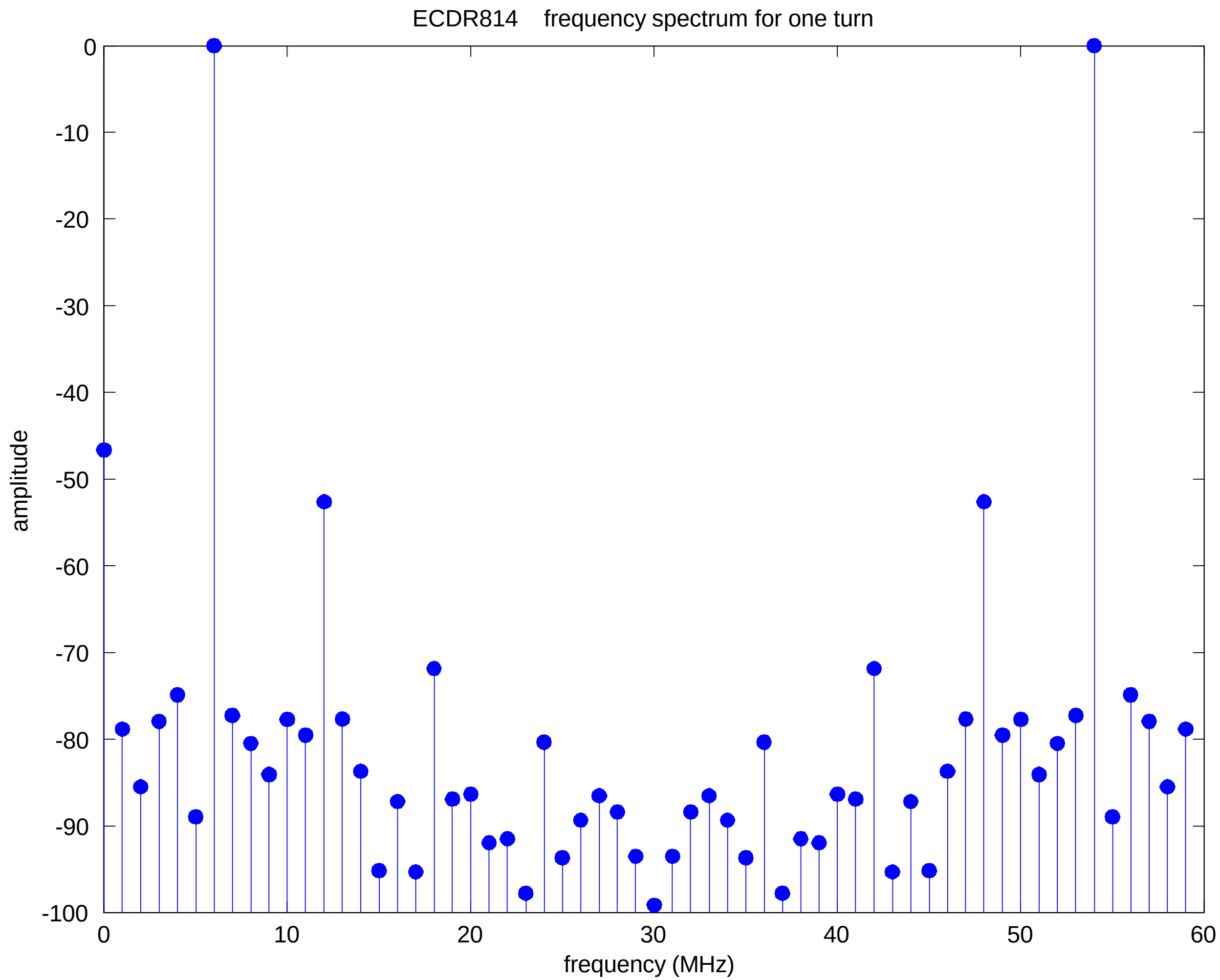
- We achieve the desired results if we program the digital receiver filters to be a single channel FFT
  - Beam generated signals are harmonics of f_{REV}
  - Generate clock that is another harmonic of f_{REV}
  - Sample for a revolution (or subharmonic) and obtain eigenvector

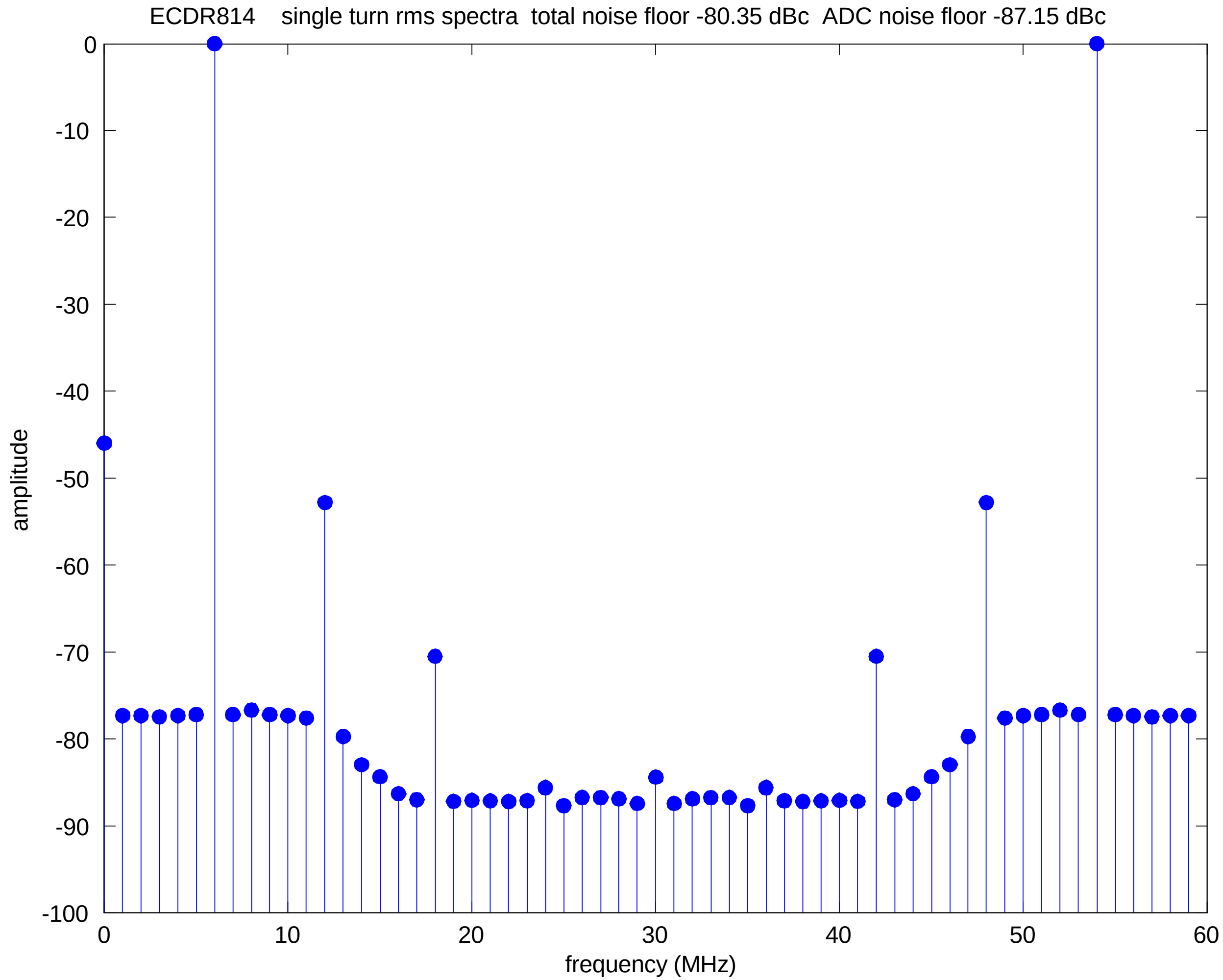
6.5 Numerical Issues

- Digitizer
  - 14 bits (greater than 13 bit accuracy)
  - $74\,\mathrm{dB}\;SNR$
- Complex multiplier numbers for I and Q are stored with 18 bit resolution
- Accumulator sum is calculated to 23 bit resolution
- AD6620 outputs 16 bits
- Numerical orthogonality of eigenvectors better than $-90\,\mathrm{dB}$

6.6 Digitizer Implementation

- Purchased 8 channel digital receiver module from Echotek Corporation
- Test data shows $6\,\mathrm{MHz}$ signal sampled at $60\,\mathrm{MHz}$ ($18\,\mathrm{dB}$ of processing gain)

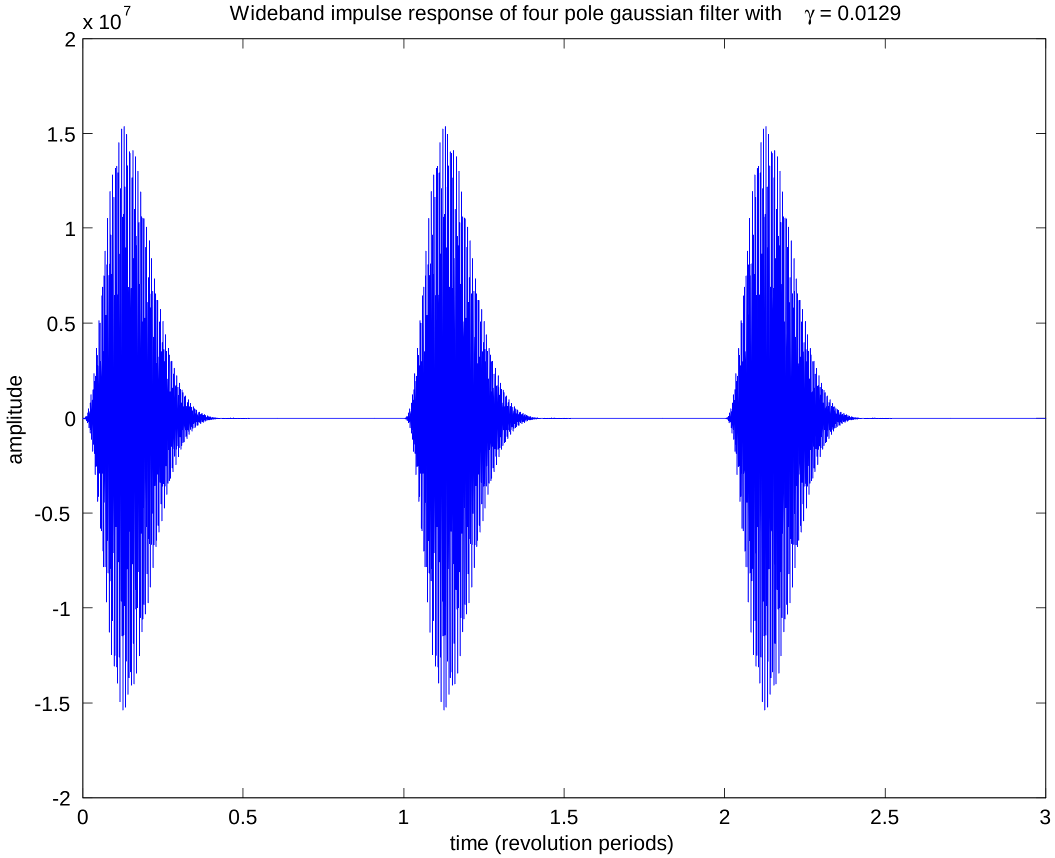




- $87\,\mathrm{dB}\ SNR$ gives $1\,\mu\mathrm{m}$ resolution in a $2\,\mathrm{cm}$ radius chamber each turn

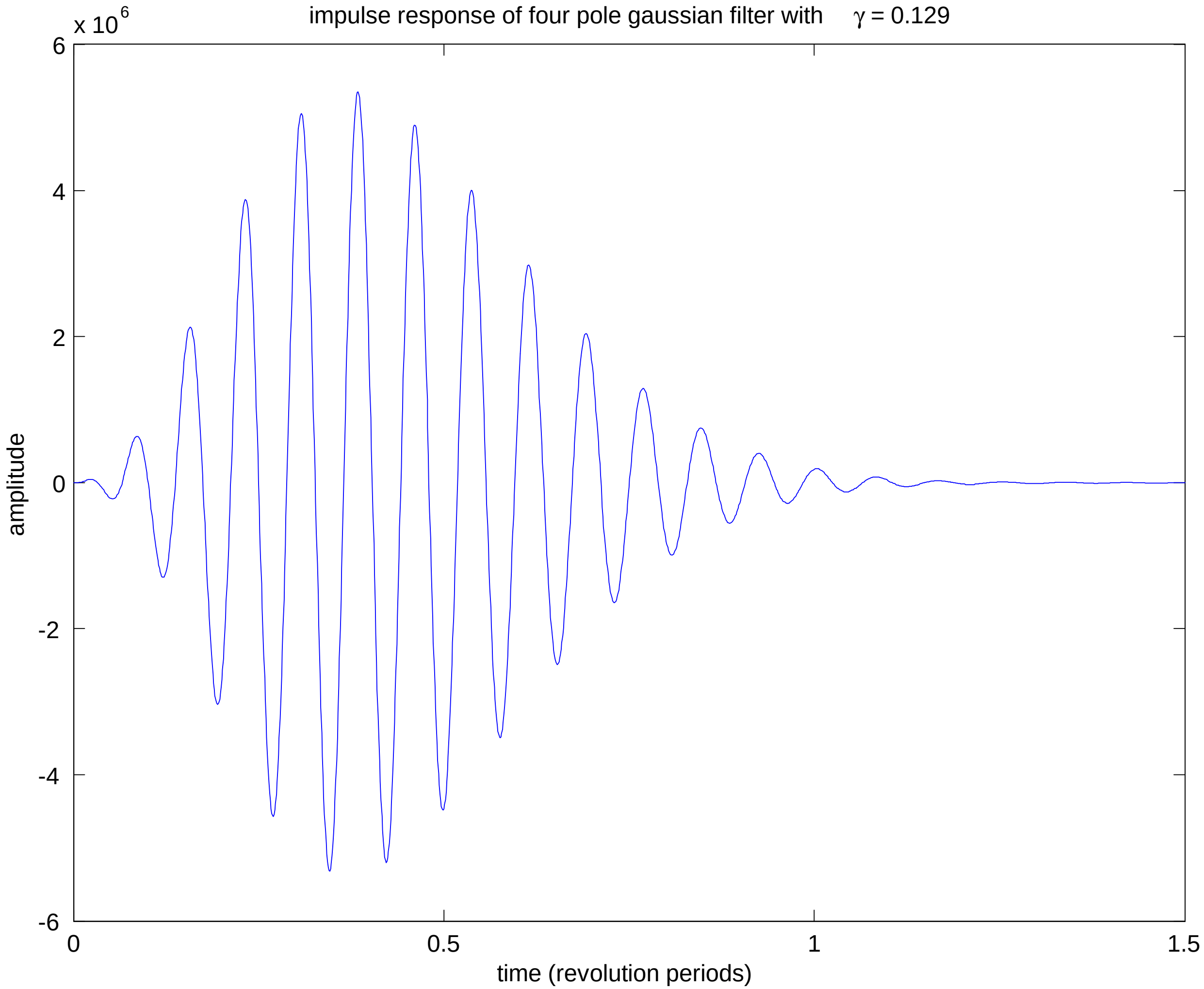
7 Analog Filtering Issues

- Our bunches are short, so time response of unprocessed bunch is short
- Need some RF bandpass filter to keep down peak voltages at the processor, but impulse response of a single bunch is not optimal for digitizing single bunch fills

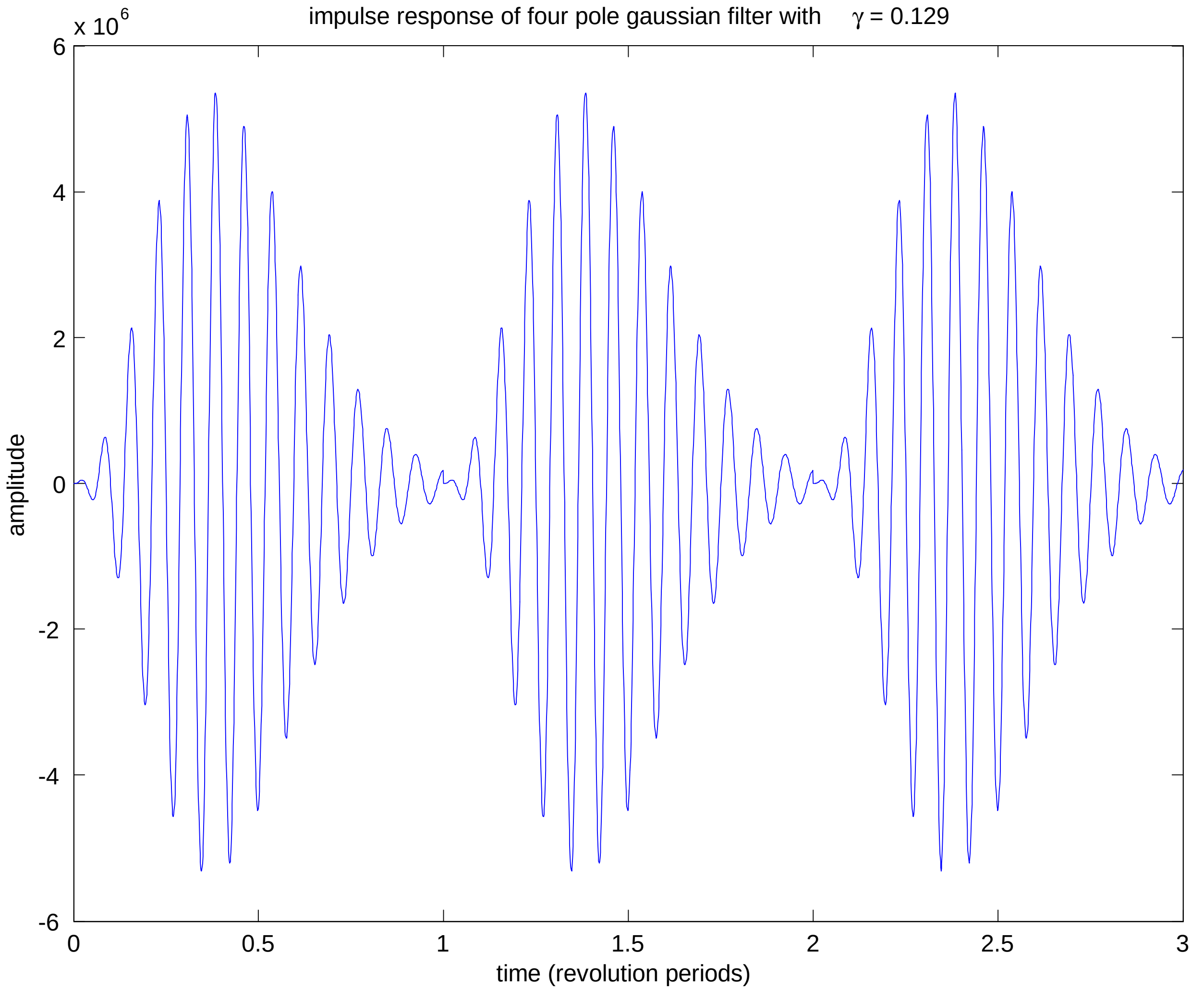


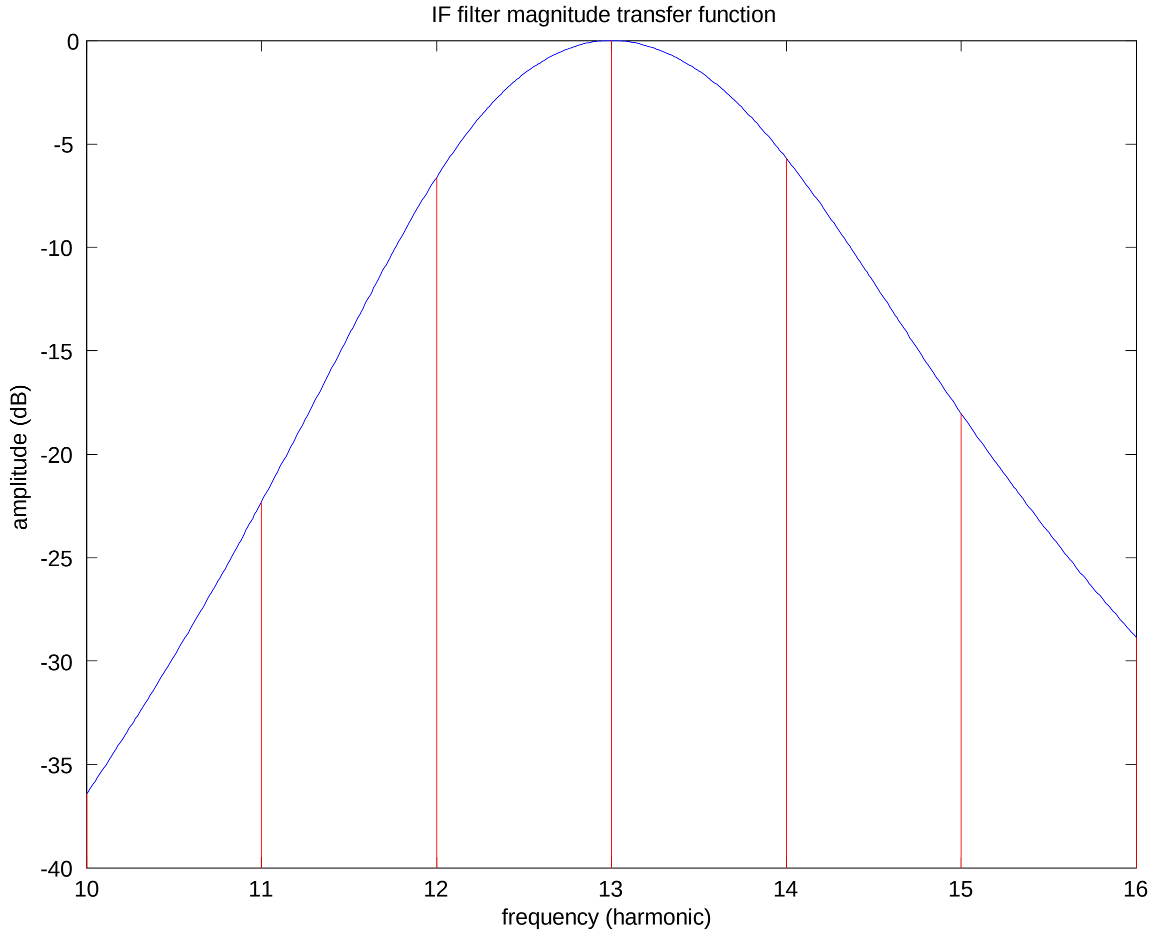
- Meaningful data exists for less than 25% of each sample for a single bunch measurement.

- Construct an IF filter that rings the signal at the desired frequency, and rings out over each turn

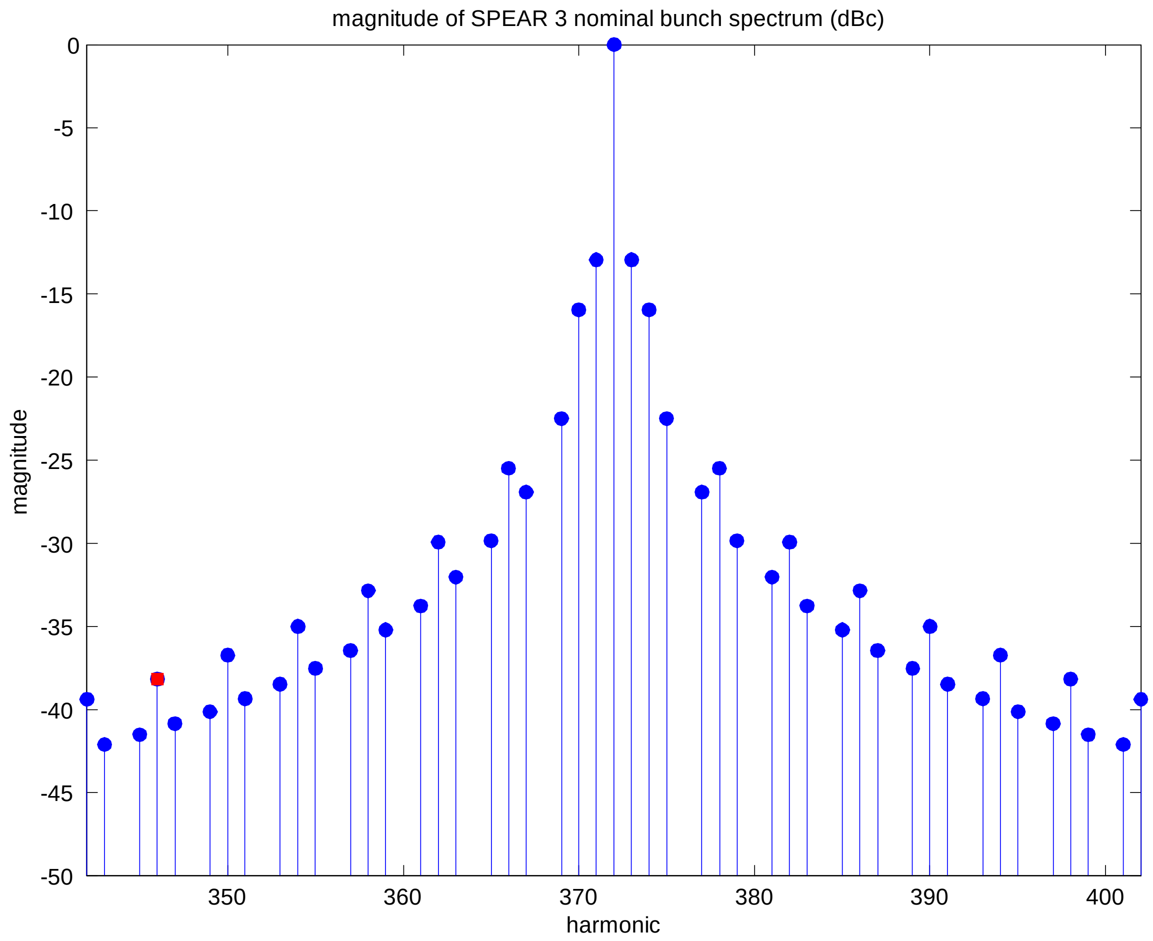


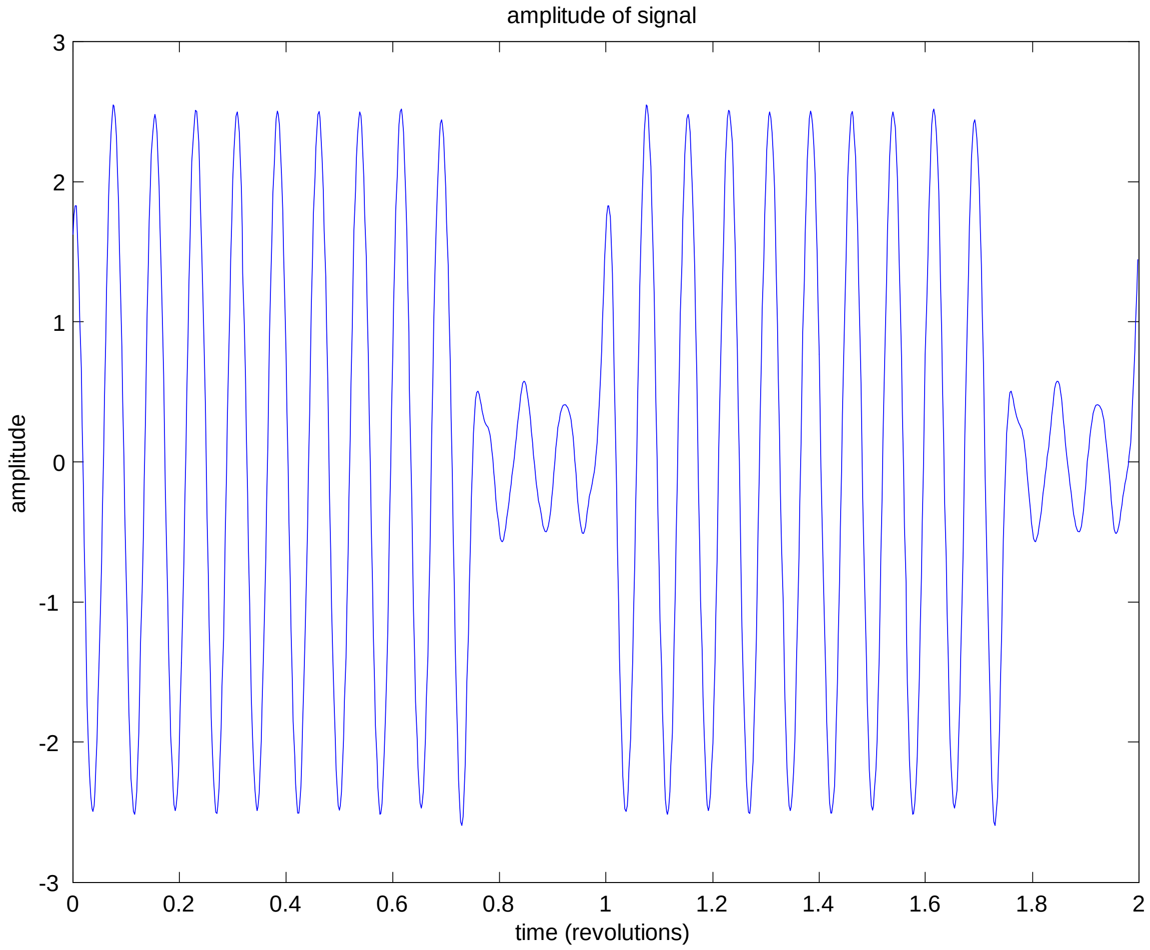
- Periodic impulses of beam gives periodic signal at f_{IF} that utilizes all samples, yet allows turn by turn position measurement.




IF filter magnitude transfer function
0
-5
-10
-15
-20
-25
-30
-35
-40
amplitude (dB)
10
11
12
13
14
15
16
frequency (harmonic)

- For normal fill pattern (279 out of 372 consecutive bunches equally filled), the fill pattern provides a filter




amplitude of signal
amplitude
time (revolutions)
3
2
1
0
-1
-2
-3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

8 SPEAR 3 Implementation

- Mix RF signal down to IF
- Modest analog signal processing (gain and filtering) to remove aliases
- Frequencies all synchronous
  - $f_{RF} = 476.3\,\mathrm{MHz}$
  - $h = 372$
  - $f_{REV} = 1.28\,\mathrm{MHz}$
  - $f_{IF} = 15 f_{REV}$
  - $f_{CLK} = 50 f_{REV}$
- 50 samples per revolution gives another processing gain of $\sqrt{50}$
- Digital stage gives 14 bit number per revolution
- Amplitude: $\sqrt{I^2 + Q^2}$
- Phase: $\arctan Q/I$
- Average amplitude for more accurate position information
- Use turn by turn data to calculate the betatron and synchrotron frequencies

9 FNAL Implementation

- M. Ross set up test of system at FNAL
- D. McCormick, M. Ross, T. Straumann, and J. Sebek went to test system out
- In two trips set up two different systems (two days each)
  - Partitioned transfer line system into 28 equal sections
    - Took data at injection of four bunches
    - Recorded turn by turn data from which we recovered machine tunes
  - Partitioned Tevatron into 159 equal sections
    - Took data at injection of full bunch pattern
    - Recorded turn by turn data, bunch by bunch