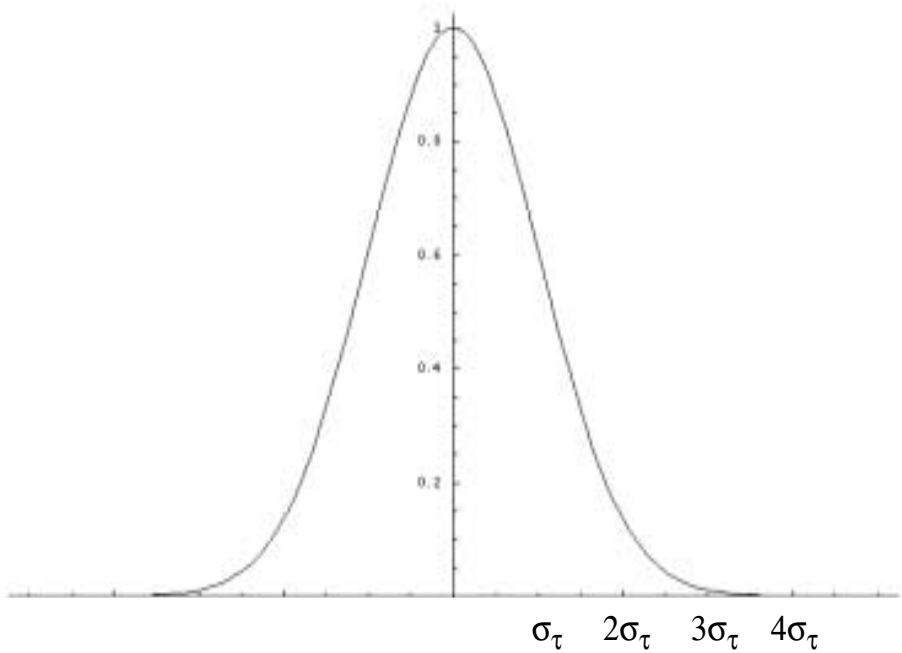
Time

Bunch structure

Time Scales

Instrumentation

Bunch Length σ_τ



Source Type	σ_τ
Storage Rings	10's of picoseconds
UV SASE FEL*	< 1 ps
X-ray FEL*	10's of femtoseconds
Energy recovery linacs*	100 fs - few ps

* Linac-based sources generate very non-Gaussian bunches

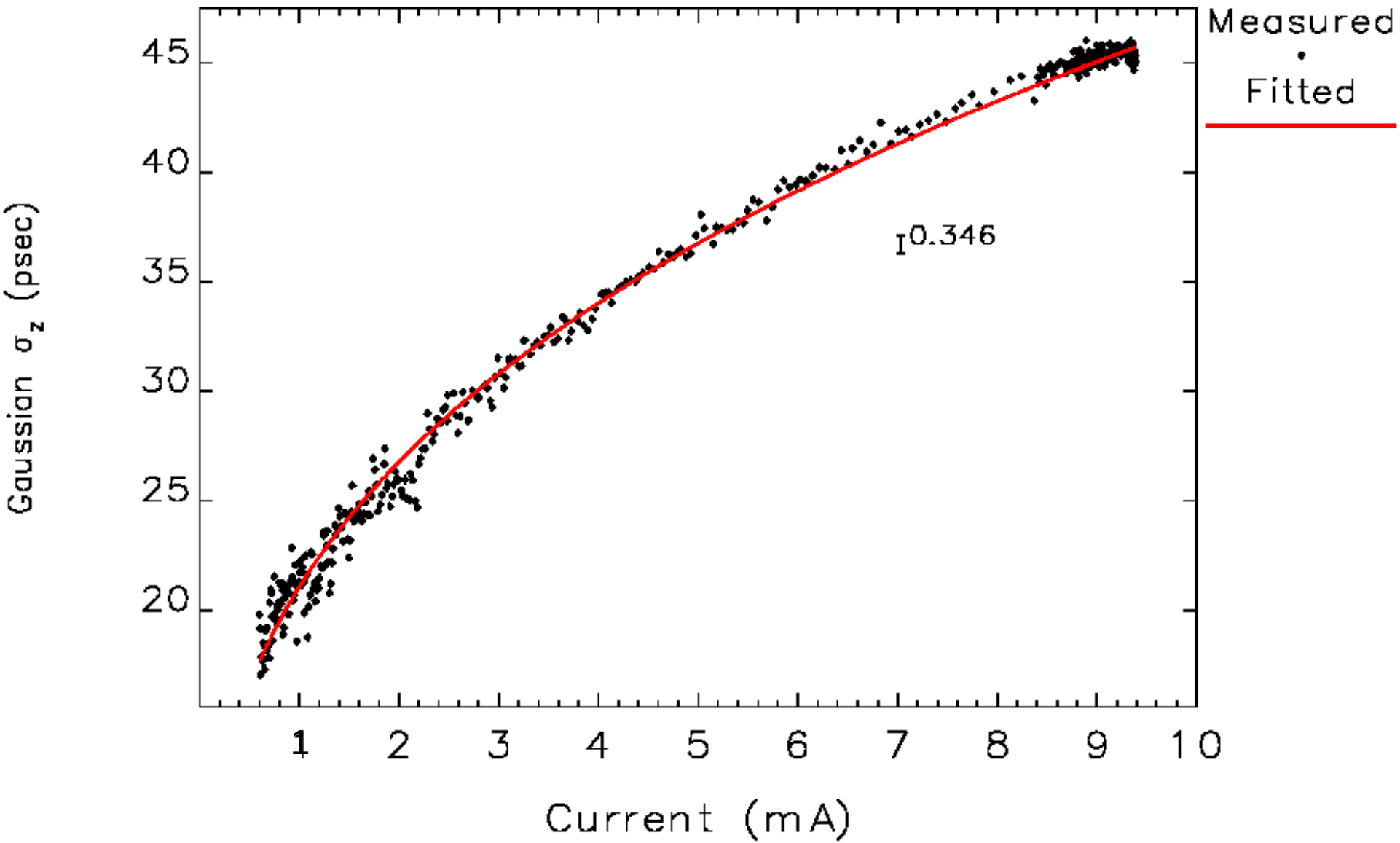
Equilibrium Zero-current bunch length - storage ring

$$\sigma_\tau = \{ 2 \pi \alpha h E / \omega_{RF}^2 \cos \phi_s e V_{RF} \}^{1/2} \sigma_E$$

α = momentum compaction
h = harmonic number = f_{rf} / f_{rev}
E = Energy
$\omega_{RF} = 2 \pi f_{rf}$

ϕ_s = Synchronous phase
V_{RF} = RF voltage
σ_E = Energy spread,
(proportional to energy)

APS Bunch Length Data



Time Scales for Modern Storage Ring - Based Light Sources

	APS	SPEAR-3	SRC - Aladdin	NSLS UV ring
Beam Energy	7 GeV	3 GeV	1.0 GeV	750 MeV
RF Period	2.84 ns	2.10 ns	19.8 ns	18.9 ns
Circumference L	1104 meters	234 meters	88.9 meters	51 meters
Revolution period T_{rev}	3683 ns	781 ns	297 ns	170 ns
Harmonic number h	$1296 = 2^4\ 3^4$	372	15	9
RF Frequency f_{RF}	352 MHz	476 MHz	50.582 MHz	53 MHz
Revolution freq. f_{rev}	271 kHz	1.28 MHz	3.37 MHz	5.88 MHz
Bunch Length σ_τ	35 ps	19 ps	479 ps	162 - 500 ps

$f_{rev} = c / L$

$T_{rev} = L / c$

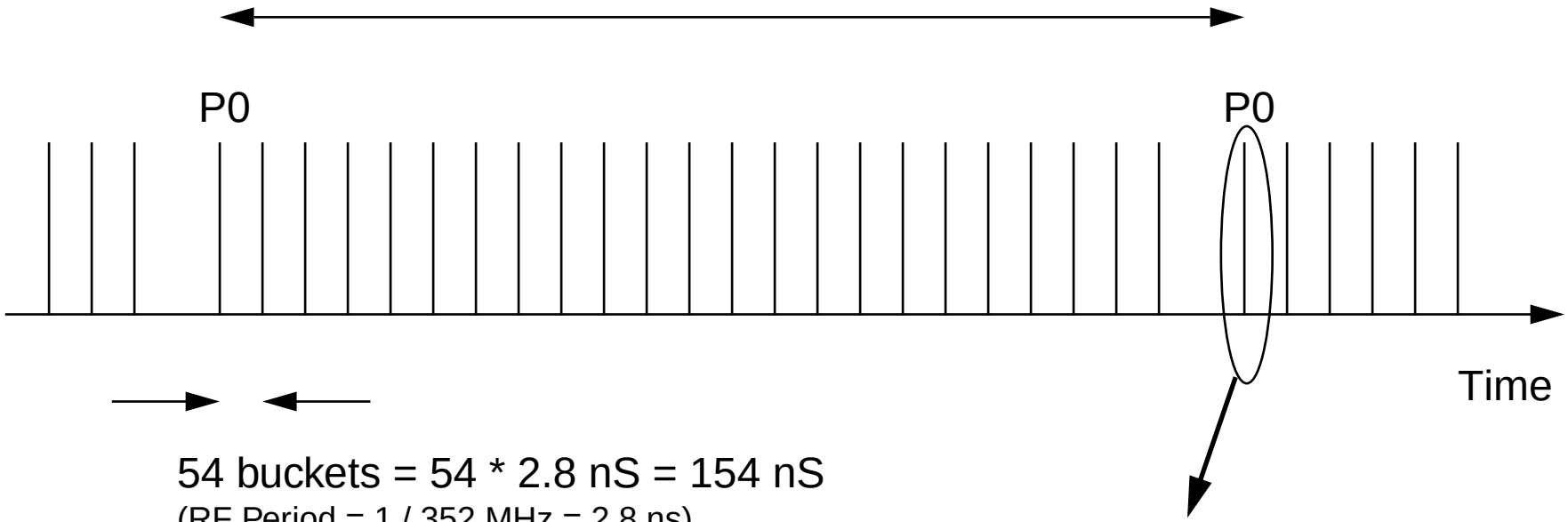
$c = 2.9979e8$ m / s

h = Harmonic number
= Maximum number of bunches
$= f_{RF} / f_{rev}$

RF Period $= 1 / f_{RF}$
= Minimum bunch spacing

APS 24 -1 Singlets Fill Pattern

Revolution period = 3.68 microseconds = 1296 buckets



54 buckets = 54 * 2.8 nS = 154 nS

(RF Period = 1 / 352 MHz = 2.8 ns)

$T_{rev} / \sigma_{\tau} = 100{,}000$

Comparison of Time Scales for Different Light Source Technologies

	APS	LCLS	TESLA	ERL
Beam Energy	7 GeV	15 Gev	25 GeV	5.3 GeV
RF Period	2841 ps	350 ps	769 ps	769 ps
Pulse Rep. Period	11 - 154 ns	8.3 milliseconds	17.7 µs**	769 ps
Bunch Length FWHM	73 ps	230 femtosec.	90 femtosec.	300 femtosec.
RF Frequency	352 MHz	2856 MHz	1300 MHz*	1300 MHz*
Pulse Rep. Frequency	6.5 - 88 MHz	120 Hz	56575	1300 MHz
Charge / pulse	14 nC	1 nC	1 nC	8 - 77 pC
Average Current	100 mA	72 nanoAmps	63 microAmps	10 - 100 mA

* Superconducting RF

** 11315 buckets * 93 nsec * 1 nC Bunch Trains * 5 Hz

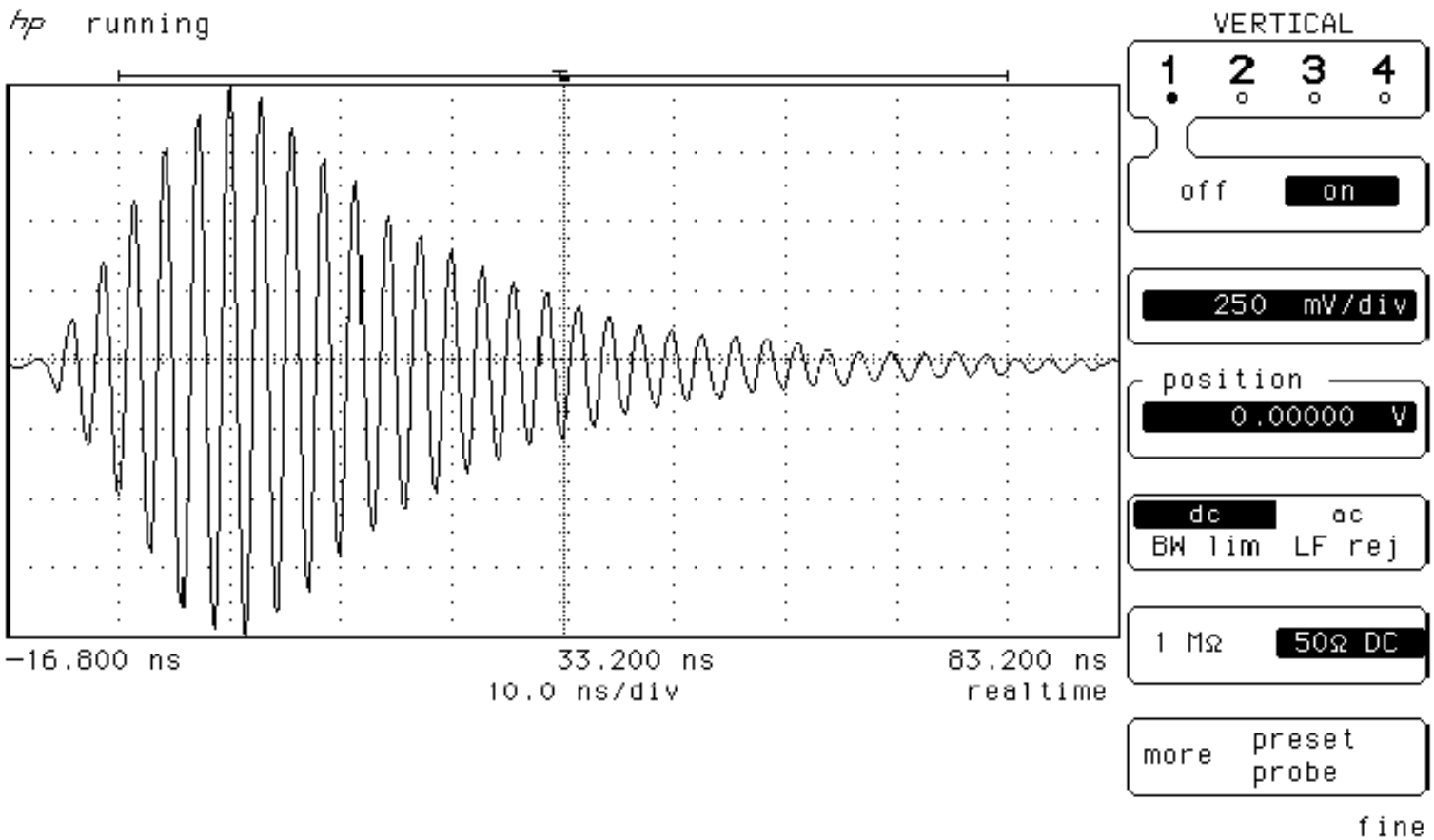
http://erl.chess.cornell.edu/papers/ERL_Study.pdf

Oscilloscopes

Specifications

Analog Bandwidth up to > 6 GHz
Sample Rate up to 20 GS / sec
Sub-ps jitter, rise time
Common PC / windows operating systems
Most often use 8-bit resolution
Some units provide convenient interface to network.

Cavity filters with 6 bunches @ 1.67 ma/bunch sum input



Transient Digitizers

Trade off speed for resolution - 14 vs. 8 bits for typical fast oscillo-scope

Usually requires additional digital signal processor plus competent programmer

Special-purpose digital radios available with a lot of on-board firmware.

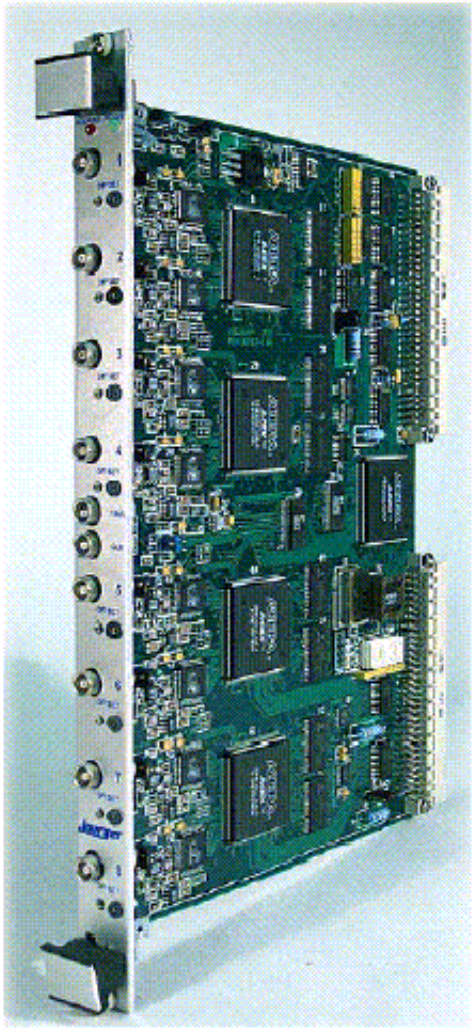
Joerger ENTERPRISES, INC.

MODEL VTR8014

EIGHT CHANNEL, 80 MHZ, 14 BIT "VME" ANALOG DIGITIZER WITH OSCILLOSCOPE CHARACTERISTICS

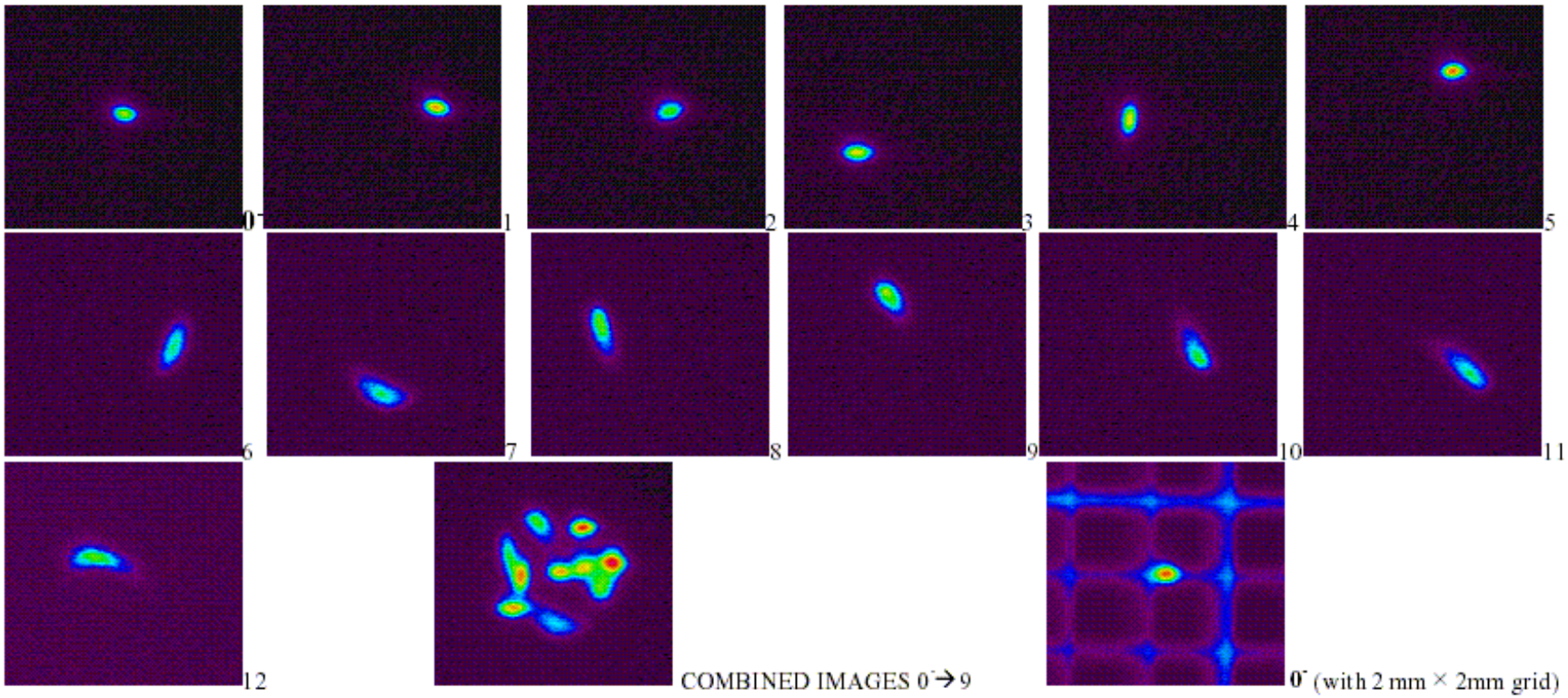
FEATURES:

- 1, 2, 4 OR 8 INDIVIDUAL CHANNELS
- 80 MHZ CLOCK SPEED
- 14 BIT RESOLUTION PLUS SIGNAL AVERAGING FOR IMPROVED SNR
- ***"OSCILLOSCOPE"*** TYPE INPUTS FEATURING:
  - HIGH INPUT IMPEDANCE, 10 MΩ's
  - FULL SCALE OFFSET CONTROL
  - SINGLE ENDED OR DIFFERENTIAL INPUTS
  - WIDE INPUT BANDWIDTH FOR GOOD WAVEFORM TRACKING
- 256K SAMPLES OF SRAM PER CHANNEL, 2M SAMPLES TOTAL
- CHANNELS CAN BE READ AT ANY TIME PROVIDING:
  - SIGNAL MONITORING
  - OFFSET ADJUST AND TEST
  - GAIN TESTING
- READ OUT OF MODULE TYPE AND IT'S SERIAL NUMBER
- RECORDING MODES:
  - POST TRIGGER
  - MULTIPLE POST TRIGGER
  - PRE/POST TRIGGER
  - MULTIPLE PRE/POST TRIGGER
- GLOBAL COMMANDS FOR MULTIPLE MODULE OPERATION
- ALL TRIGGER ADDRESSES STORED
- REAL TIME TRIGGER ARRIVAL STORED
- TRIGGER COUNTER PROVIDED
- NUMBER OF EVENTS REGISTER
- INDIVIDUAL FILTERING OF EACH CHANNELS POWER AND GROUND
- HIGH NOISE IMMUNITY AND LOW CHANNEL CROSSTALK



Injection Transient

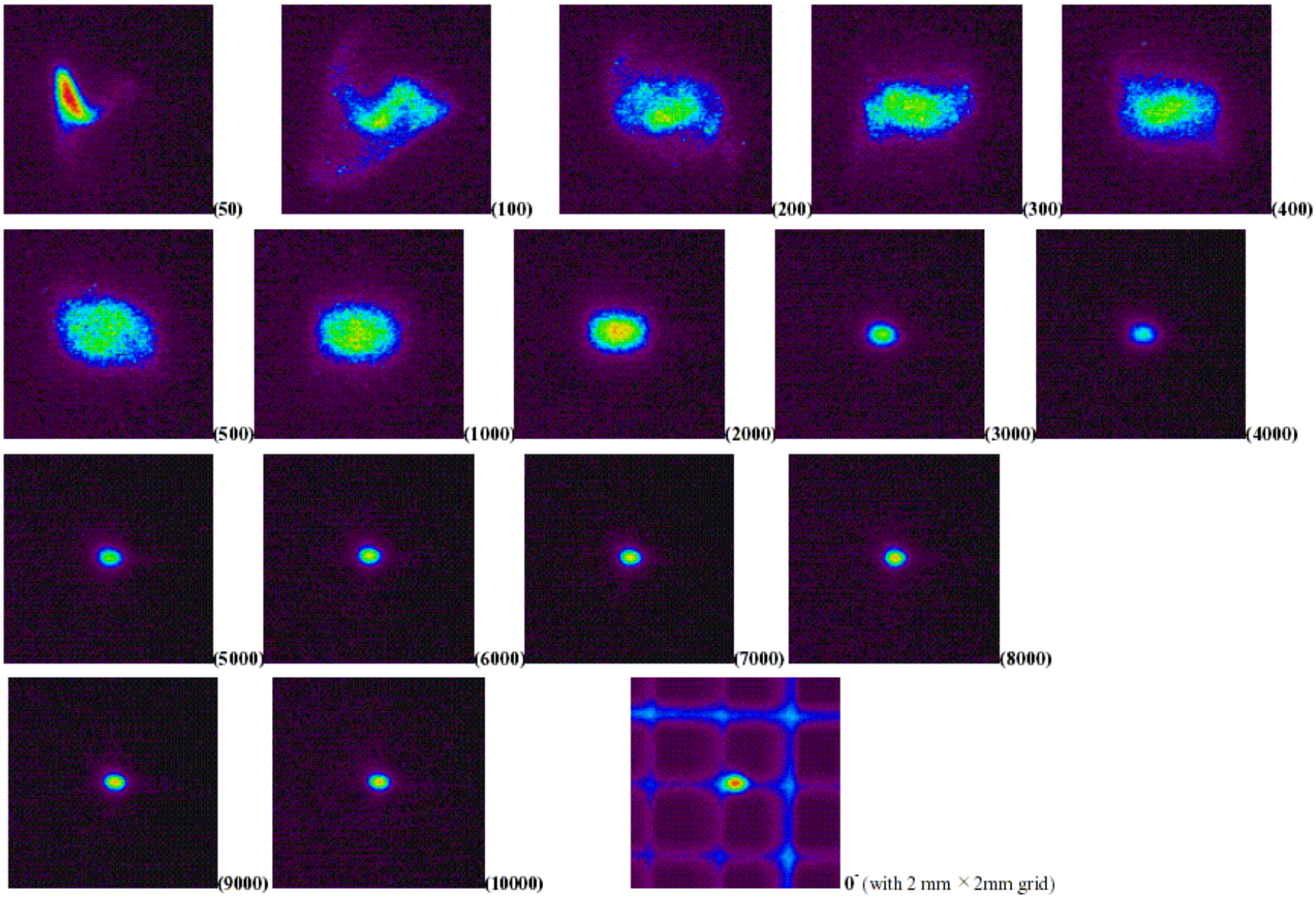
Gated camera single turn images (1/19/98)



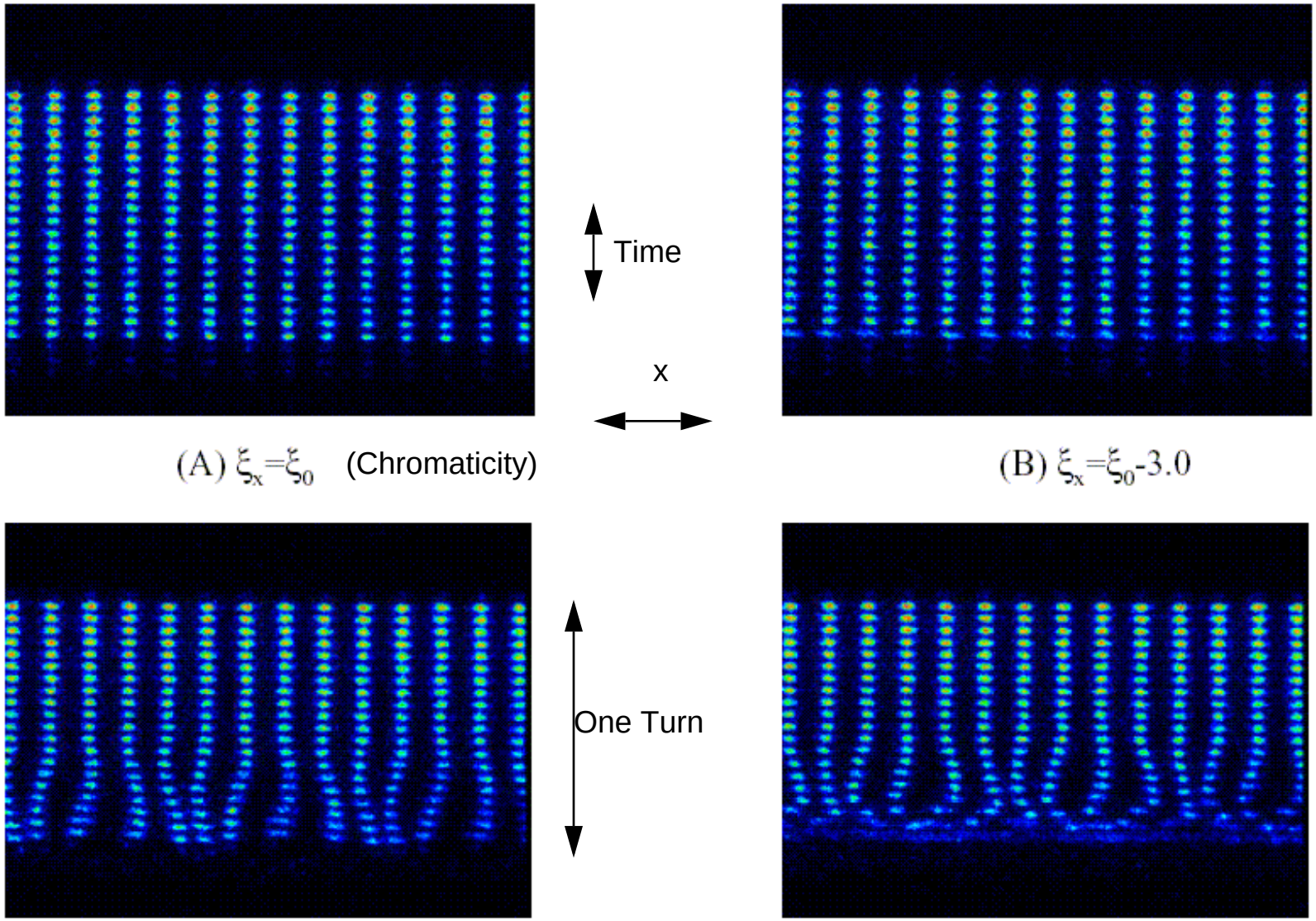
Turn after injection

Transverse Damping

Gated camera single turn images (1/19/98)



Streak Camera Image of Bunch Train Instability



(A) $\xi_x=\xi_0$ (Chromaticity)

(B) $\xi_x=\xi_0-3.0$

(C) $\xi_x=\xi_0-4.8$

(D) $\xi_x=\xi_0-5.2$

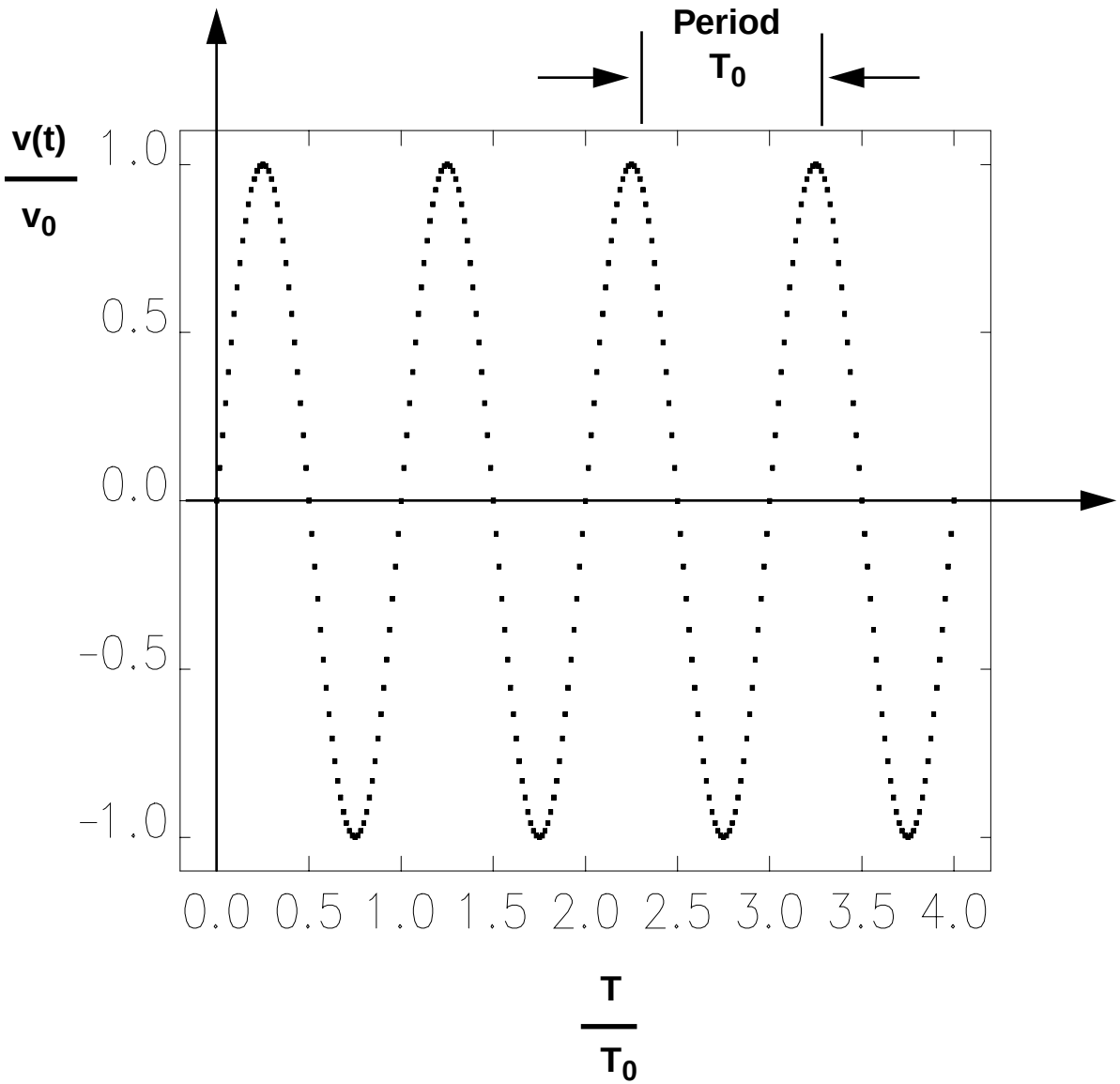
Frequency

Introduction to sampling theory and Fourier transforms
-- Digital Fourier transform
- Cross correlation function
- Auto correlation function
- Parseval's theorem

Accelerator characteristic frequencies
- RF frequency
- Bunch frequency
- Revolution frequency
- Sampling frequency
- Betatron frequencies
- Synchrotron frequency
- Line frequencies

Instrumentation
Spectrum Analyzer
Vector voltmeter
Vector Signal Analyzer
Network Analyzer

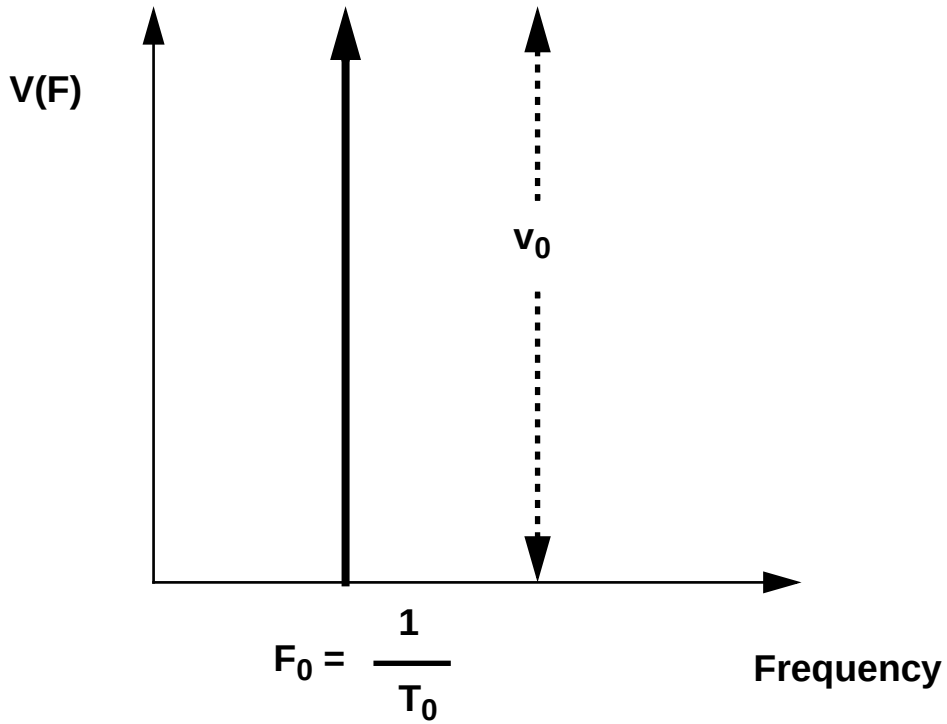
Simple Sine Wave



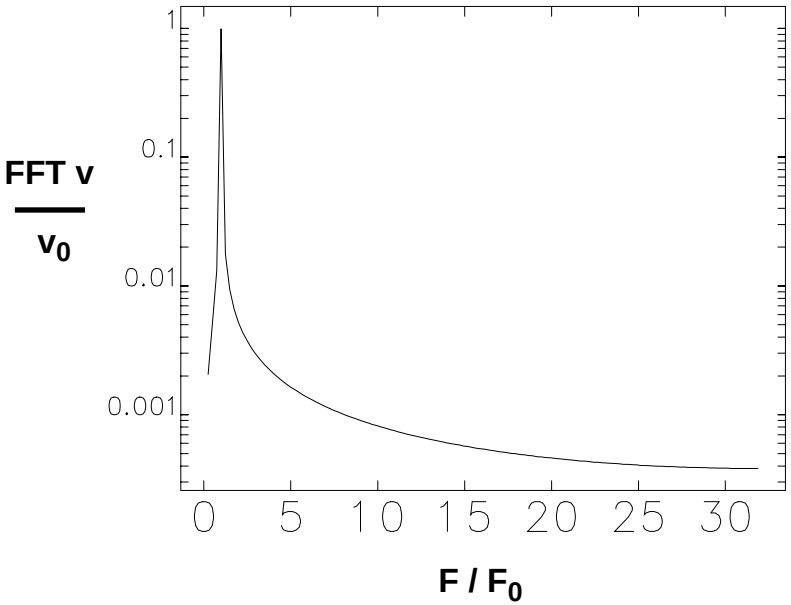
$$v(t) = v_0 \, \mathrm{Sin}\left\{ \frac{2\pi t}{T_0} \right\}$$

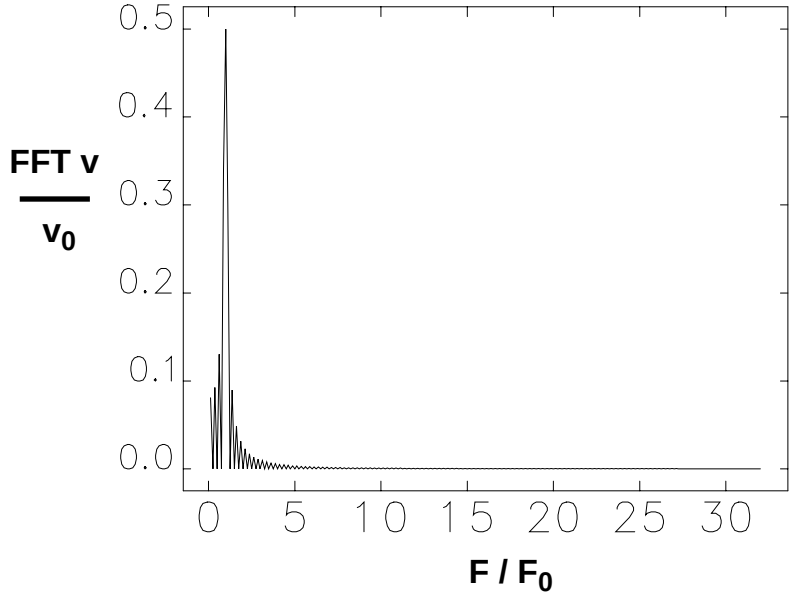
$$\text{Frequency } F_0 = \frac{1}{T_0}$$

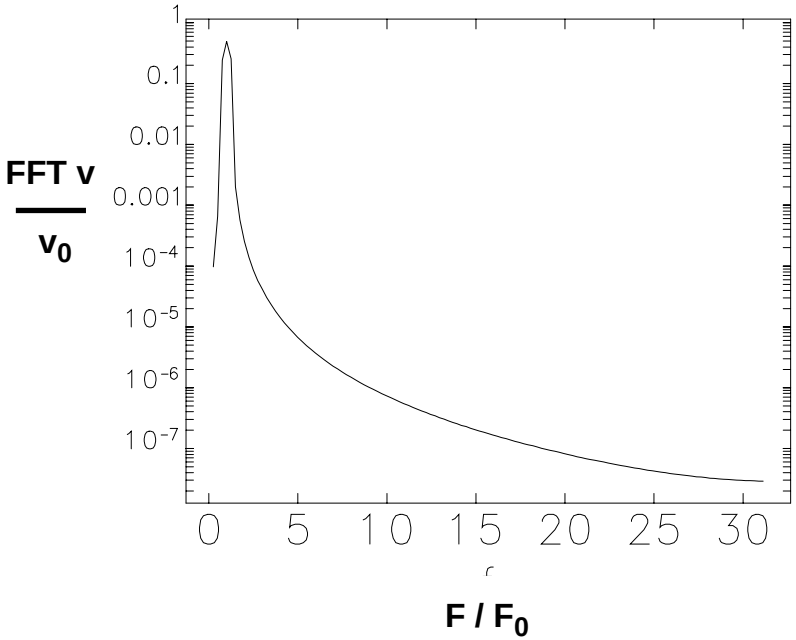
Simple Sine Wave Spectrum

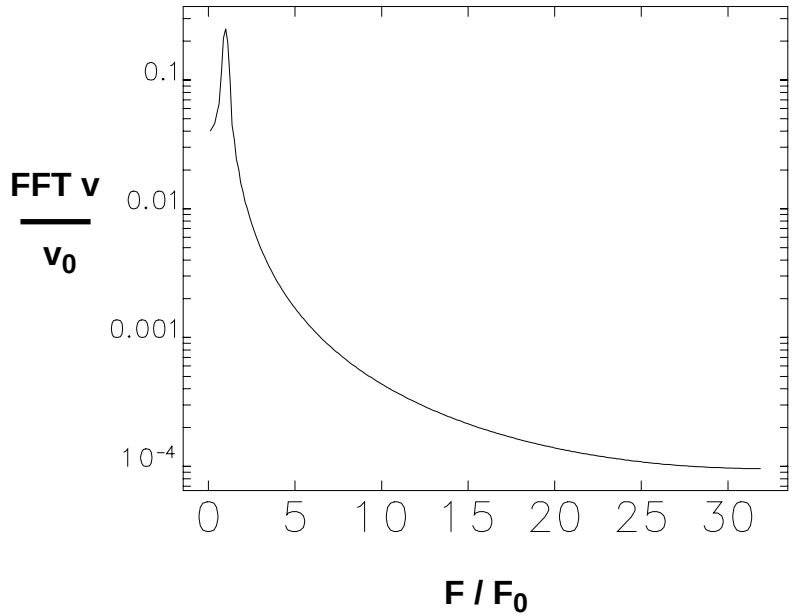


“Simple” Sine Wave Spectrum

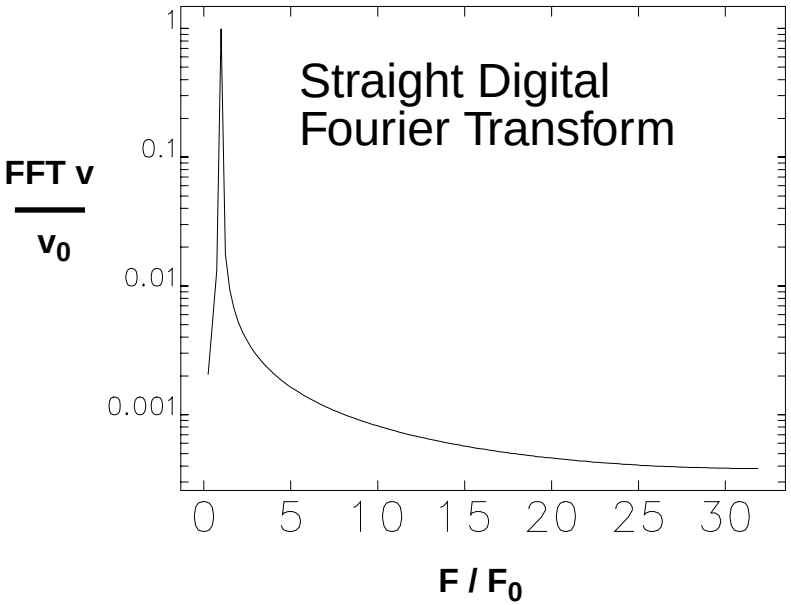


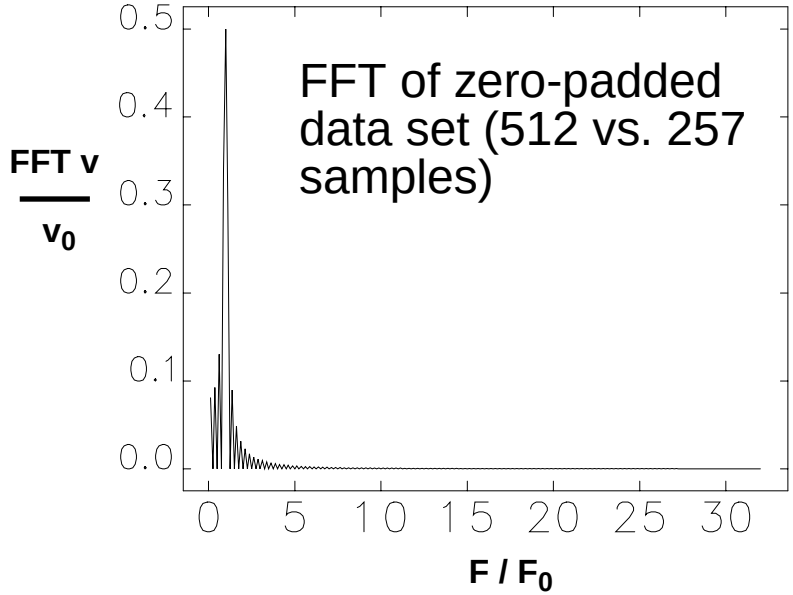


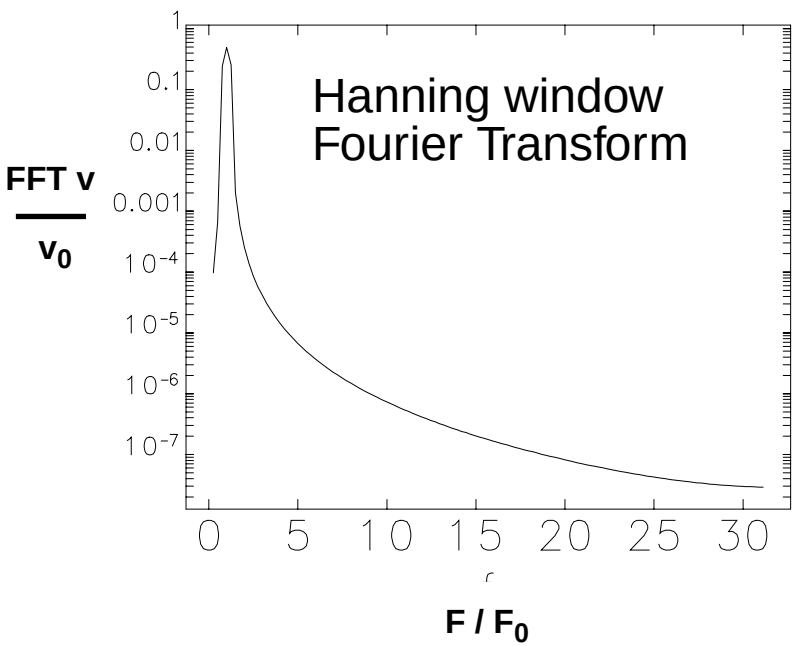


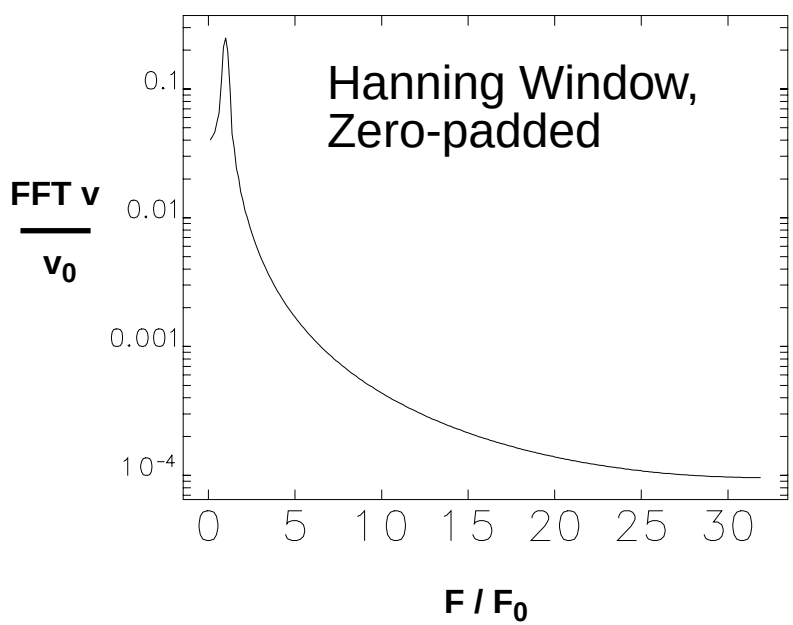


“Simple” Sine Wave Spectrum





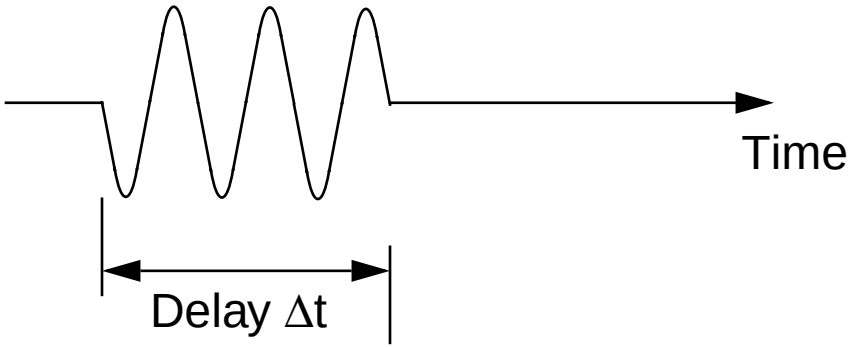




The Cross-Correlation Function

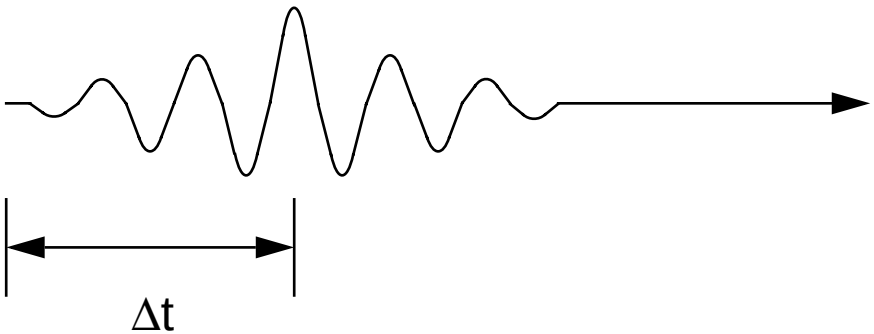
Used for radar range finding:

Transmitted Signal *x(t) :*



Received signal *y(t)* (∗1000) :

Correlation $r_{xy}(t)$:



$$r_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)y(t+\tau)\,\mathrm{d}t$$

White Noise

Noise Signal One *x(t) :*

Uncorrelated Noise Signal Two *y(t)* :

Cross-correlation $r_{xy}(t) = 0$, (by definition of white noise)

Autocorrelation of White Noise

Noise Signal One *x(t) :*

Copy of Noise Signal One *x(t) :*

Autocorrelation function $r_{xx}(t) = \int_{-\infty}^{\infty} x(\tau)x(\tau+t)_d\tau$ = Const. $*$ δ(t)

Sampled Data

$$x_n = x(t_n), \qquad t_n = n\Delta, \qquad n = 0, 1, 2, 3, \ldots \text{N-1}$$

$$y_n = y(t_n), \qquad t_n = n\Delta, \qquad n = 0, 1, 2, 3, \ldots \text{N-1}$$

Δ = Sampling period

Cross-correlation $r_{xy}(m) = \frac{1}{N}\sum_{n=0}^{N-1} x_n y_{n+m}$

Autocorrelation $r_{xx}(m) = \frac{1}{N}\sum_{n=0}^{N-1} x_n x_{n+m} = \langle x^2 \rangle \, \delta_{m0}$ For random noise

$r_{xx}(0) = \frac{1}{N}\sum_{n=0}^{N-1} x_n x_n = \langle x^2 \rangle$ in general <---

= mean square signal

(it's square root is the rms signal)

Discrete Fourier Transform Pair

$$X(f_k) = X_k = \frac{1}{N}\sum_{n=0}^{N-1} x_n e^{2\pi i n k/\mathrm{N}} \qquad f_k = \frac{k}{N\Delta} = 2 f_c \frac{k}{N}, \qquad k = 0, 1, \ldots (N-1)$$

Nyquist Frequency $f_c = \dfrac{1}{2\Delta}$

$$x(t_n) = x_n = \sum_{k=0}^{N-1} X_k e^{-2\pi i n k/\mathrm{N}} \qquad t_n = n\Delta, \qquad n = 0, 1, 2, 3, \ldots (N-1)$$

Note: $X_0 = \dfrac{1}{N}\sum_{n=0}^{N-1} x_n$ $= \langle x \rangle$, the mean value of the signal x_n

Note: X_k and x_n have the same dimensions

DFT of Cross-correlation

$$R_{xy}(k) = \frac{1}{N}\sum_{n=0}^{N-1} r_{xy}(n)e^{2\pi ink/\mathrm{N}} = X_k^{*}\, Y_k$$

DFT of Autocorrelation

$$R_{xx}(k) = \frac{1}{N}\sum_{n=0}^{N-1} r_{xx}(n)e^{2\pi ink/\mathrm{N}} = X_k^{*}\, X_k = |\, X_k\,|^{\mathbf{2}}$$

Parseval's Theorem

$$< x^2 > = \boxed{\frac{1}{N}\sum_{n=0}^{N-1}\left|x_n\right|^2 = \sum_{k=0}^{N-1}\left|X_k\right|^2}$$ = Mean-square signal amplitude

$$= X_0^2 + \sum_{k=1}^{N-1}\left|X_k\right|^2 = < x >^2 + \sum_{k=1}^{N-1}\left|X_k\right|^2$$

$$\boxed{< x^2 > - < x >^2 = \sum_{k=1}^{N-1}\left|X_k\right|^2 = \text{The variance } \sigma_x^2 \ ,}$$

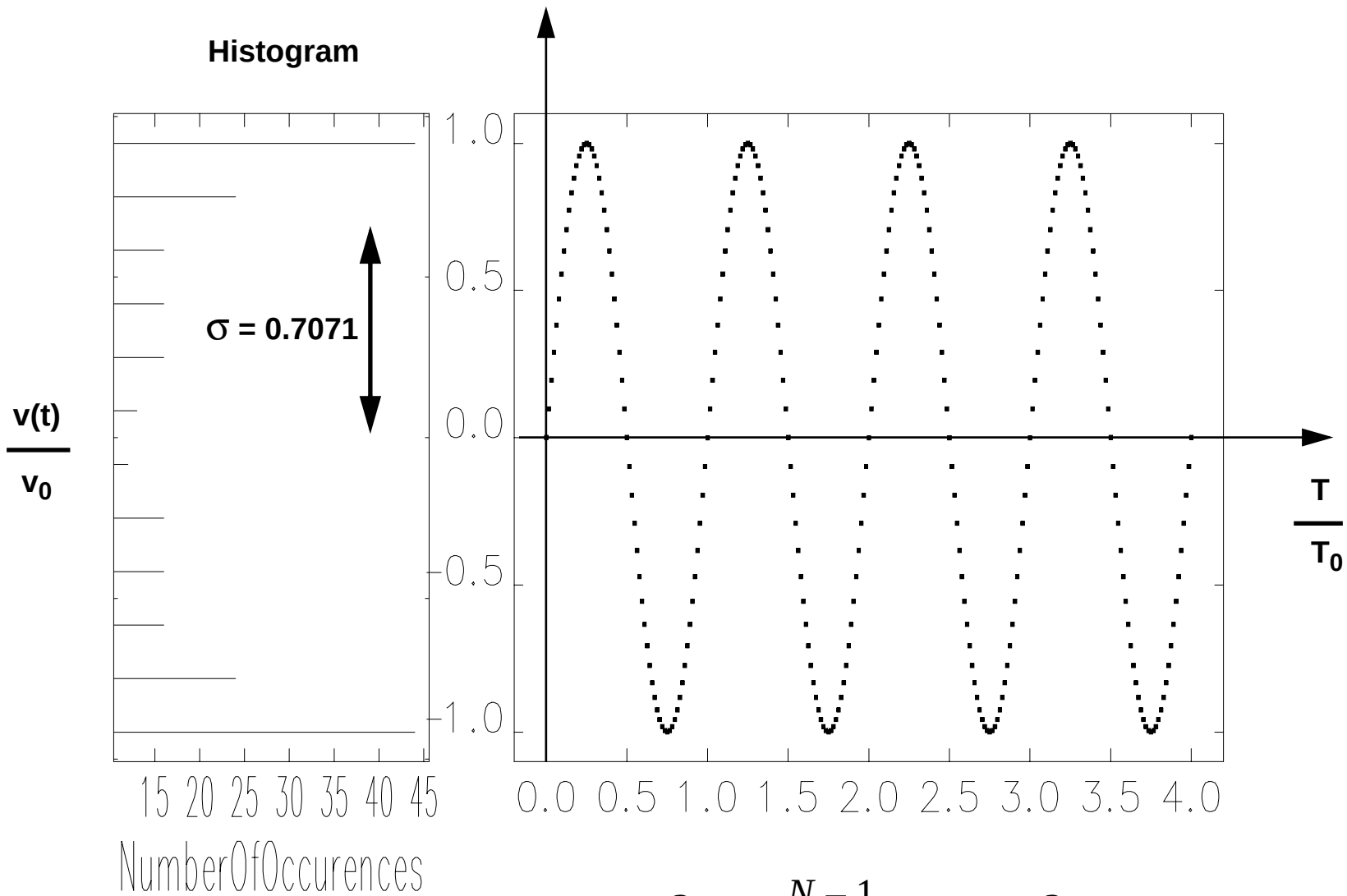
Can also be written*

$$\sigma_x^2 = \frac{1}{N}\sum_{n=0}^{N-1}(x_n - \langle x \rangle)^2$$

σ_x = the Standard Deviation

* The factor $\frac{1}{N}$ is commonly seen to be $\frac{1}{N-1}$, for obtuse reasons explained in, e.g., W. H. Press, etal, "Numerical Recipies in C," New York, Cambridge University Press, p. 611 - The definition given here makes more sense to me, and shouldn't make a hill of beans' difference

Statistics



$$\sigma_v^2 = \frac{1}{N}\sum_{n=0}^{N-1}(v_n - \langle v \rangle)^2$$

Noise Power

$$\sigma_x^2 = \frac{1}{N}\sum_{n=0}^{N-1}(x_n - \langle x\rangle)^2 = \langle x^2\rangle - \langle x\rangle^2 = \sum_{k=1}^{\mathrm{N-1}}\left|X_k\right|^2$$

Note:

$$X_{N\text{-}k} = X_{\text{-}k} = \frac{1}{N}\sum_{n=0}^{N-1} x_n e^{-2\pi i n k/\mathrm{N}} = X_k^*$$

So

$$\boxed{\sigma_x^2 = \sum_{k=1}^{\mathrm{N-1}}\left|X_k\right|^2 = 2\sum_{k=1}^{\frac{N}{2}-1}\left|X_k\right|^2 + \left|X_{N/2}\right|^2}$$

Power Spectral Density

$$x_n = x(t_n), \qquad t_n = n\Delta, \qquad n = 0, 1, 2, 3, \ldots \text{ N-1}$$

$$X_k = X(f_k), \quad f_k = \frac{k}{N\Delta}, \qquad k = 0, 1, \ldots \frac{N}{2}$$

Variance of signal

$$\boxed{\sigma_x^2 = 2\sum_{k=1}^{\frac{N}{2}-1} \left|X_k\right|^2 + \left|X_{N/2}\right|^2}$$

The amount of mean square signal per unit of frequency

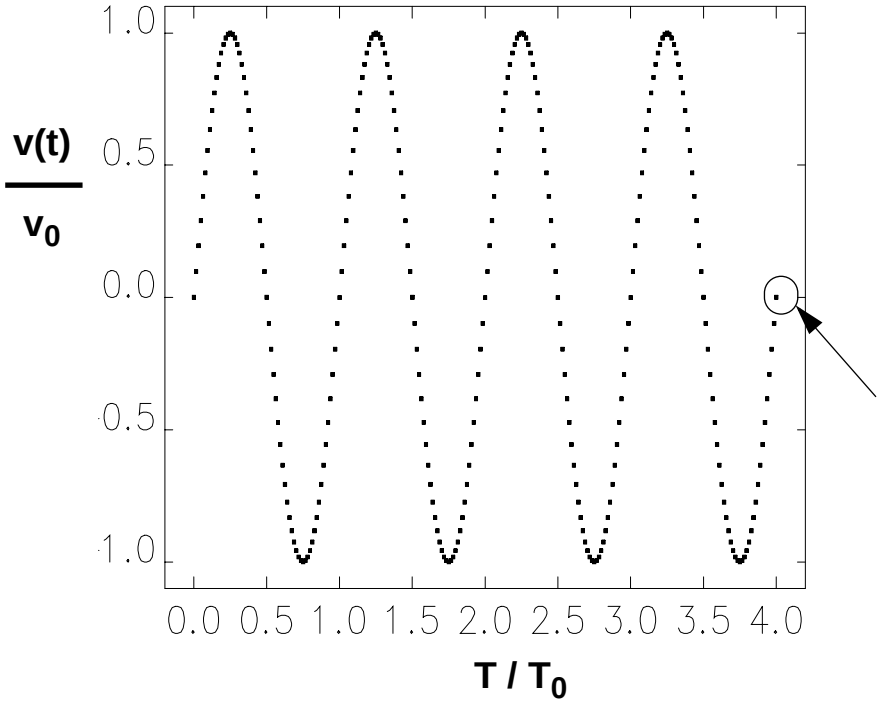
in the frequency span from $f_k - \frac{1}{2N\Delta}$ to $f_k + \frac{1}{2N\Delta}$ is approximately

$$P(f_k) = 2\left|X_k\right|^2/(\Delta f) = 2N\Delta\left|X_k\right|^2, \qquad k = 1, 2, \ldots \frac{N}{2} - 1$$

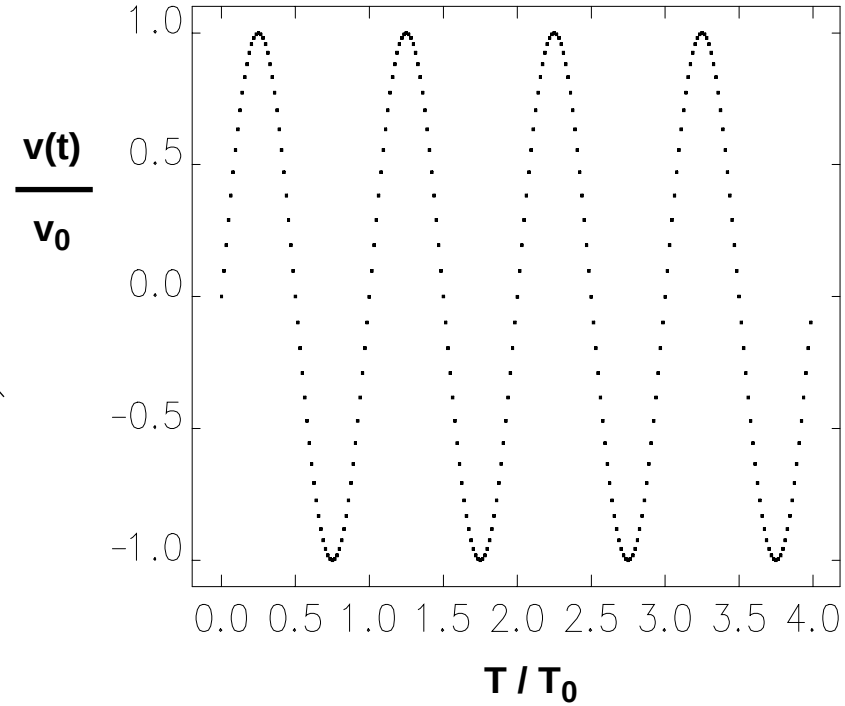
$$= N\Delta\left|X_{N/2}\right|^2, \qquad k = \frac{N}{2}$$

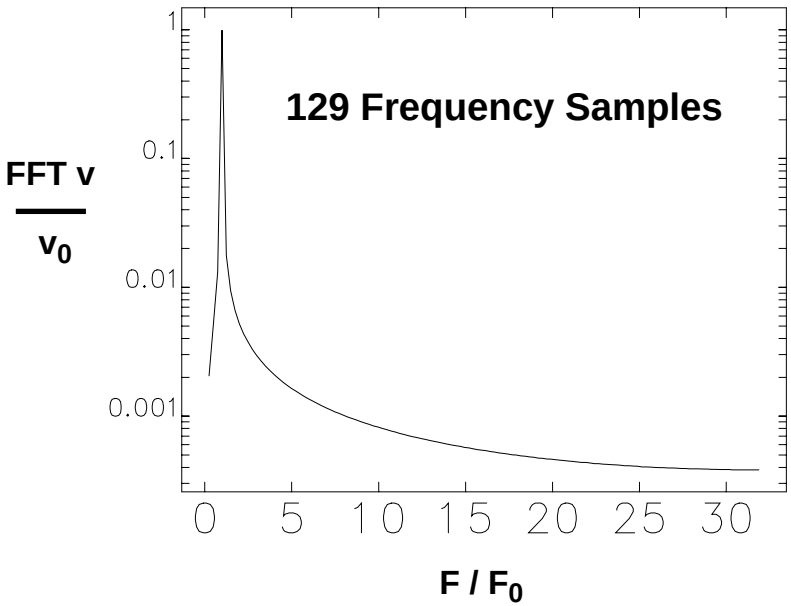
Note: The power spectral density P is measured in units = (dimension of x)^2 / frequency

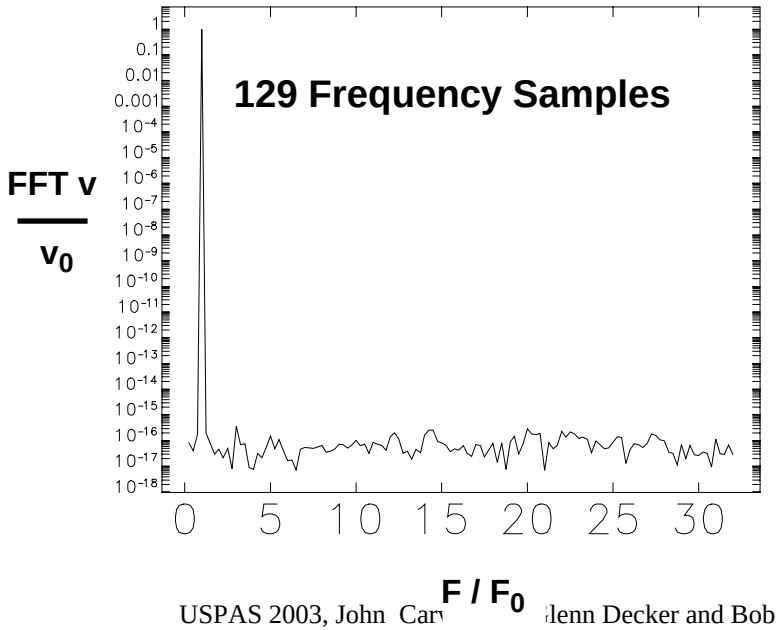
257 Data Points,
σ = **0**.7071067811865476

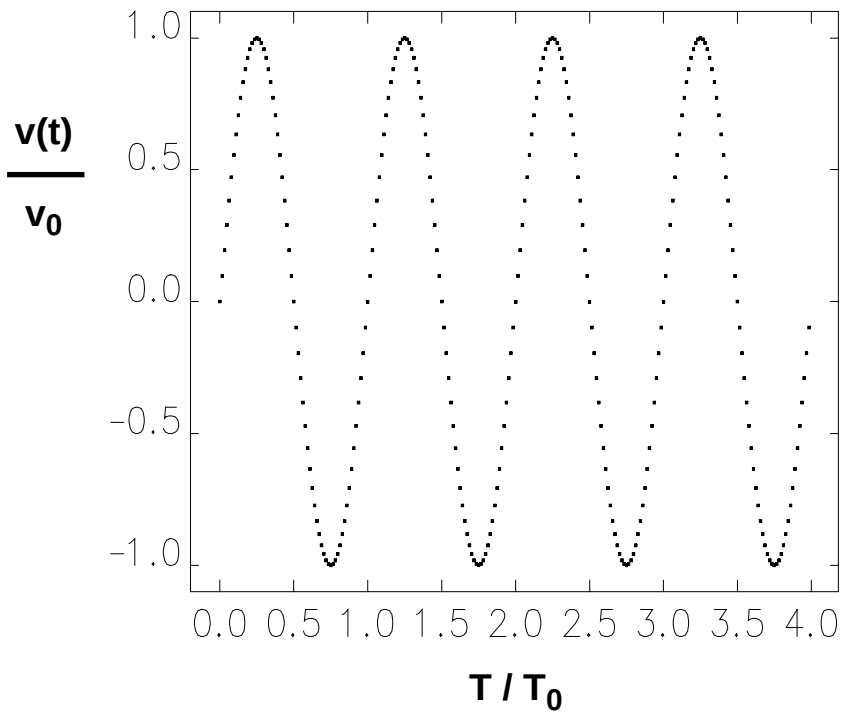


256 Data Points,
σ =7.084919084320763









$$\frac{\mathbf{v}(t_n)}{\mathbf{v_0}} = \frac{\mathbf{v}_n}{\mathbf{v_0}} = Sin\left(\frac{2\pi t_n}{T_0}\right) \qquad t_n = n\Delta$$

$$N = 256$$

Exactly 4 Periods = N = 256 Samples, so

$$T_0 = 64\Delta \longrightarrow \frac{2\pi t_n}{T_0} = 2\pi\frac{n}{64}$$

$$\frac{\mathbf{v}_n}{\mathbf{v_0}} \equiv v_n = \frac{e^{i\left(\frac{2\pi n}{64}\right)} - e^{-i\left(\frac{2\pi n}{64}\right)}}{2i}$$

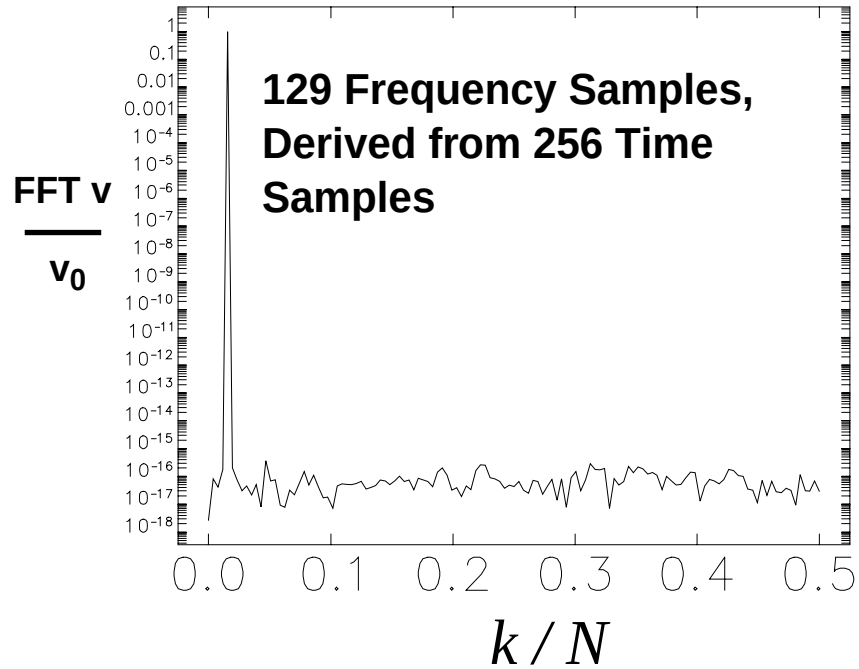
$$V_k = \frac{1}{N}\sum_{n=0}^{N-1} v_n e^{2\pi i n k/\mathrm{N}} = \frac{1}{N}\sum_{n=0}^{N-1}\left[\frac{e^{i\left(\frac{2\pi n}{256}\right)4} - e^{-i\left(\frac{2\pi n}{256}\right)4}}{2i}\right] e^{i\left(\frac{2\pi n}{256}\right)k}$$

Orthogonality Condition:

$$\sum_{n=0}^{N-1} e^{-i\left(\frac{2\pi n}{N}\right)k} e^{i\left(\frac{2\pi n}{N}\right)m} = N\delta_{k,m}$$

$$0 \le k \le N-1 \qquad \delta_{k,m} = \begin{cases} 1, (k = m) \\ 0, \text{Otherwise} \end{cases}$$

$$0 \le m \le N-1$$



so

$$V_k = \left(-\frac{\delta_{k,4}}{2i} + \frac{\delta_{-k,4}}{2i}\right) = \left(-\frac{\delta_{k,4}}{2i} + \frac{\delta_{N-k,4}}{2i}\right)$$

$$f_k = \frac{k}{N\Delta} \quad , \quad \Delta = \text{sampling period}$$

“Simple” Sine Wave Spectrum

$$V_k = -\frac{\delta_{k,4}}{2i}$$

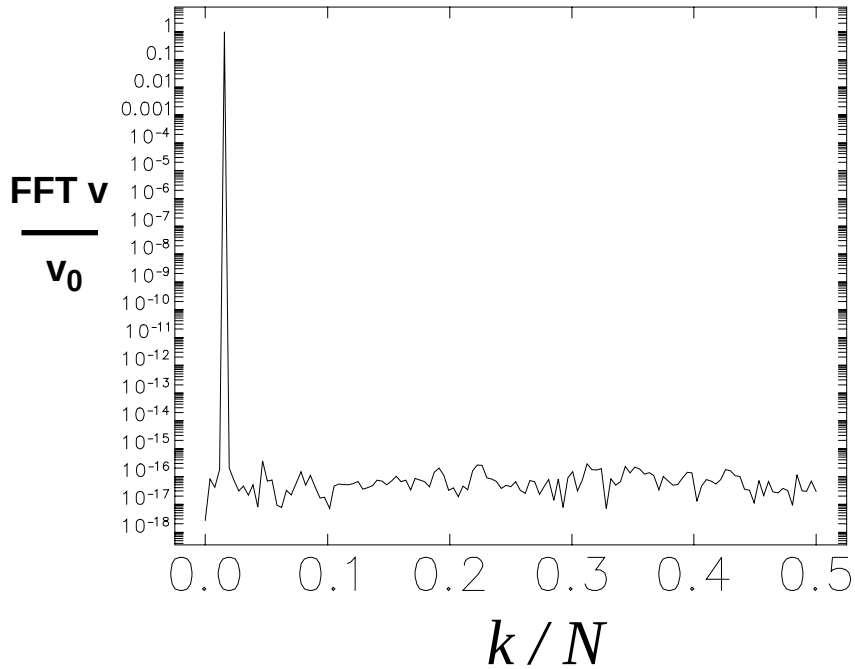
$$0 \le k \le \frac{N}{2}$$

$$\sigma_v^2 = 2\sum_{k=1}^{\frac{N}{2}-1} \left|V_k\right|^2 + \left|V_{N/2}\right|^2 = \frac{1}{2}$$

$$\longrightarrow \quad \sigma_v = \frac{1}{\sqrt{2}} = 0.7071067811865477$$

$$P(f_k) = 2N\Delta \left|V_k\right|^2 = \frac{N\Delta}{2}\delta_{k,4}$$

$$\sum_{k=1}^{\frac{N}{2}-1} P(f_k)\Delta f = \sum_{k=1}^{\frac{N}{2}-1} P(f_k)\frac{1}{N\Delta} = \frac{1}{2}$$



$$f_k = \frac{k}{N\Delta}$$

$$V_k = -\frac{\delta_{k,4}}{2i} \qquad 0 \le k \le \frac{N}{2}$$

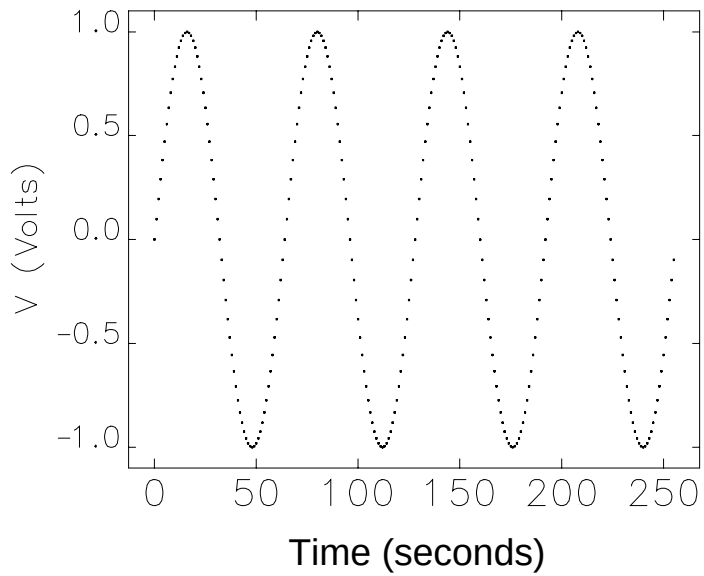
$$\sigma_v^2 = 2\sum_{k=1}^{\frac{N}{2}-1} |V_k|^2 + |V_{N/2}|^2 = \frac{1}{2}$$

$$\sqrt{2}|V_k| = \frac{1}{\sqrt{2}}\delta_{k,4} = \text{RMS Signal}$$

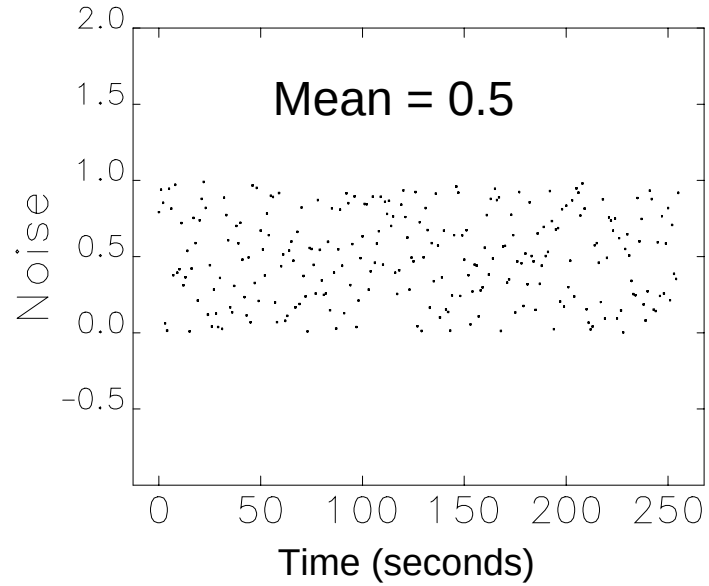
Finally $\dfrac{\text{FFT v}}{\text{v}_0} = \sqrt{2} * \text{RMS Signal} = 2|V_k| = \delta_{k,4} = \text{Peak Signal}$

A (slightly) more realistic example

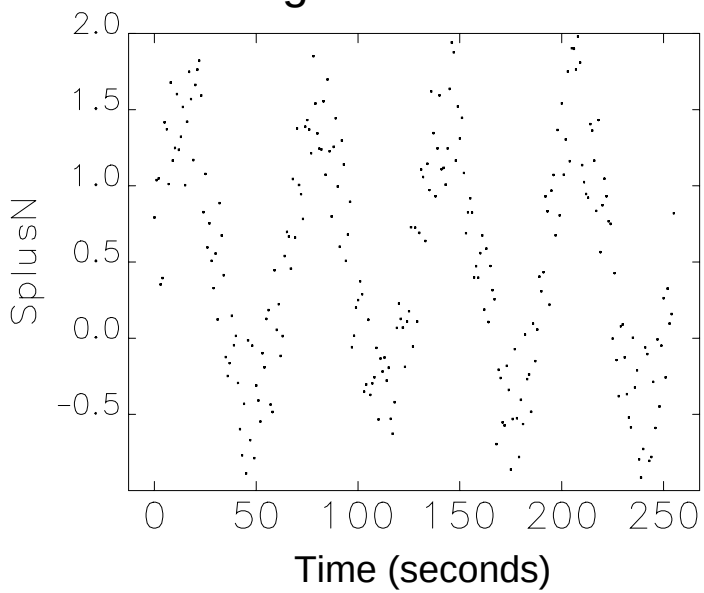
Signal



Noise



Signal + Noise

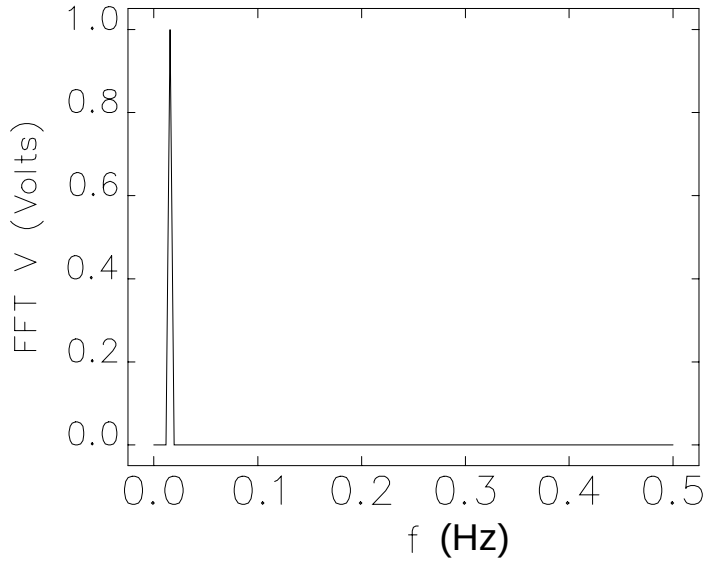


VMean (Volts) = -6.245005e-17
VRms (Volts) = 7.071068e-01
VStDev (Volts) = 7.084919e-01

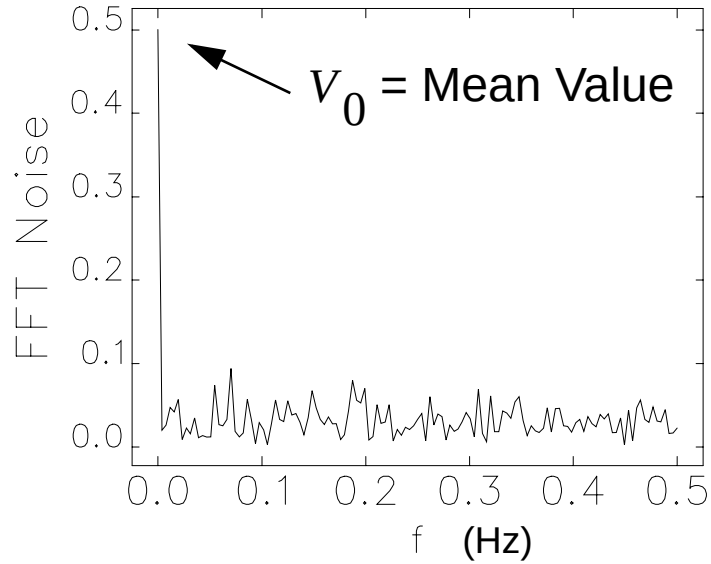
NoiseMean = 5.005493e-01
NoiseRms = 5.769418e-01
NoiseStDev = 2.874632e-01

SplusNMean = 5.005493e-01
SplusNRms = 8.932156e-01
SplusNStDev = 7.412359e-01

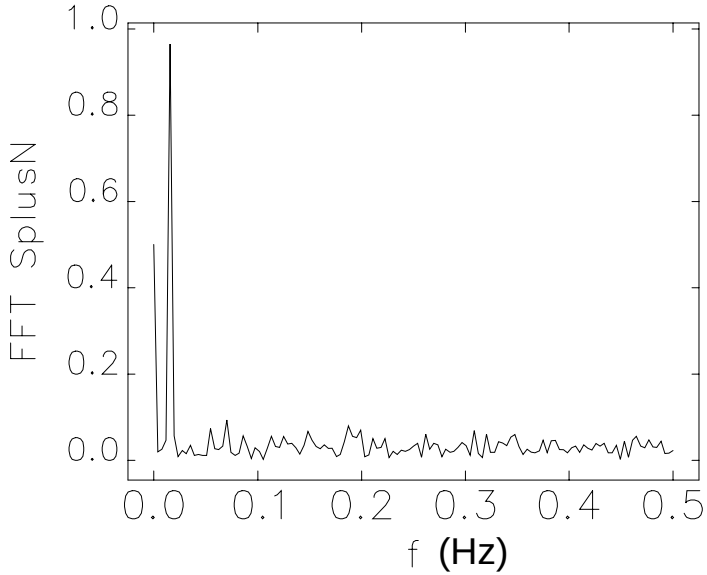
Peak Signal Spectrum $2|V_k|$



Peak Noise Spectrum



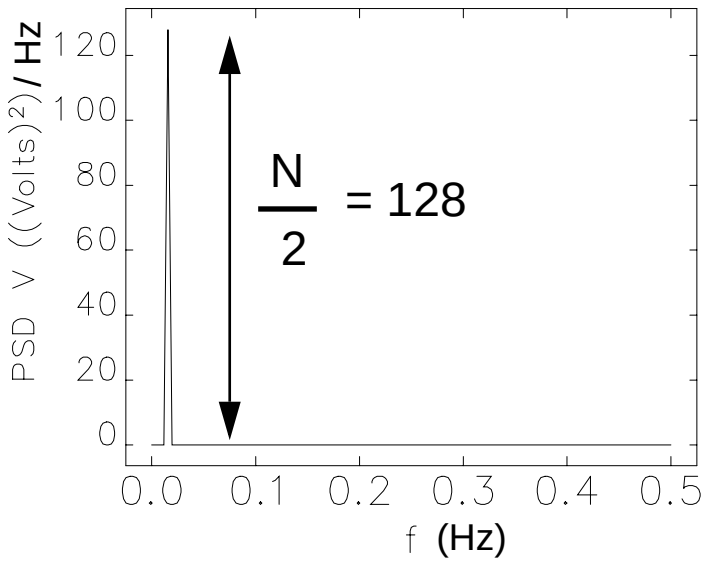
Signal + Noise



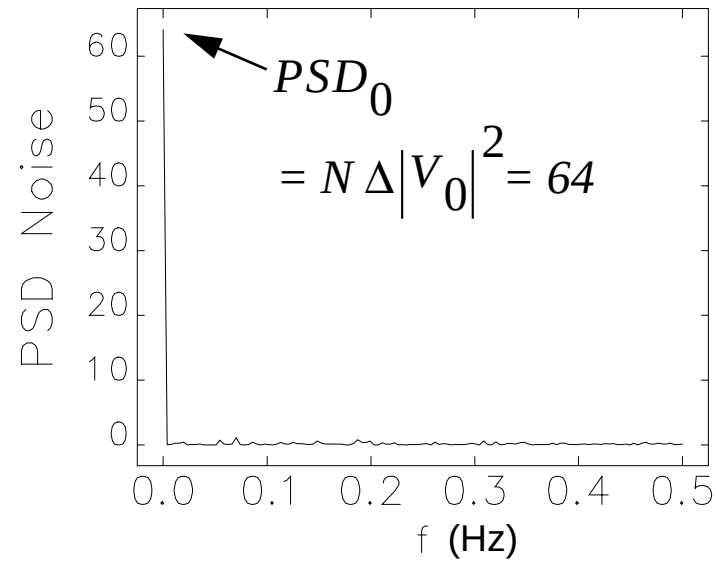
$$f_k = \frac{k}{N\Delta}$$, Δ = sampling period
= 1 second for this example

Power Spectral Densities

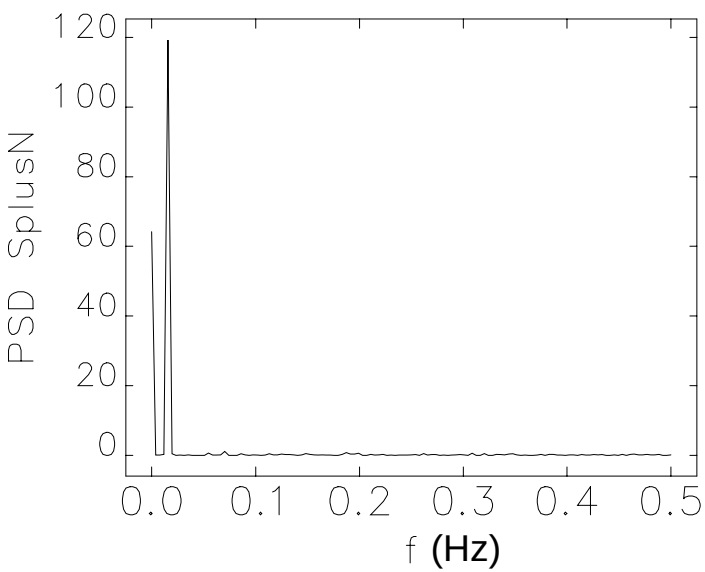
Signal



Noise



Signal + Noise

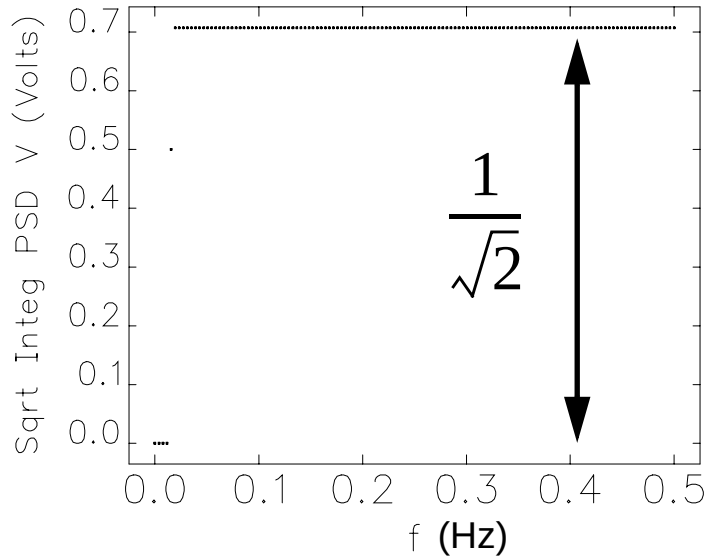


$$P(f_k) = 2N\Delta \left|V_k\right|^2, \quad \Delta = 1$$

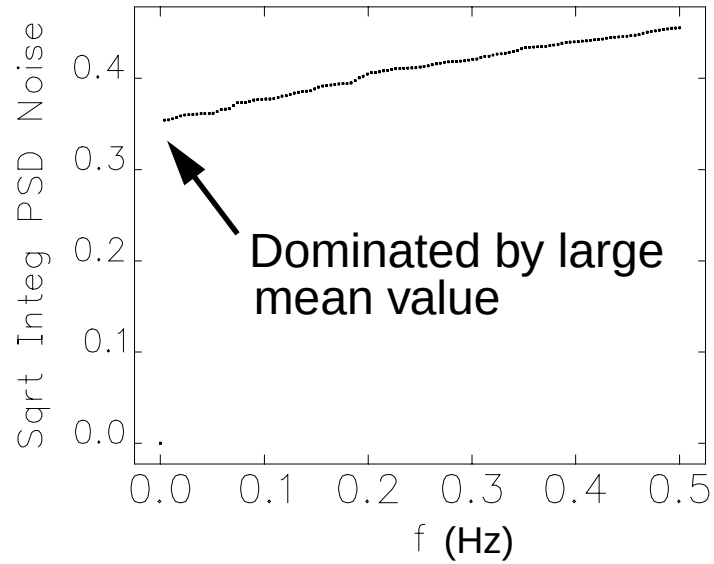
(except k = 0 and k = N / 2)

Integrated Power Spectral Densities

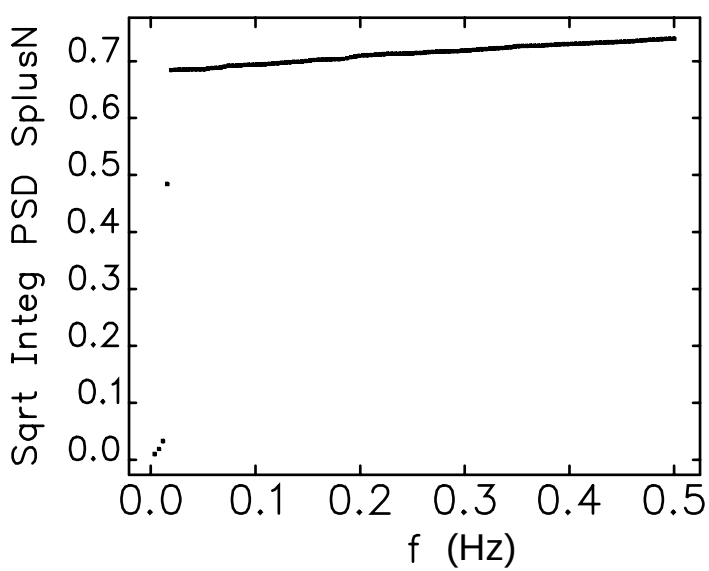
Signal



Noise



Signal + Noise



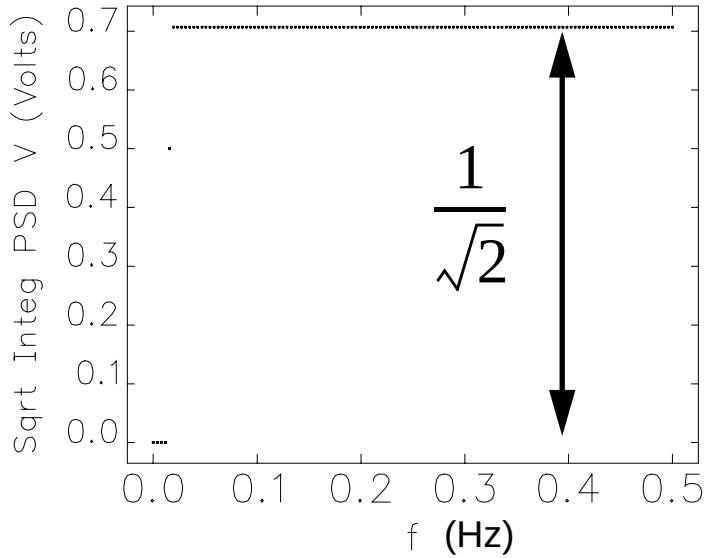
$$Vrms = \sqrt{\int_0^f P(f)\,df}$$

Note: $\int_0^f P(f)\,df$ *is shorthand for*

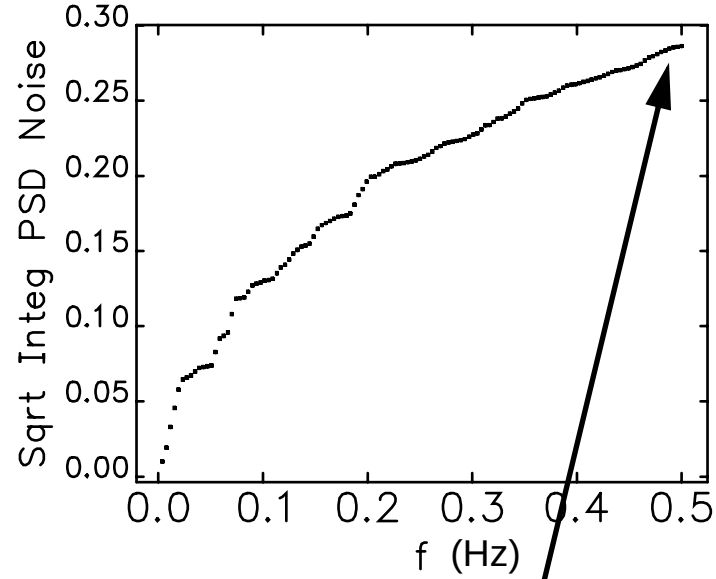
$$\sum_{k'=0}^{k} P(f_{k'})\Delta f$$

Integrated Mean-Suppressed Power Spectral Densities

Signal

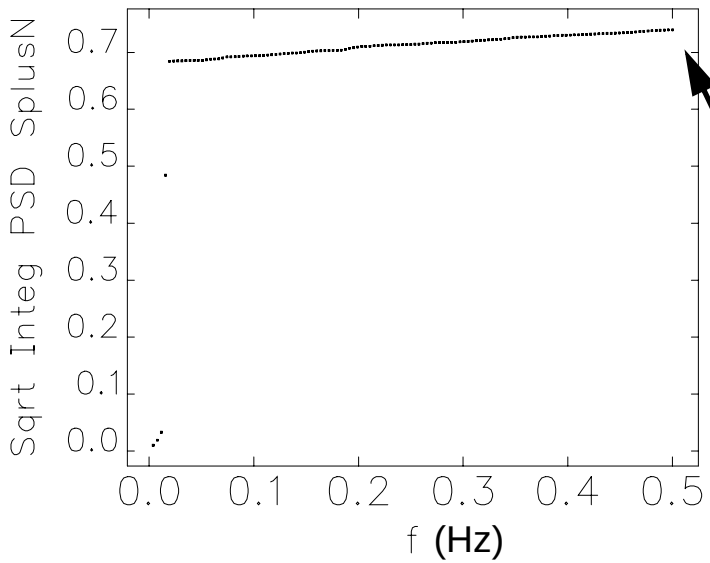


Noise



NoiseStDev = 2.874632e-01

Signal + Noise

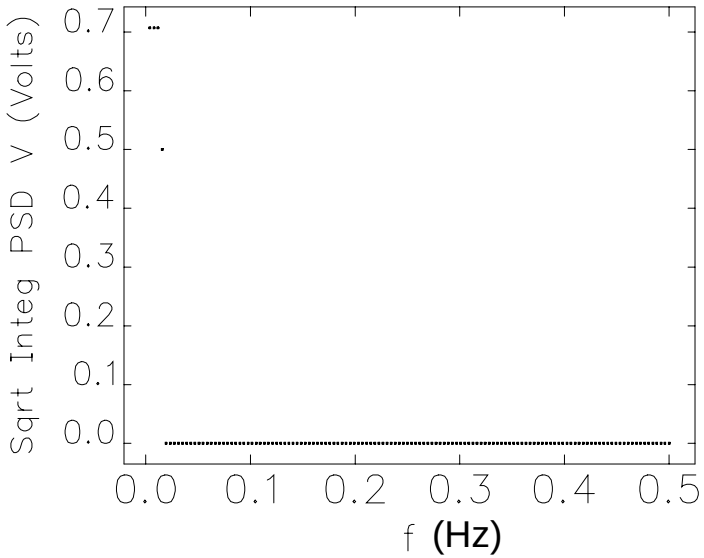


S+N StDev = 0.74 < (SignalStDev^2+NoiseStDev^2)$^{1/2}$ = 0.76

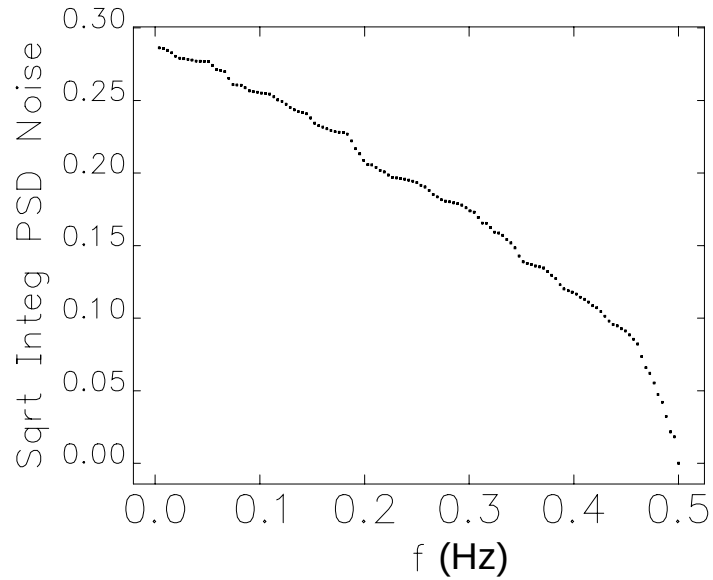
(Both Signal + Noise are non-Gaussian)

Reverse-Integrated Mean-Suppressed Power Spectral Densities

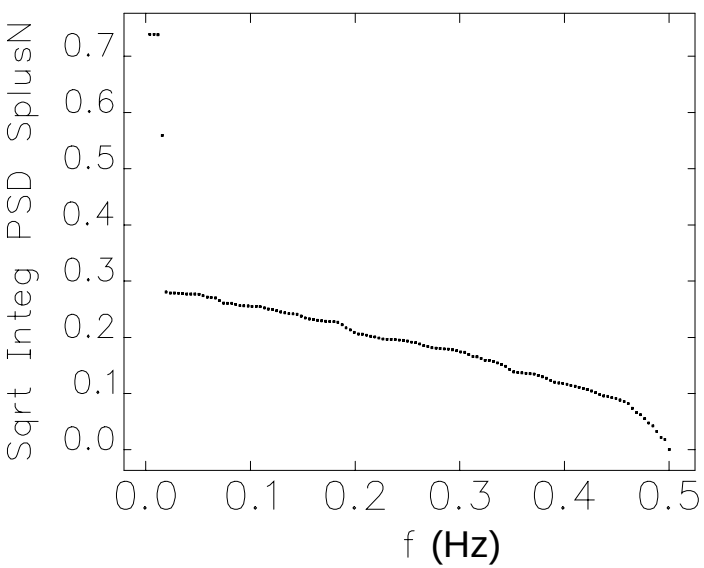
Signal



Noise



Signal + Noise



$$Vrms = \sqrt{\int_{f_c}^{f} P(f)\,df}$$

$$f < f_c = \frac{1}{2\Delta} = \frac{1}{2}\ \text{Hz}$$

Useful for resolving high frequency components in the presence of large signal at low frequency.

Homework Problem - Computer Lab

Given the power spectrum shown, together with the “voltage spectrum” (.xls file) -

1) Is this a peak voltage spectrum, rms, or something else?
Hint 0 dBm = 1 mW rms into a 50 ohm load.

2) What is the relation between the given voltage spectrum and $|V_k|$?

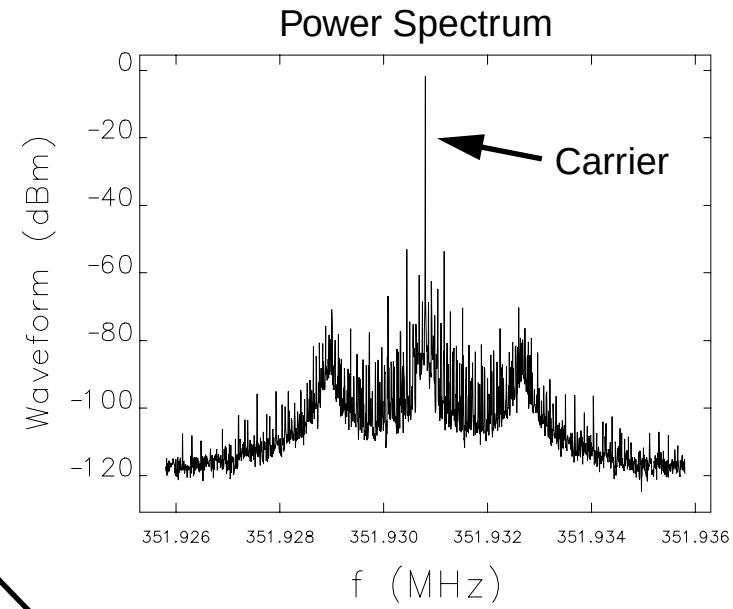
3) Plot the power spectral density in (Volts^2 / Hz) vs. deltaFreq

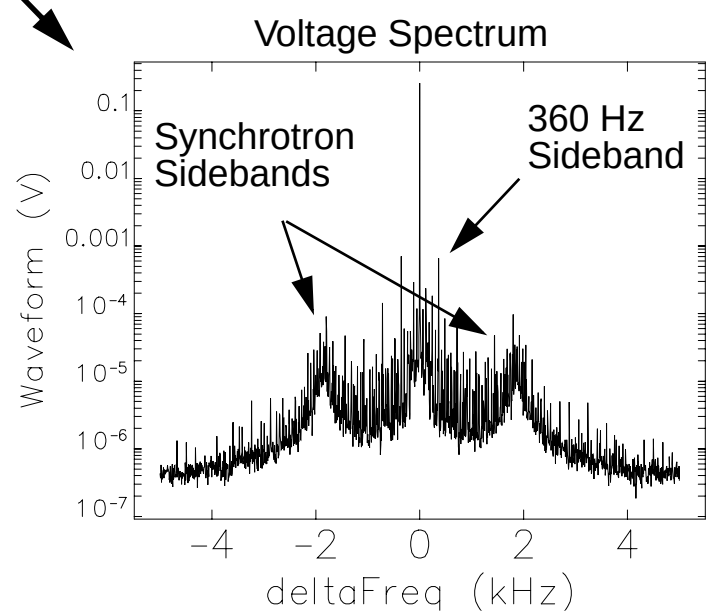
4) What is the voltage variance and standard deviation in the full 10 kHz frequency span, not counting the carrier.?
Note - deltaFreq < 0 does not imply V_k -> V_k * = V_{-k}

5) What is the ratio of rms voltage in the upper 360 Hz sideband to the overall rms voltage corresponding to deltaFreq > 0 ?

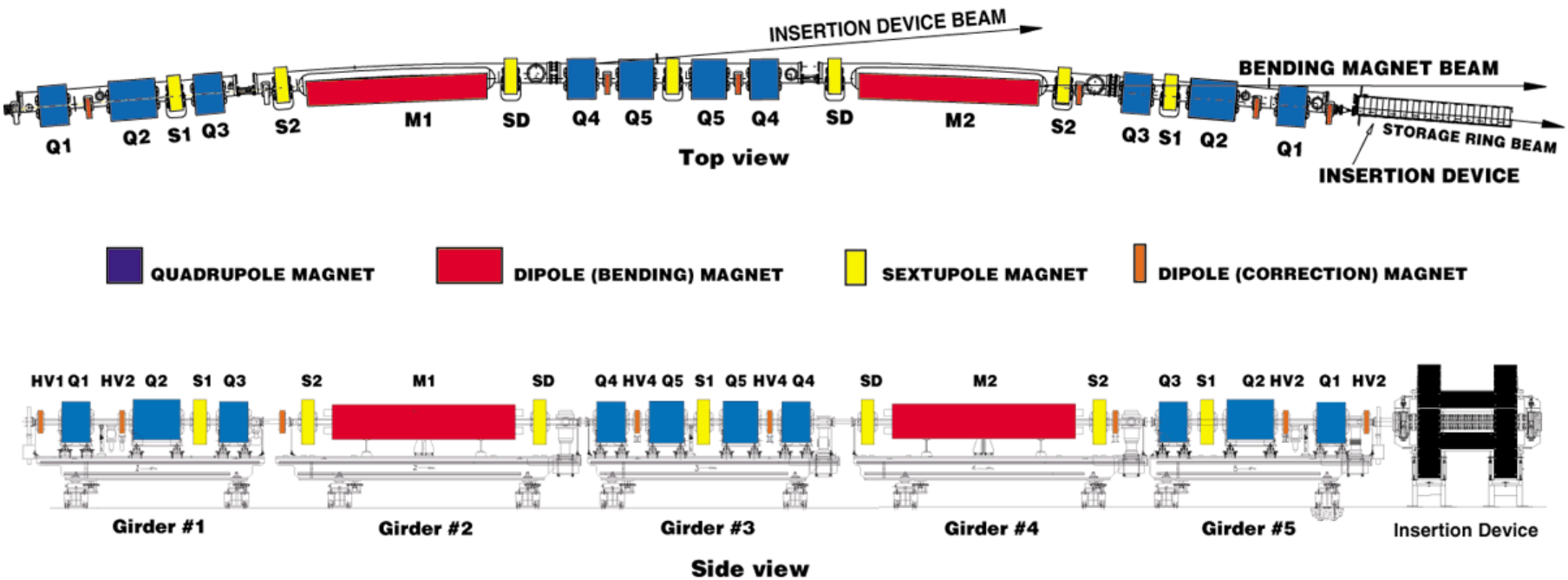
6) What is the ratio of the rms voltage in the 360 Hz sidband to the rms voltage associated with the upper synchrotron sideband between 1.5 and 2.5 kHz.

7) Plot the square root of the reverse-integrated power spectral density from 5 kHz down to, but not including deltaFreq=0, in volts rms. Simpson rule integration is good enough.





Betatron and Synchrotron Oscillations in Space and Time



One of the 40 Sectors of the Advanced Photon Source

Betatron Oscillations

Observations

$$x(s) = [W_x \, \beta_x(s)]^{1/2} \cos[\psi_x(s) - \psi_{0x}]$$

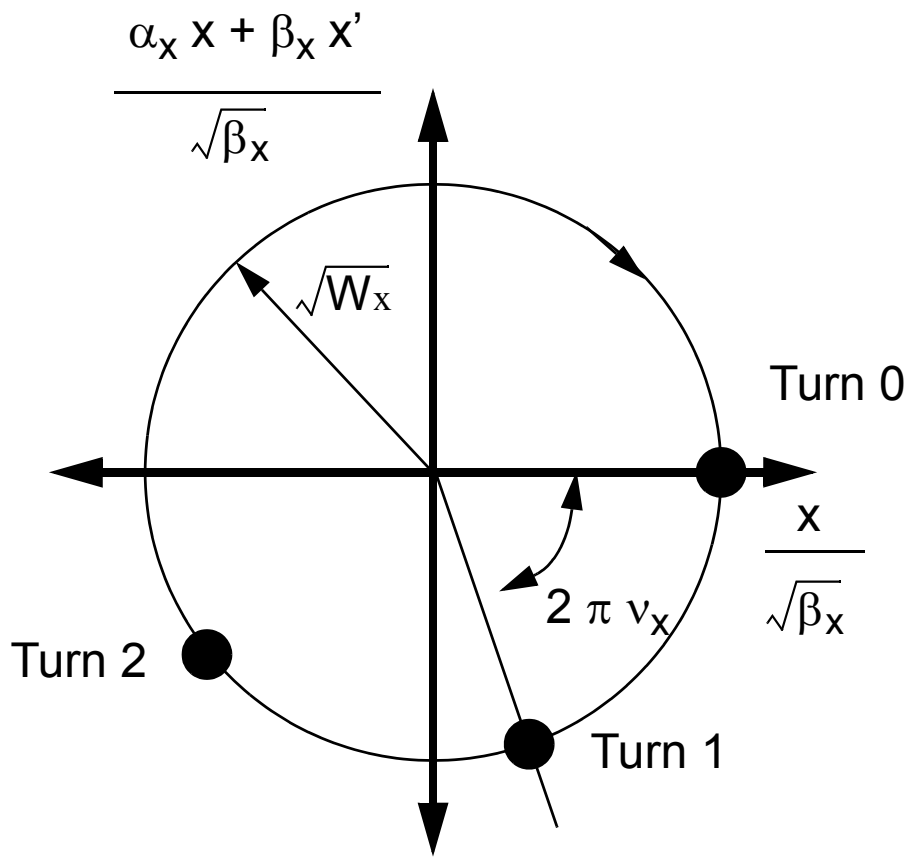
1) To first order, a bunch of electrons behaves like one gigantic super-electron satisfying the above relation.

2) The curvilinear coordinate s in the above can equally well be understood to represent time, according to the relation s = vt = ct

3) Since no human can be in all places at all times, measurements of x(s) are observable only in a quantized fashion dictated by the bunch structure of the beam. For any particular bunch, any detector bolted to the vacuum chamber can observe the bunch's transverse position x(s) only at times corresponding to $s = s_0+cT_0$, $s+2cT_0$,... etc, where T_0 = the revolution period, cT_0 = the ring circumference L.

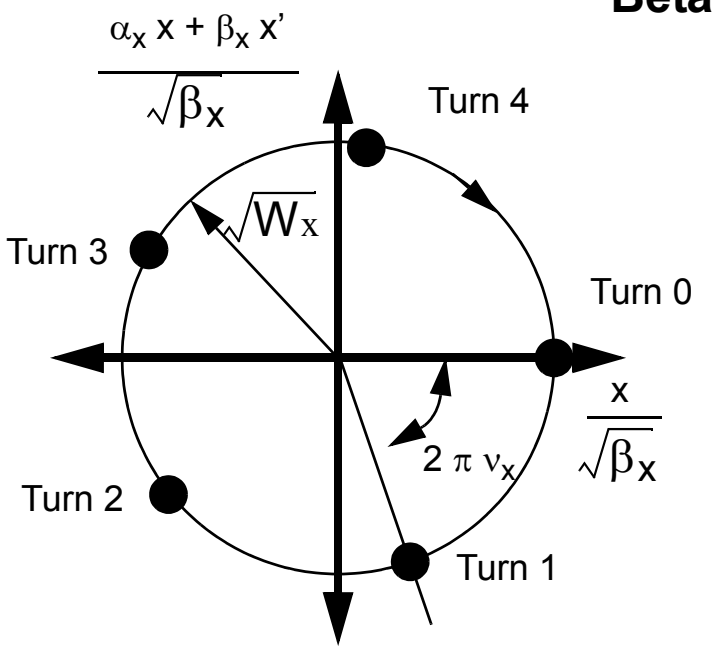
4) $\beta_x(s_0+L) = \beta_x(s_0) = \beta_x$, i.e. every detector gets its own unique β_x; and
$\psi_x(s_0+L) = \psi_x(s_0) + 2\pi\nu_x$,
i.e. the tune ν_x sets the time scale for betatron oscillations

Normalized Electron Phase Space at Fixed Location s



Unlike the angle x', the position variable x is directly observable at a fixed location along the ring circumference

Betatron Oscillations



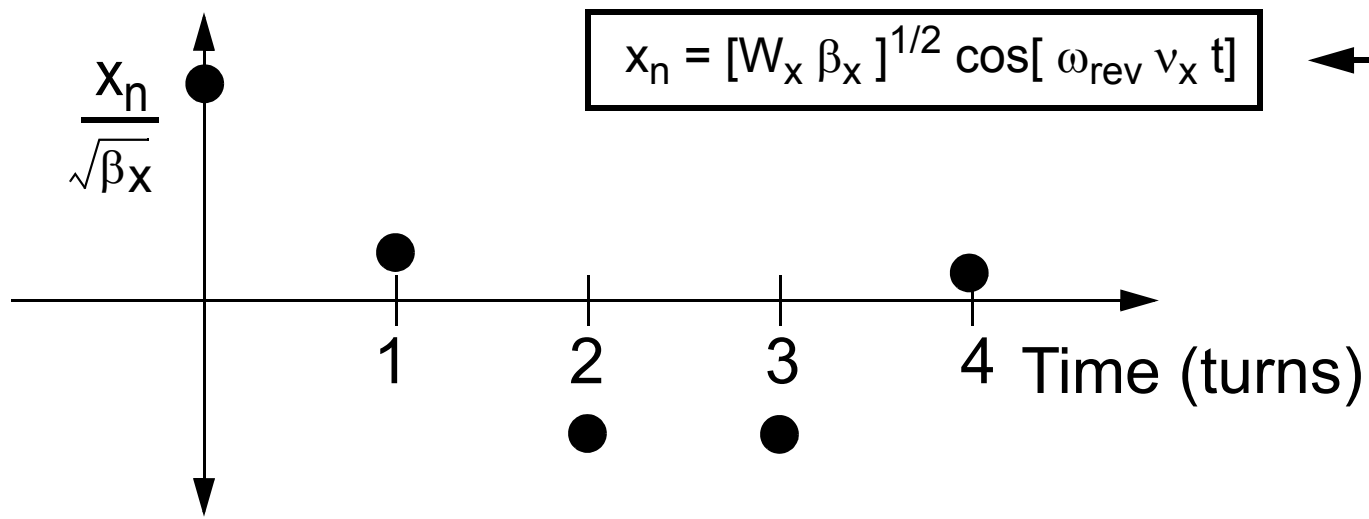
$$x(s) = [W_x\, \beta_x(s)]^{1/2} \cos[\psi_x(s) - \psi_{0x}]$$

$$x_n = [W_x\, \beta_x]^{1/2} \cos[2\pi\, \nu_x\, n]$$

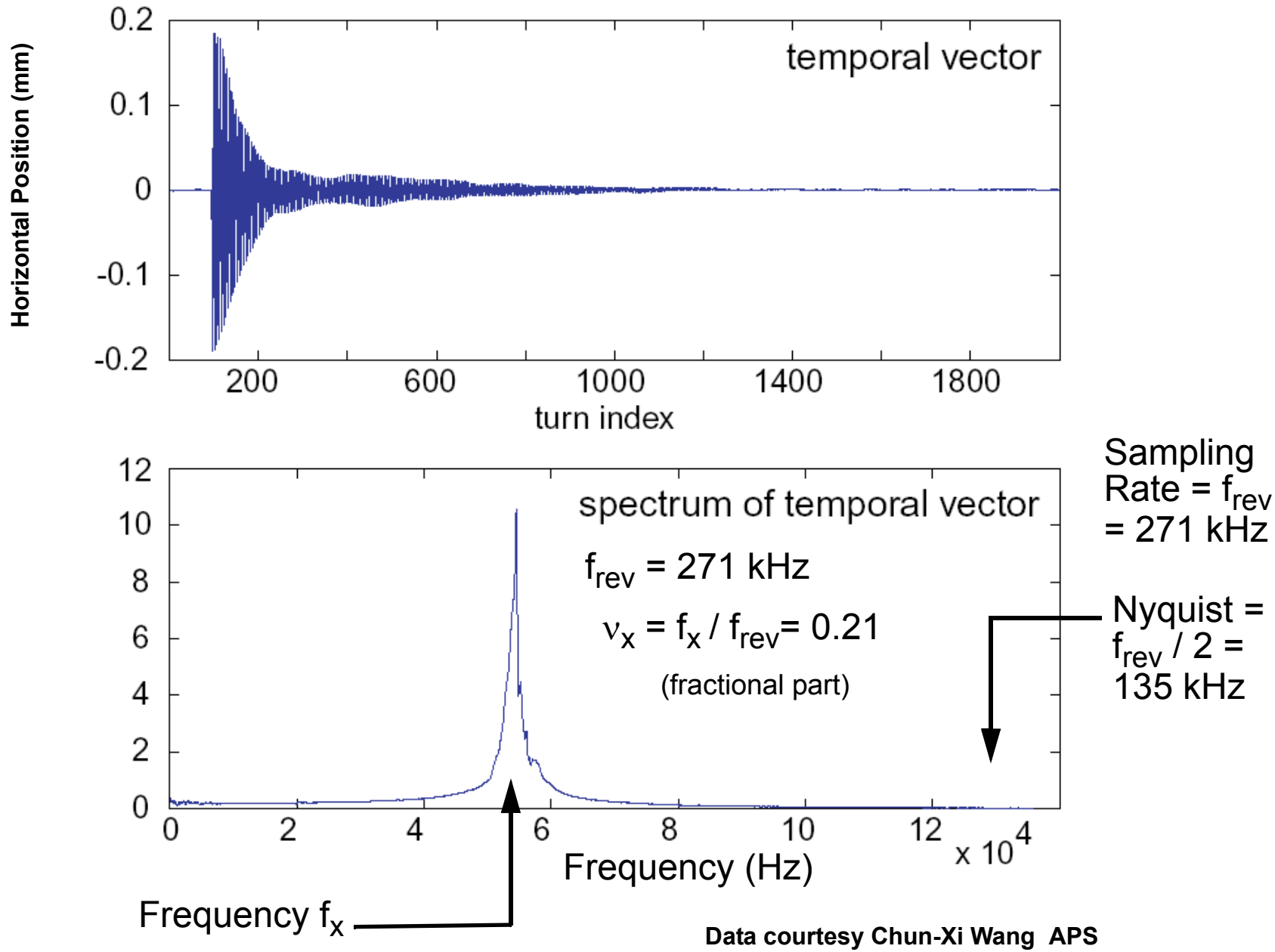
$$n = t / T_0 = f_{rev}\, t$$

$$x_n = [W_x\, \beta_x]^{1/2} \cos[2\pi\, f_{rev} \nu_x\, t]$$

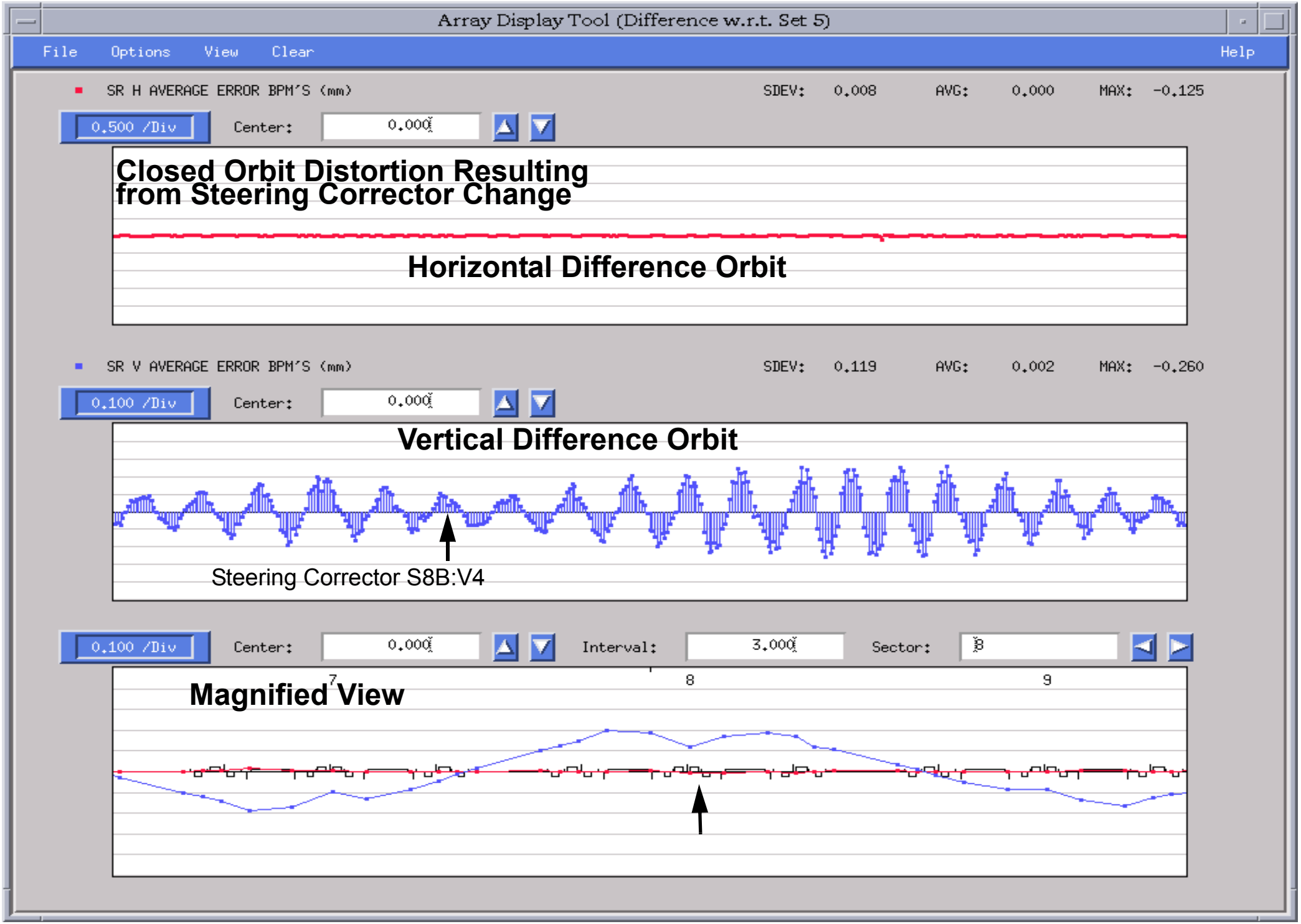
$$\boxed{x_n = [W_x\, \beta_x]^{1/2} \cos[\omega_{rev}\, \nu_x\, t]}$$



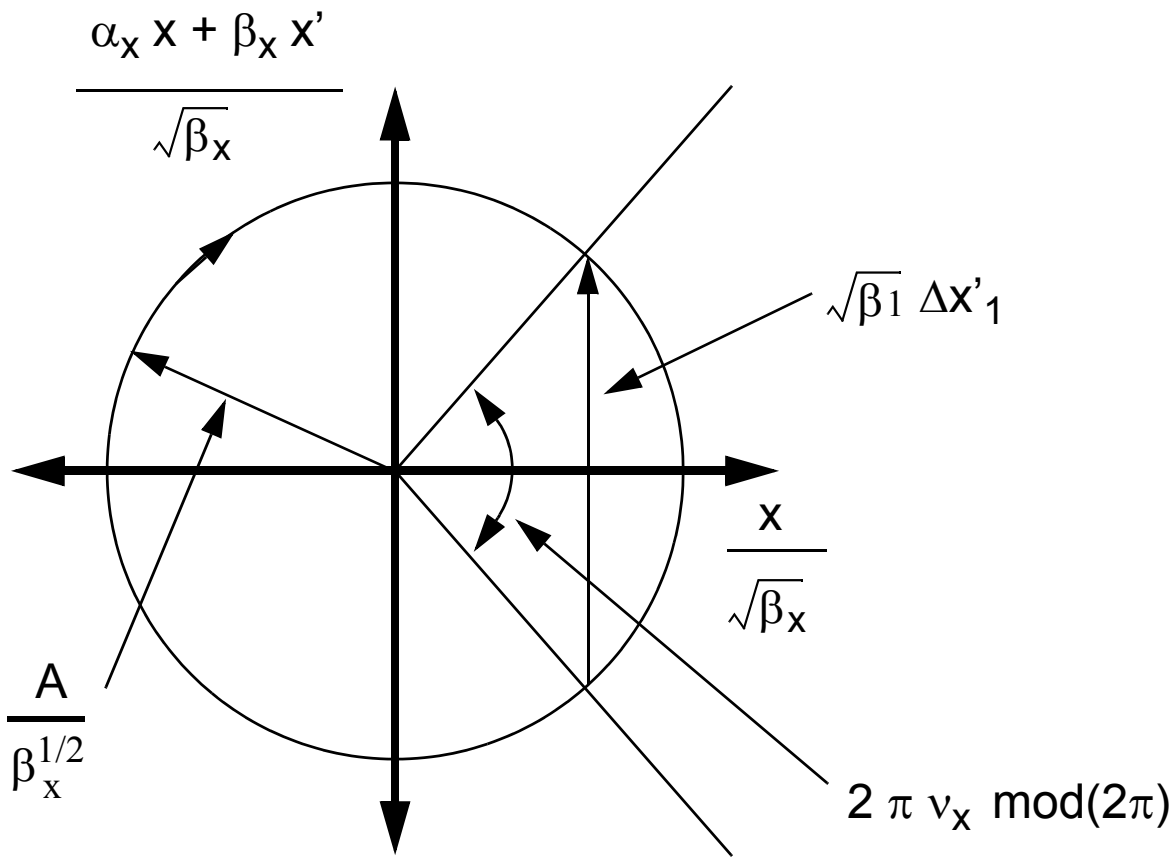
Betatron oscillation resulting from firing injection kicker



Data courtesy Chun-Xi Wang APS


Array Display Tool (Difference w.r.t. Set 5)
File
Options
View
Clear
Help
SR H AVERAGE ERROR BPM'S (mm)
SDEV: 0.008
AVG: 0.000
MAX: -0.125
0.500 /Div
Center: 0.000
Closed Orbit Distortion Resulting from Steering Corrector Change
Horizontal Difference Orbit
SR V AVERAGE ERROR BPM'S (mm)
SDEV: 0.119
AVG: 0.002
MAX: -0.260
0.100 /Div
Center: 0.000
Vertical Difference Orbit
Steering Corrector S8B:V4
0.100 /Div
Center: 0.000
Interval: 3.000
Sector: 8
7
8
9
Magnified View

Closed Orbit Distortion Resulting from Single Corrector Change $\Delta x'_1$ Phase Space Representation



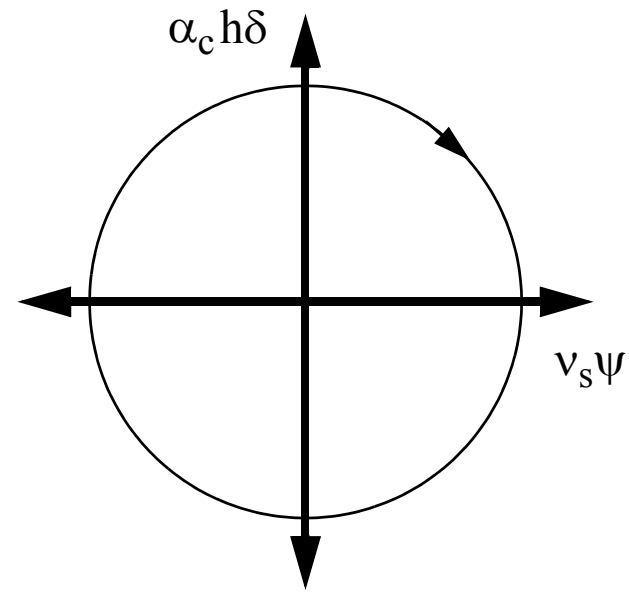
Synchrotron Oscillations

$$\psi = \psi_0 \cos \Omega_s t$$

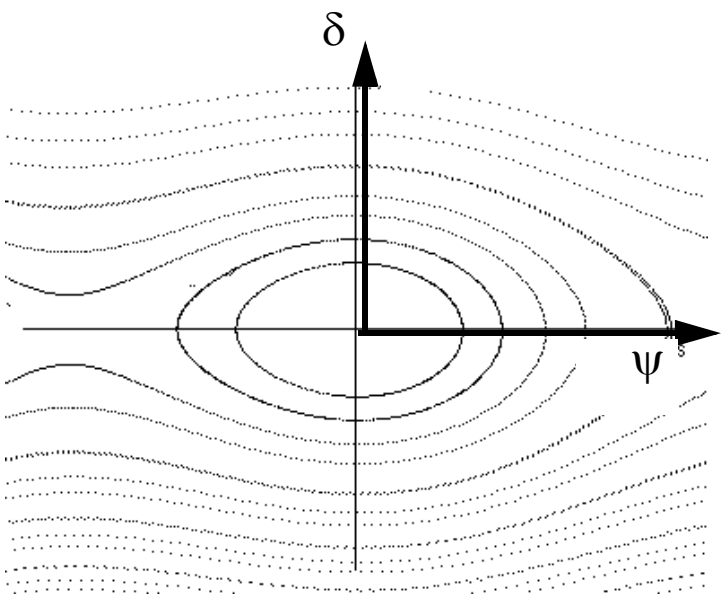
$$\Omega_s = \sqrt{\frac{\alpha_c \omega_{RF} e V_{RF}^0 \cos \phi_s}{E_0 T_0}}$$

$$\delta = \delta_{max} \sin \Omega_s t$$

Normalized Small-Amplitude Phase Space



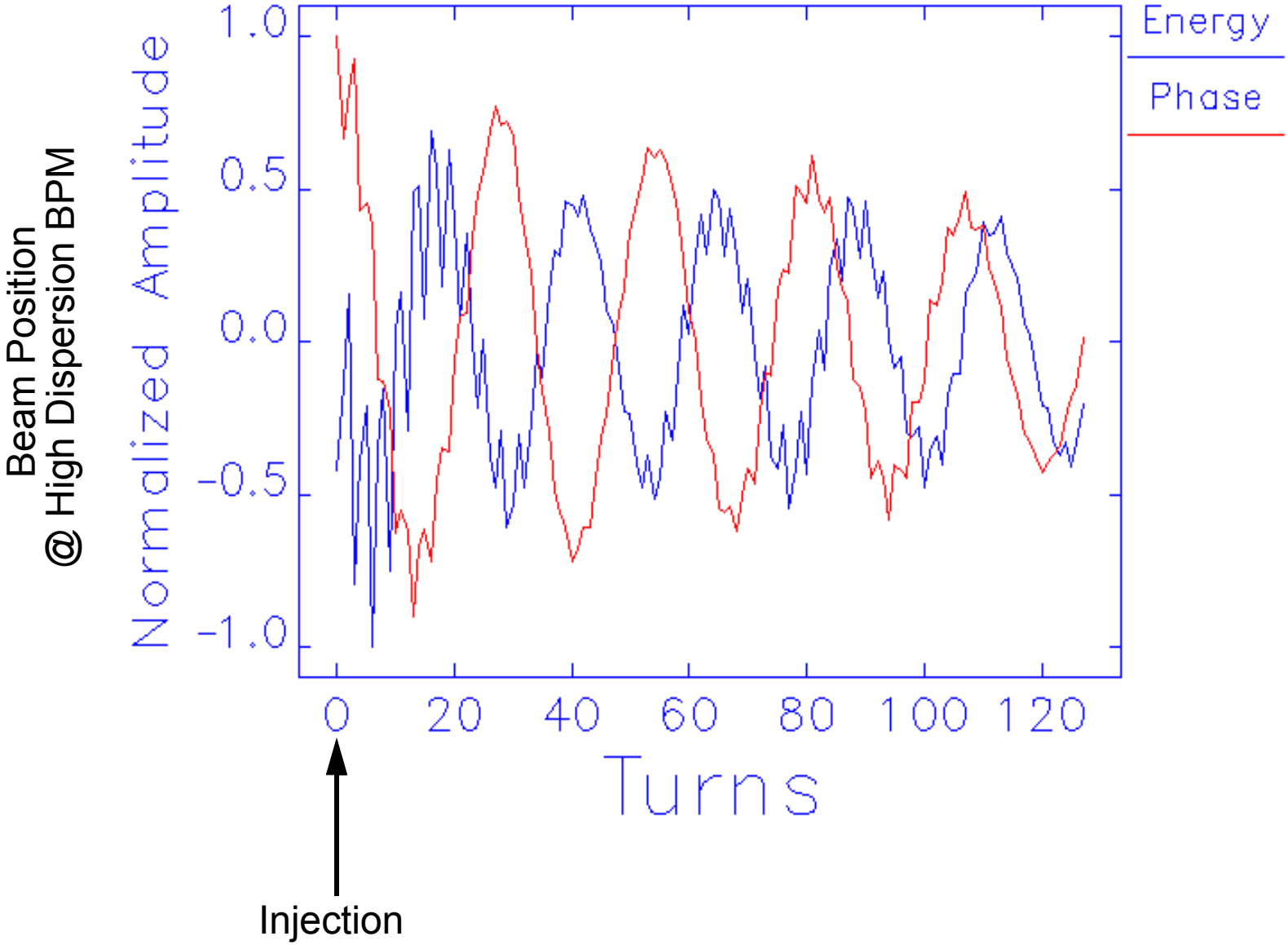
$$\left(\alpha_c{}^2 h^2\right)\delta^2 + \left(\nu_s^2\right)\psi^2 = \text{constant}$$



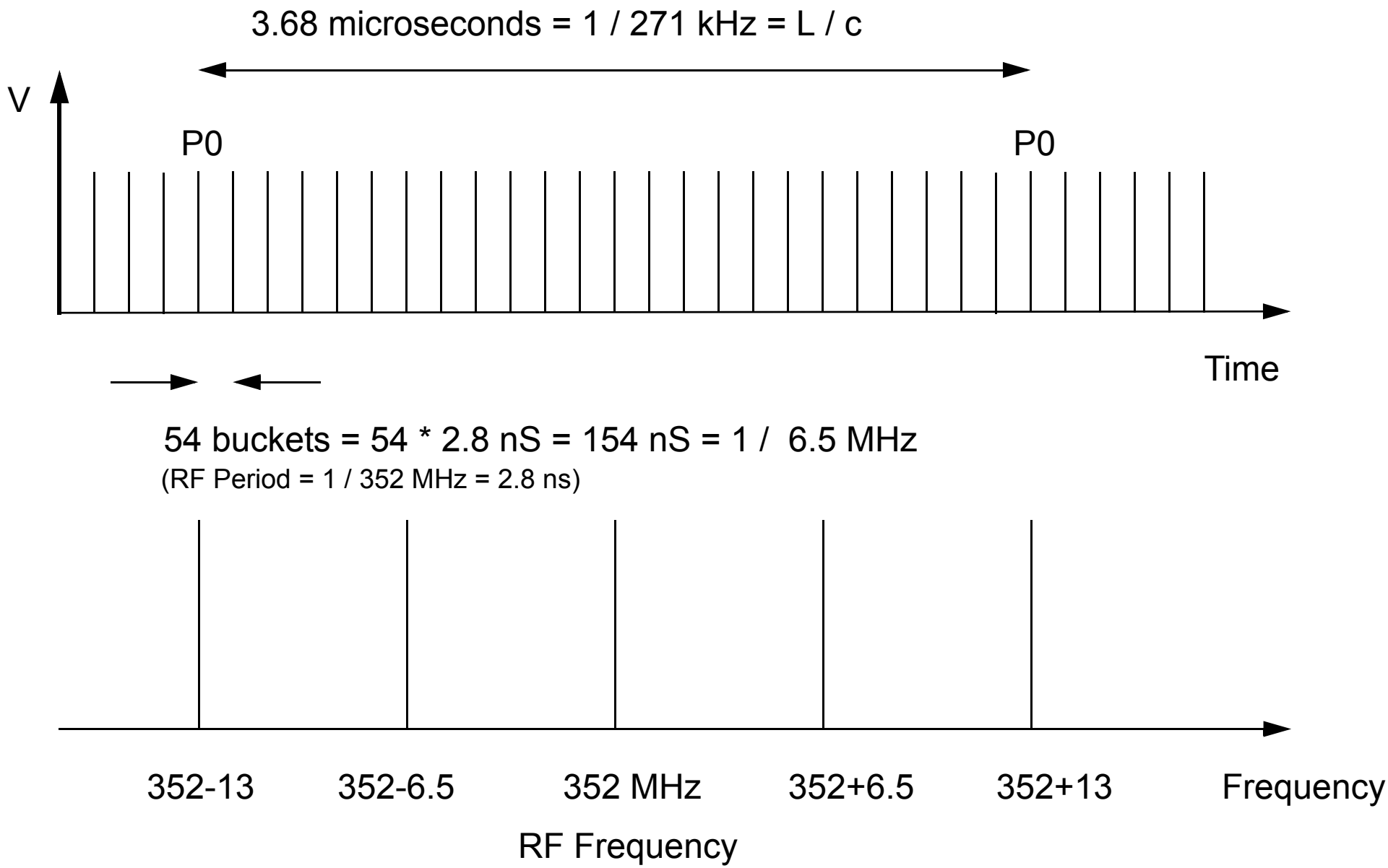
Phase Space - Separatrix and All

$$\Delta x_E(s) = \eta(s) \frac{\Delta E}{E_0} = \eta(s)\, \delta$$

Synchrotron Oscillations Resulting from Energy / Phase Mismatch at Injection APS Booster



24 Bunch Fill Pattern - APS



Impact of Synchrotron Phase Oscillations on All Beam-derived Signals

$$V(t) = A \{ \mathrm{Cos}[\omega_{RF}\, t + \psi] + \mathrm{Cos}[\, (\omega_{RF} + 2\,\pi * 6.5\ \mathrm{MHz})t + \psi\,] + \ldots \}$$

But $\psi = \psi_0 \mathrm{Cos}\, \Omega_s t$

So $V(t) = A\, \mathrm{Cos}[\omega_{RF}\, t + \psi_0 \cos \Omega_s t\,] + \ldots$ etc.

Digression - Frequency / Phase Modulation

$$v(t) = \mathrm{Cos}(\omega_c t + \beta\,\mathrm{Sin}(\omega_m t))$$

$$v(t) \cong \mathrm{Cos}(\omega_c t) - \frac{\beta}{2}\{\mathrm{Cos}[(\omega_c + \omega_m)t] - \mathrm{Cos}[(\omega_c - \omega_m)t]\}$$

ω_c = 2 π * Carrier Frequency

ω_m = 2 π * Modulation Frequency

β = Frequency Modulation Amplitude (radians)
... usually << 1