Properties of Synchrotron Radiation

Bending Magnet Synchrotron Radiation

- Space-time dependency
- Power
- Flux
- Spectral properties

Insertion Device Radiation Properties

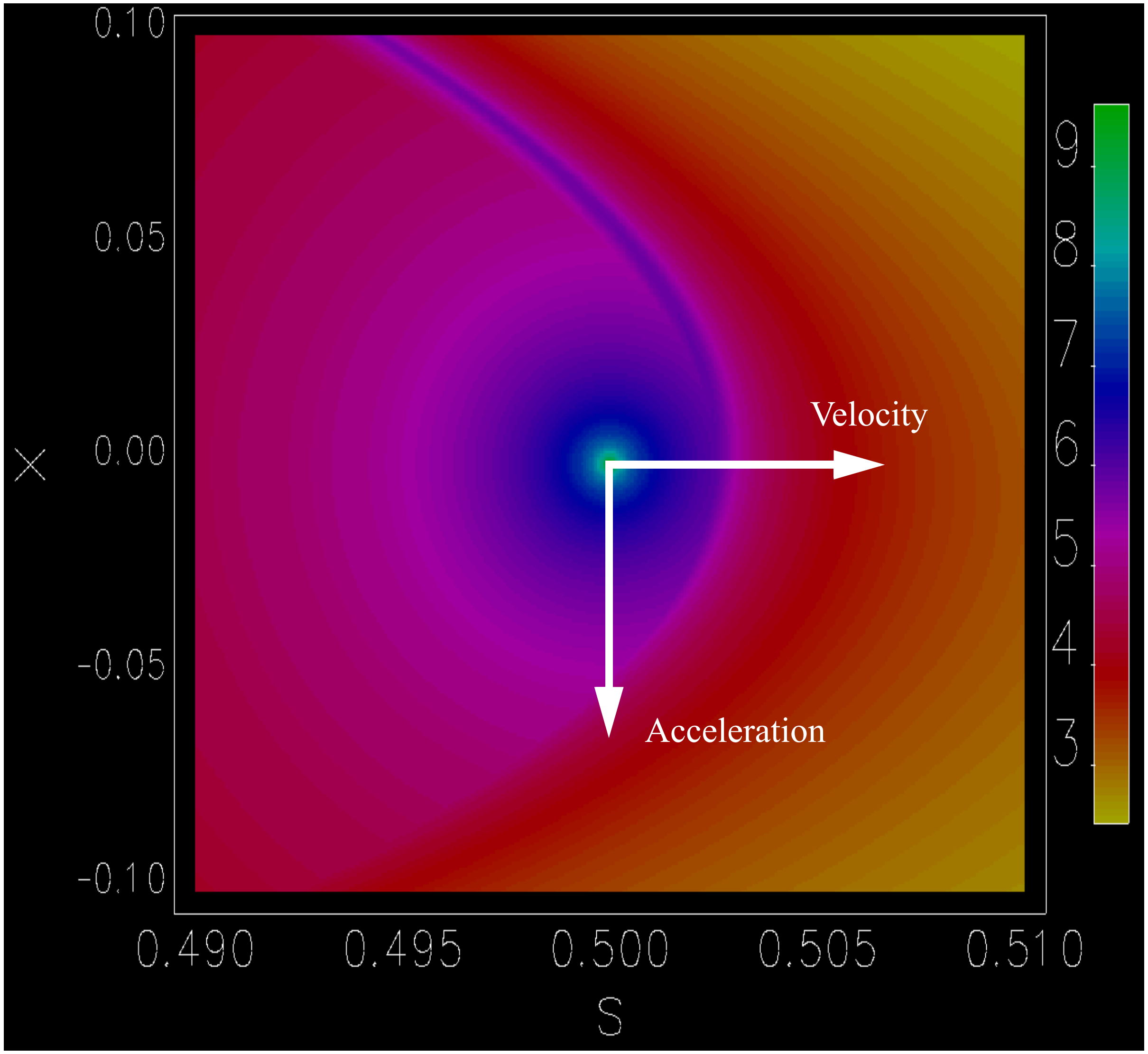
- Power
- Flux
- Brightness

Bending Magnet Synchrotron Radiation

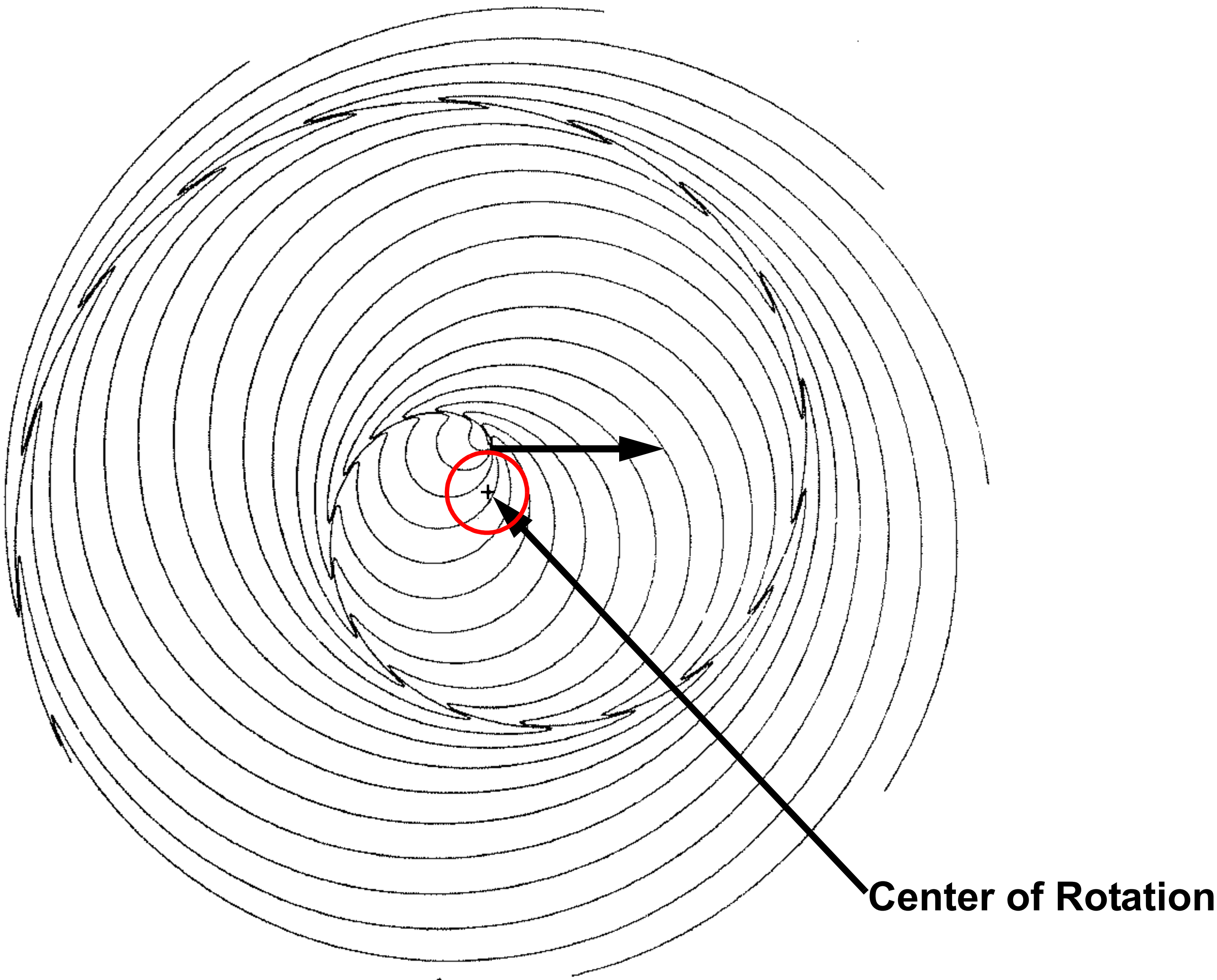
Space-Time dependency

$$\Phi(\mathbf{r},t) = \left. \frac{k\,q}{(R - \beta\bullet \mathbf{R})} \right|_{t_R}$$

v = 0.995 c

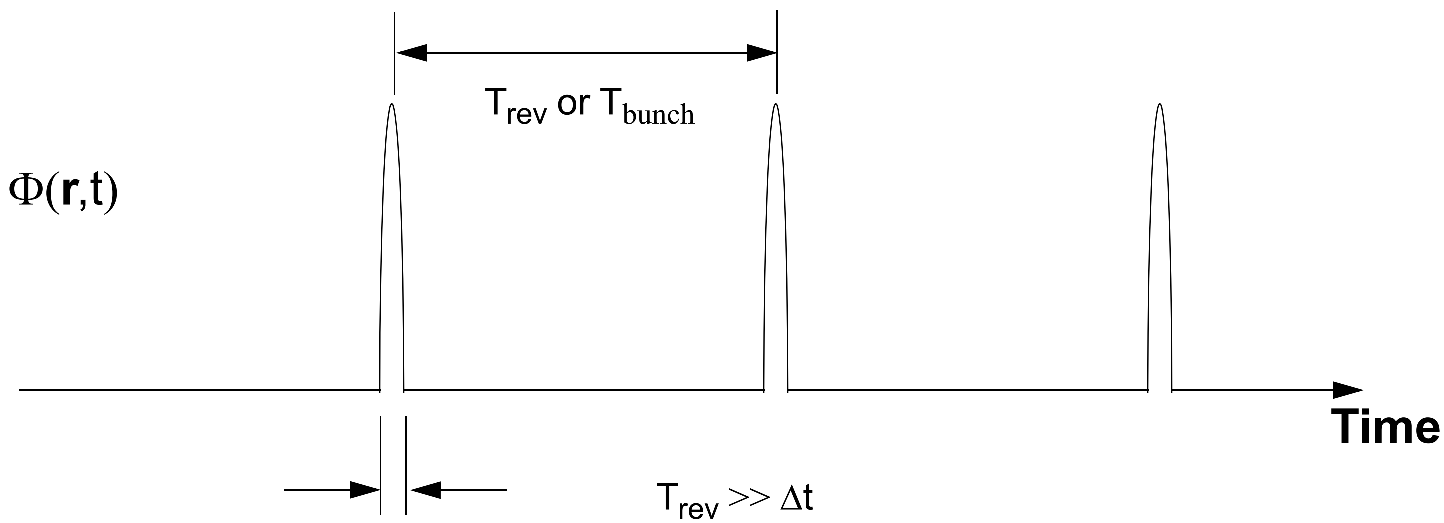


Electric Field Lines for Circular Motion, v = 0.9 c



R.Y. Tsien, AM. J. Phys. **40**, 46 (1972)

Bending Magnet Synchrotron Radiation Spectral Properties



$$\Delta t \approx \frac{\rho}{c\,\gamma^3}$$

$$\omega_c = C_c \frac{E^3}{\rho} \qquad C_c = 3.37e18 \frac{\text{meters}}{\text{sec-GeV}^3}$$

$(\gamma = E / mc^2)$

$$\varepsilon_c\ (\text{keV}) = \hbar\,\omega_c = 2.218 \frac{E^3(\text{GeV}^3)}{\rho\ (\text{meters})} = 6.65\text{e-}2\ E^2(\text{GeV}^2)\ B(\text{kG})$$

Ref. H.Wiedemann “Particle Accelerator Physics I ” Ch. 9

Bending Magnet Synchrotron Radiation Power*

The instantaneous power radiated by one electron while travelling through a magnetic field is

$$P_\gamma \text{ (GeV / sec)} = \frac{c\, C_\gamma\, E^4}{2\, \pi\, \rho^2}$$

where c = 3e8 m/s, E is the particle energy in GeV, and C_γ = 8.8575e-5 (meters / GeV^3) . The quantity ρ is the radius of curvature (in meters) of the particle's trajectory while travelling in a magnetic field B :

$$\boxed{B\, \rho = 3.335641\, \beta\, E}$$

With the magnetic field B in Tesla, ρ in meters, and E is the particle energy in units of GeV. Also, $\beta = v/c$ is essentially equal to one. The energy lost by a single electron making one circuit of a storage ring is obtained by multiplying P_γ by the time spent in the magnetic field during one revolution:

$$U_0 \text{ (GeV)} = P_\gamma\, T_{rad.} = P_\gamma\, (2\, \pi\, \rho / c) = C_\gamma\, E^4 / \rho$$

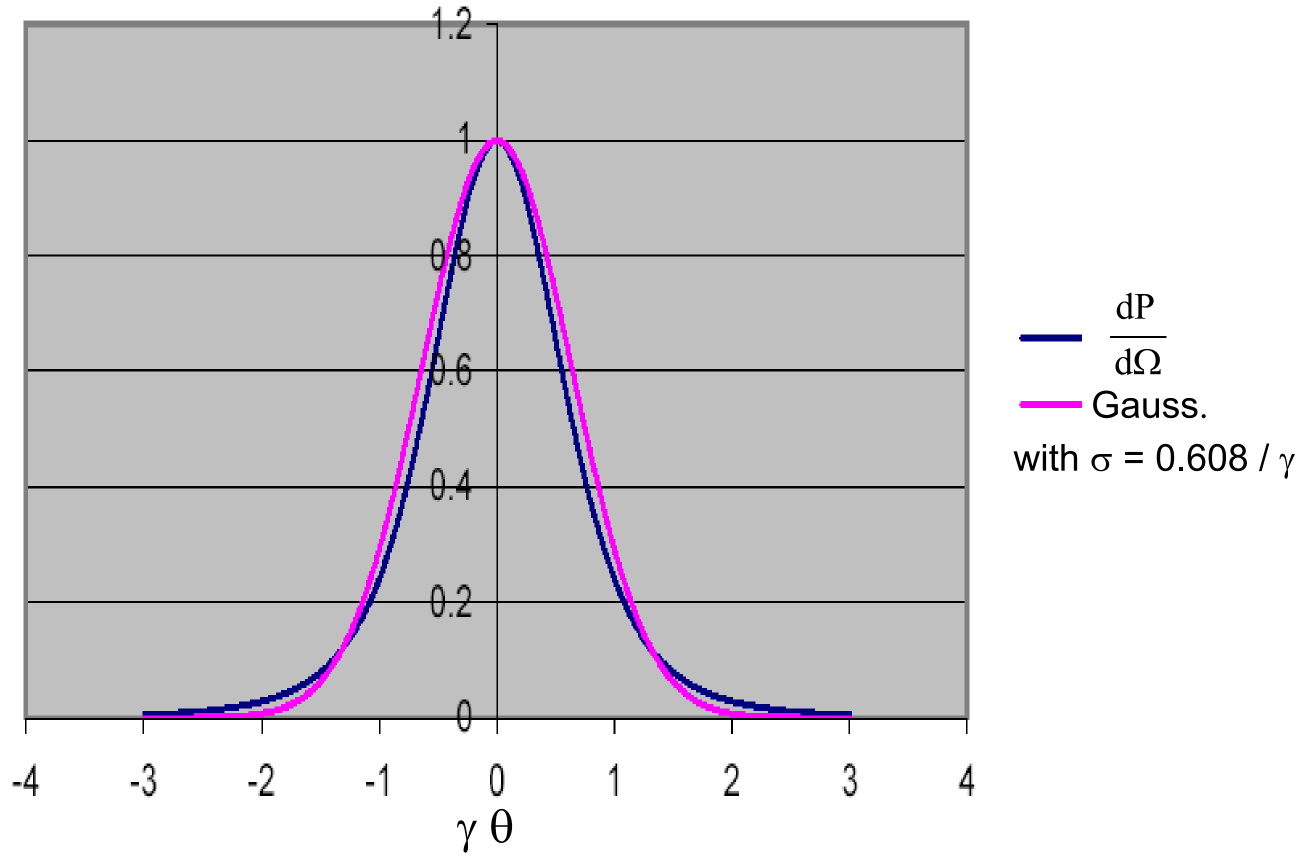
Here $T_{rad.}$ is the time the particle spent passing through the magnetic field B in one turn. Multplying this by the number of electrons per second streaming around the ring results in the total radiation power:

$$P_{tot} \text{ (MW)} = U_0\, \mathbf{I} / e = \frac{C_\gamma\, E^4\, \mathbf{I}}{\rho}$$

where E is in GeV, ρ is in meters, and **I** is the stored beam current in milliAmps.

* Ref. H.Wiedemann "Particle Accelerator Physics II " Ch. 7

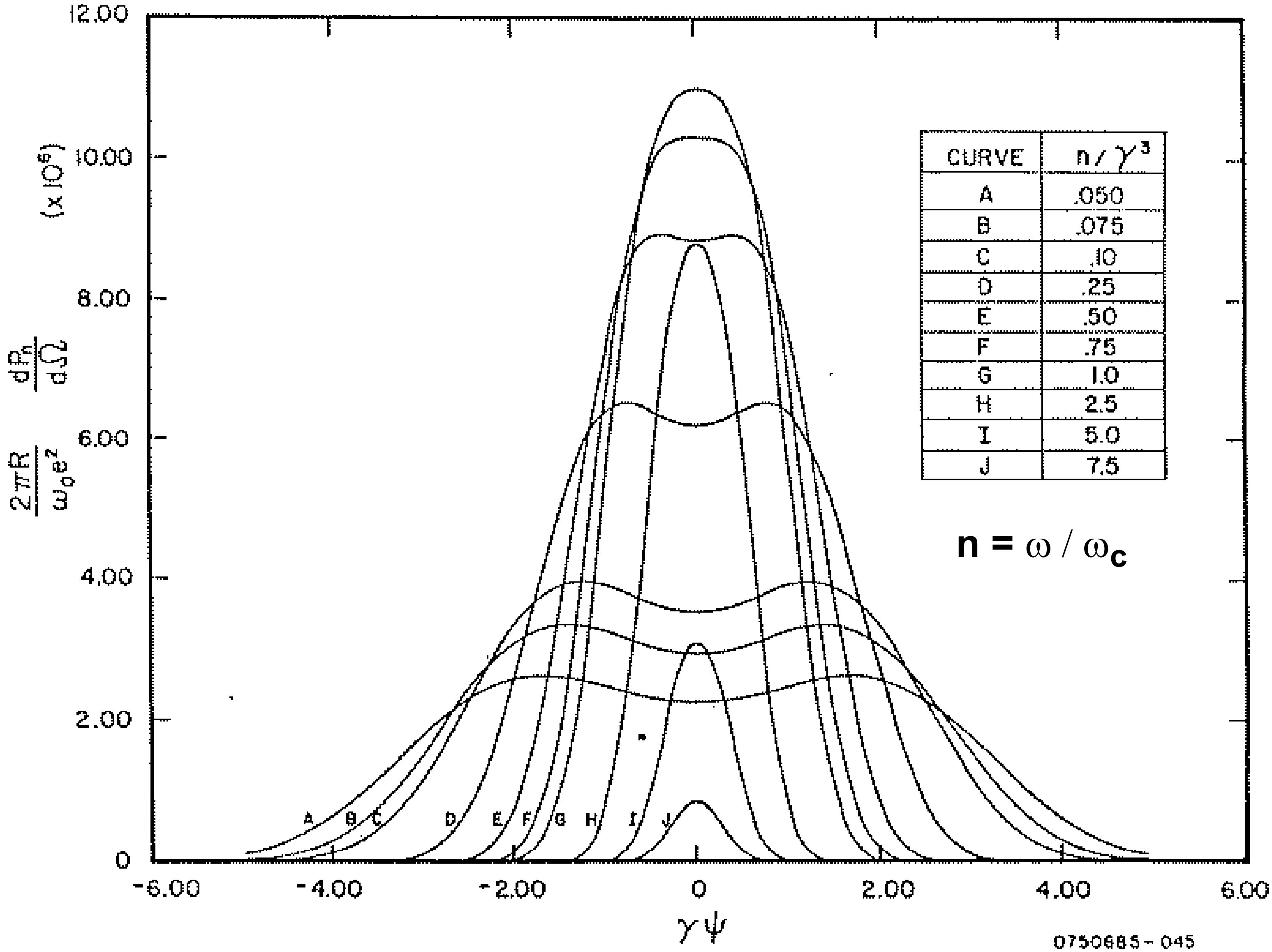
Bending Magnet Synchrotron Radiation Flux Distribution



The power per unit solid angle shows clearly the 1 / γ characteristic vertical opening angle.

G.K. Green, "Spectra and Optics of Synchrotron Radiation", BNL 50522 April, 1976

Variation of Bending Magnet Radiation with Angle and Frequency

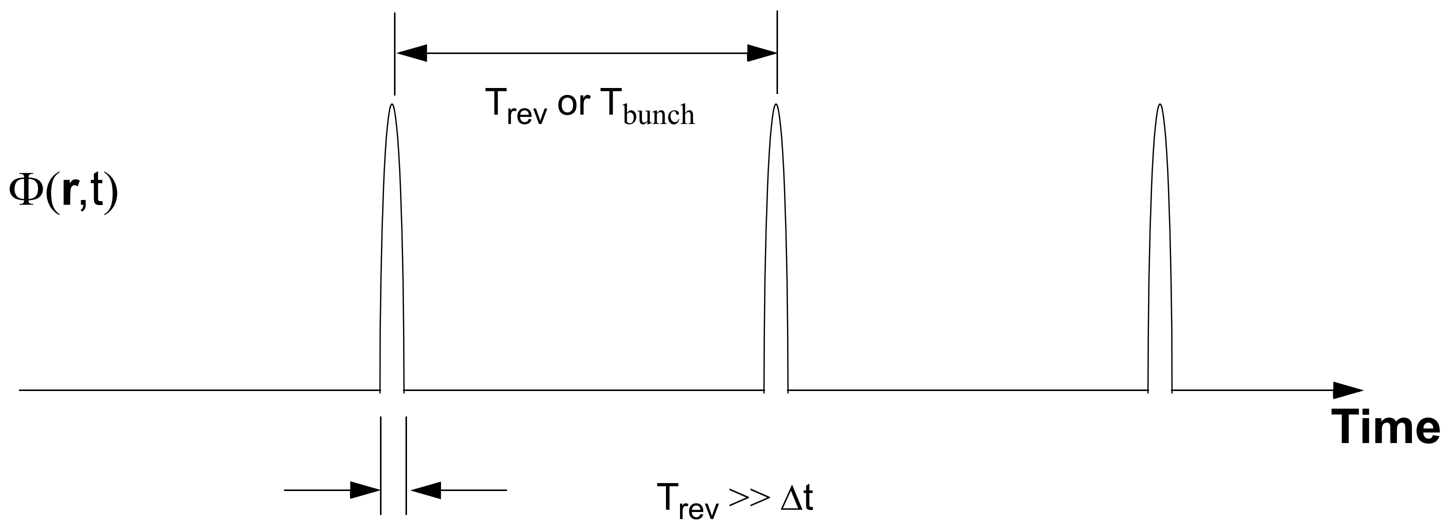


CURVE	n/γ^3
A	.050
B	.075
C	.10
D	.25
E	.50
F	.75
G	1.0
H	2.5
I	5.0
J	7.5

n = ω / ω_c

Figure 2.2 Power radiated into the n^{th} harmonic of the revolution frequency per unit solid angle.

Bending Magnet Synchrotron Radiation Spectral Properties



$T_{rev} >> \Delta t$

$$\Delta t \approx \frac{\rho}{c\,\gamma^3}$$

$$\omega_c = C_c \frac{E^3}{\rho}$$

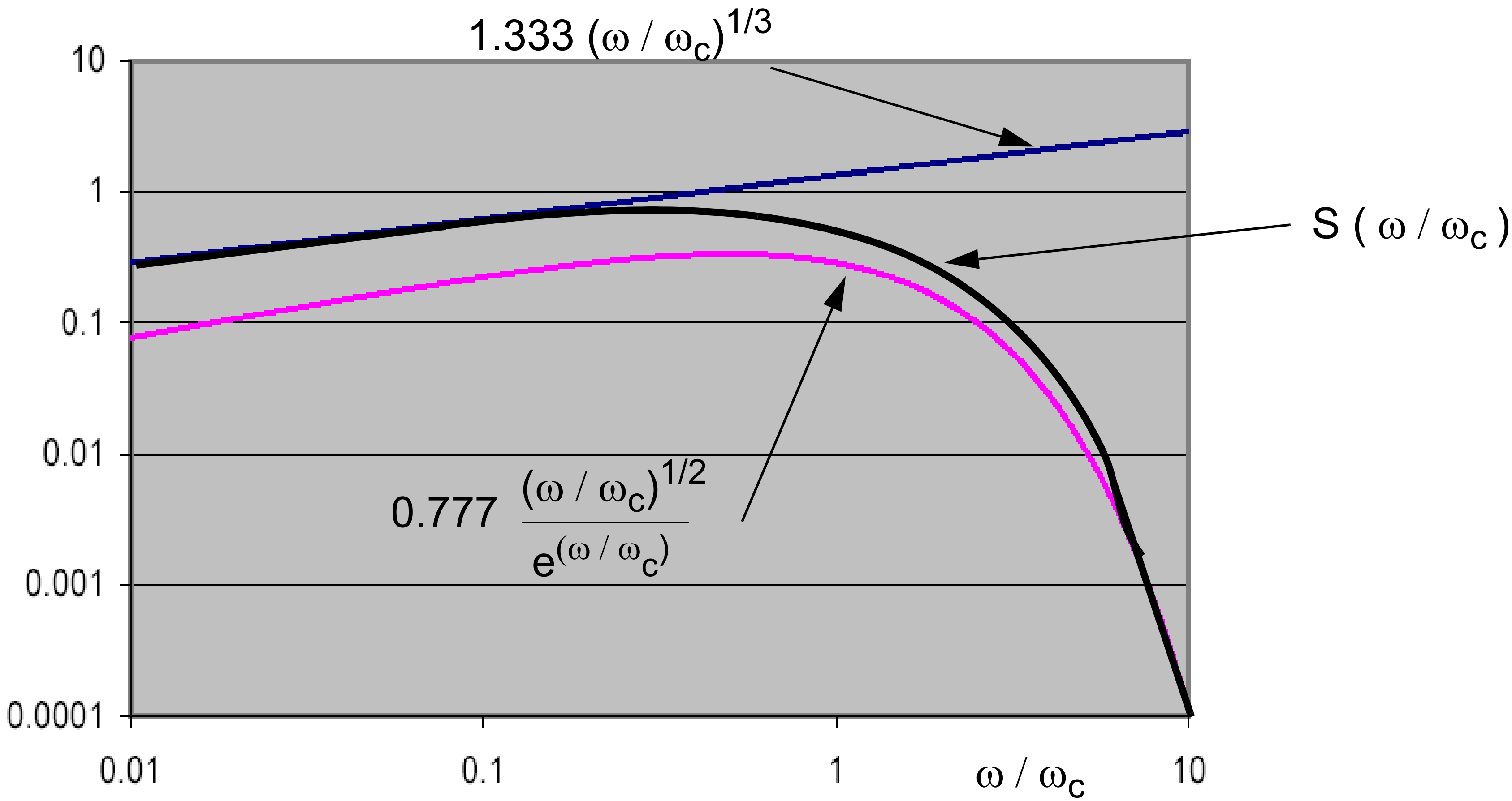
$$C_c = 3.37e18 \frac{\text{meters}}{\text{sec-GeV}^3}$$

$(\gamma = E / mc^2)$

$$\varepsilon_c\ (\text{keV}) = \hbar\,\omega_c = 2.218 \frac{E^3(\text{GeV}^3)}{\rho\ (\text{meters})} = 6.65e\text{-}2\ E^2(\text{GeV}^2)\ B(\text{kG})$$

Ref. H.Wiedemann "Particle Accelerator Physics I " Ch. 9

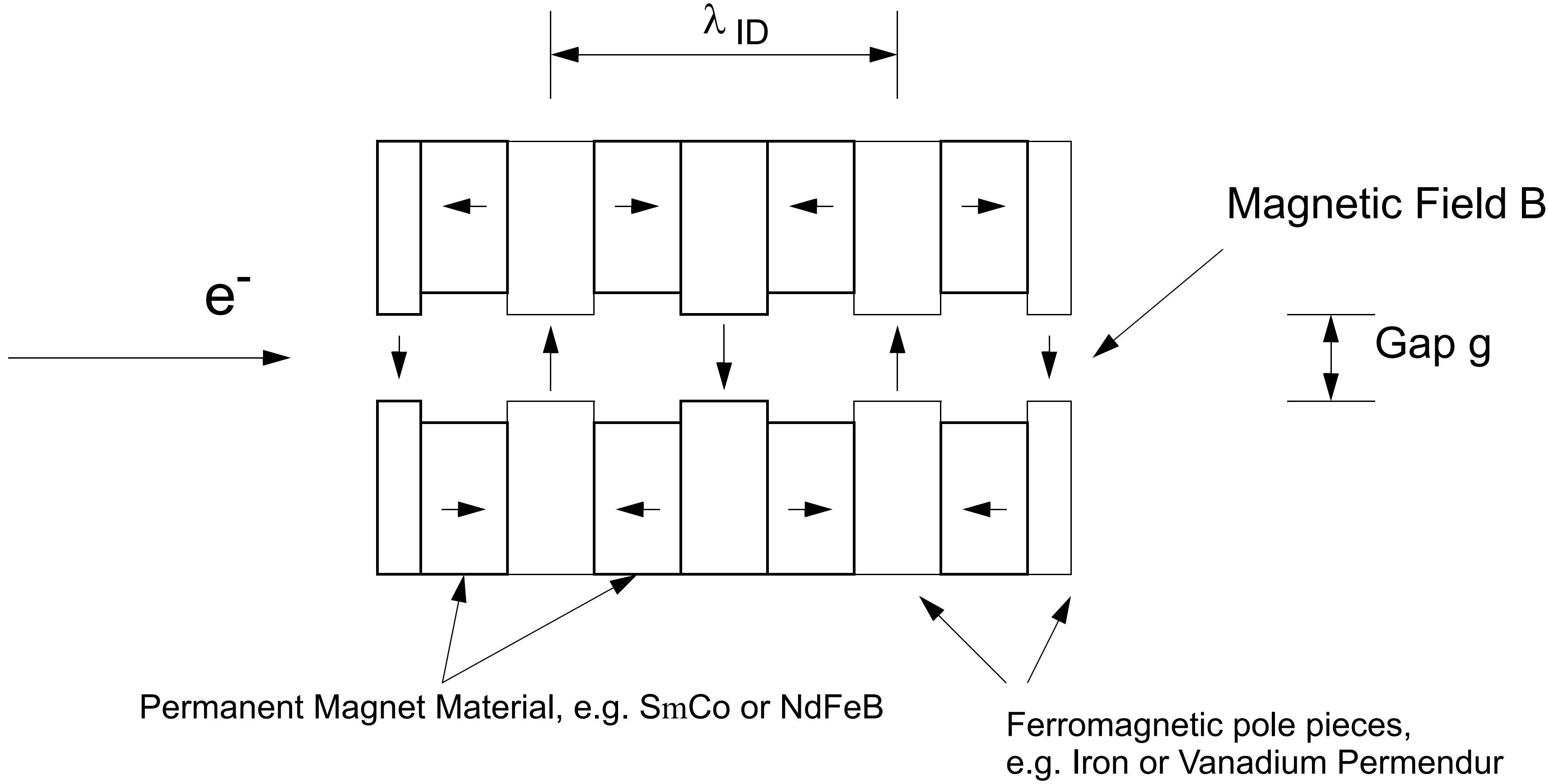
Bending Magnet Synchrotron Radiation Spectral Properties



$$\frac{dP}{d\omega} = \frac{P_\gamma}{\omega_c} S(\omega / \omega_c)$$

* Ref. H.Wiedemann "Particle Accelerator Physics II " Ch. 7

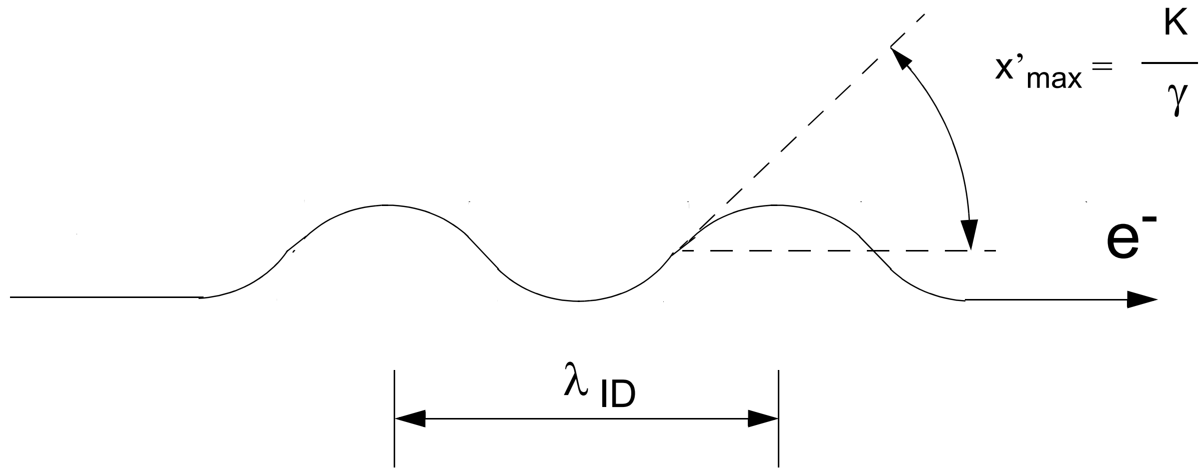
Insertion Devices



Side View

Insertion Devices

Top View of Electron Trajectory



$$K = 0.934\, B_{max}(T)\, \lambda_{ID}(cm)$$

$$B_{max}(T) = 3.33 \exp[-(g/\lambda_{ID})(5.47 - 1.8(g/\lambda_{ID})] \; *$$

* Ref. K. Halbach J. Physique, C1 Suppl. 2, **44** (February 1983)

Insertion Devices

Undulator: $K < 1.5$ or so,

Narrow Spectral Lines at wavelength

$$\lambda_\gamma (A) = 13.056 \frac{\lambda_{ID}}{E^2(GeV^2)} (1 + K^2/2 + \gamma^2 \theta^2)$$

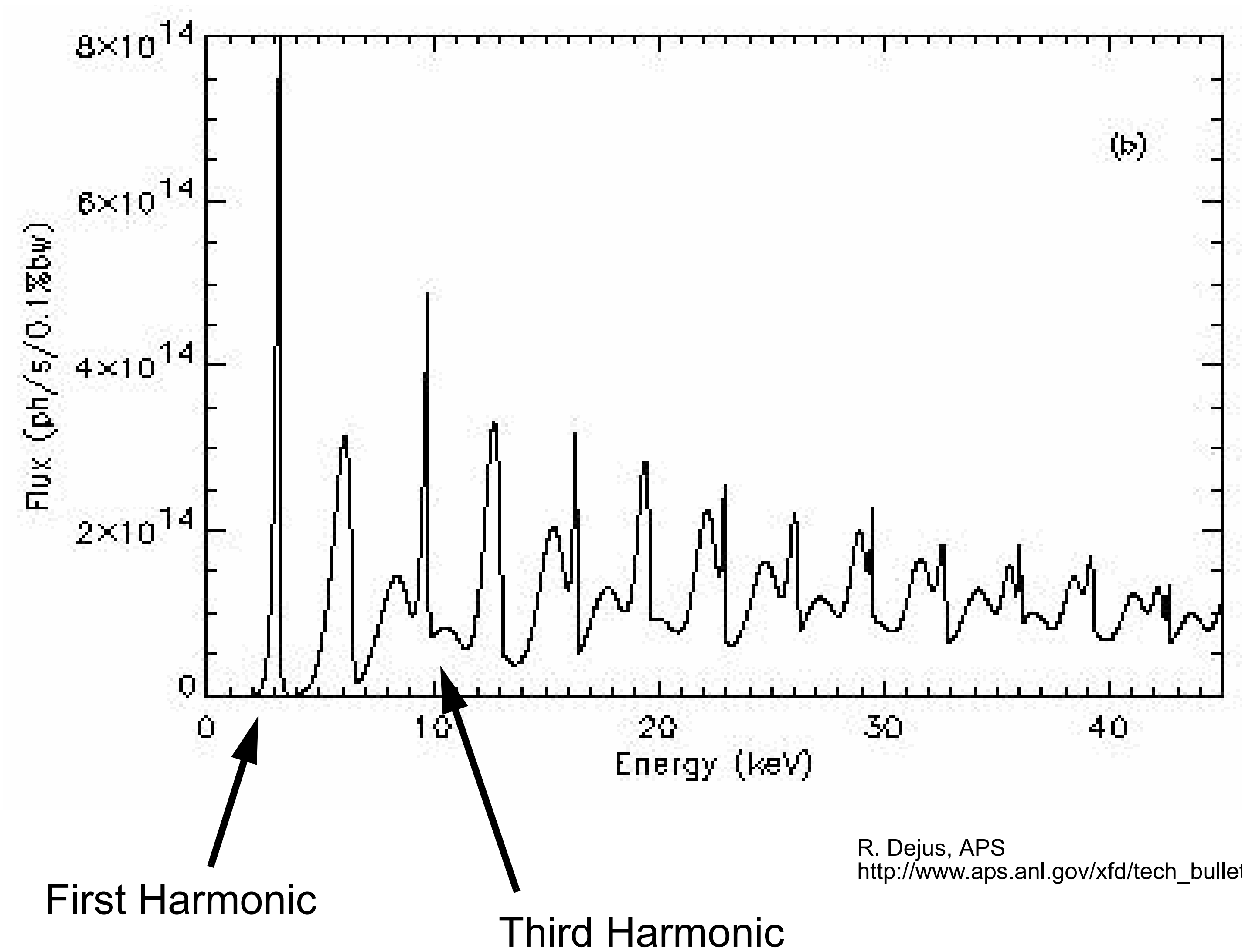
and harmonics thereof.

Wiggler: $K >> 1$, Broadband Radiation

Total Power Radiated (undulator or wiggler):

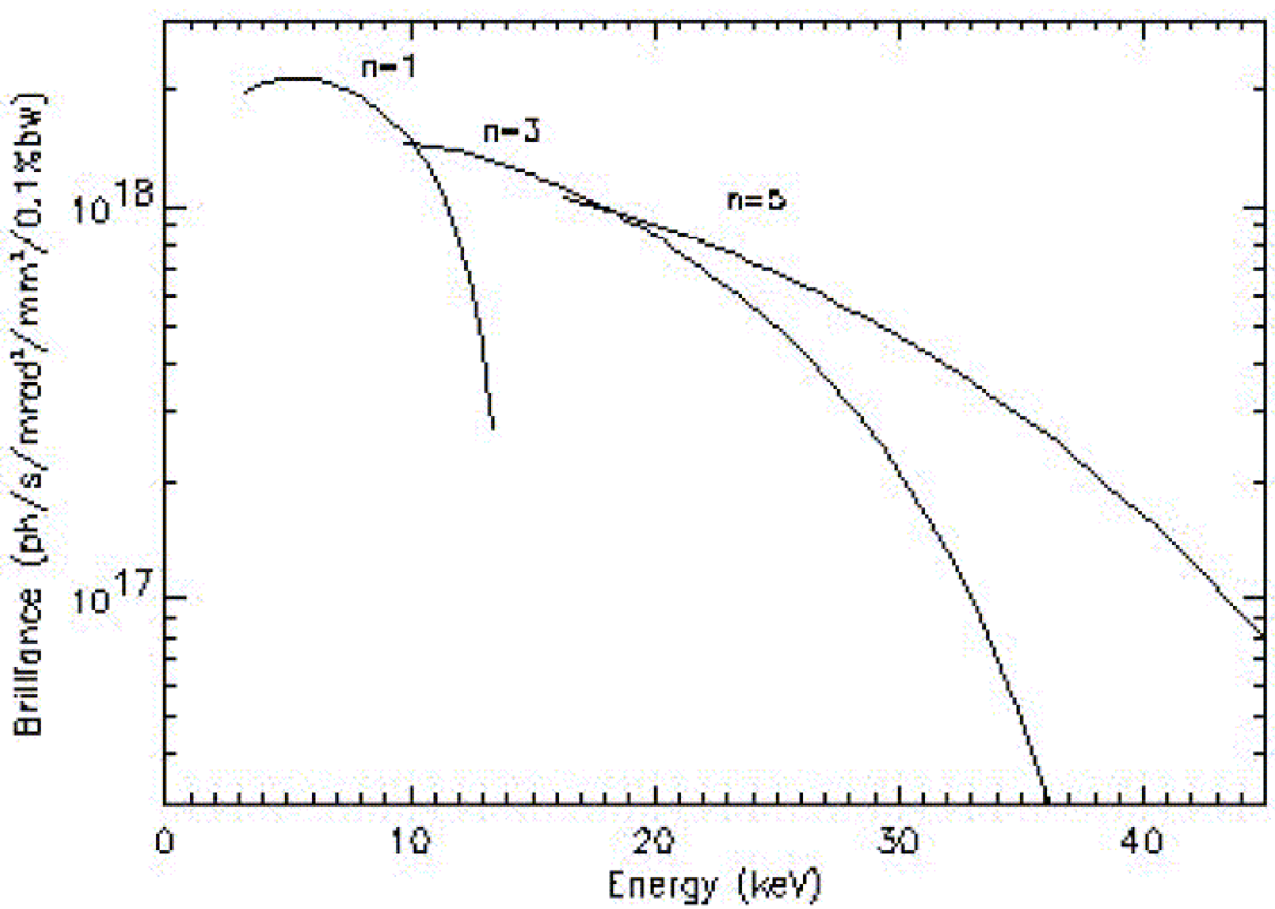
$$P_T \text{ (Watts)} = \frac{7.26 \; E^2(GeV) \; \mathbf{I}(Amps) \; N_u \; K^2}{\lambda_{ID} \text{ (cm)}}$$

Flux density vs. Photon Energy Showing Harmonics



R. Dejus, APS
http://www.aps.anl.gov/xfd/tech_bulletins/tb17.pdf

Tuning Curves for First, Third, Fifth Undulator Harmonics



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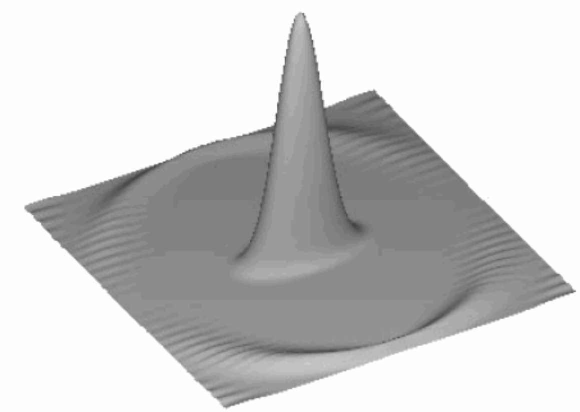
http://www.aps.anl.gov/xfd/tech_bulletins/tb17.pdf

- Undulator radiation is tightly collimated in the forward direction:

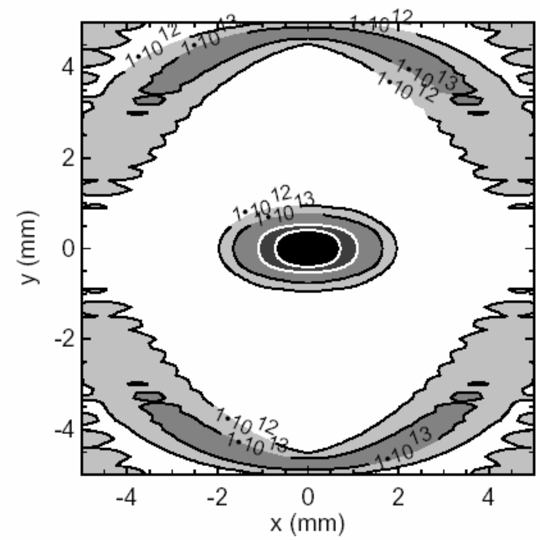
Undulator photon rms angular divergence

$$\sigma' = \frac{(1 + K^2 / 2)^{1/2}}{\gamma \, (2mN_u)^{1/2}}$$



Where m is the harmonic number, e.g. 1,3,5 etc, and N_u is the number of undulator periods

$$N_u = L / \lambda_{ID}$$



First harmonic profile 30 meters from source point

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http://www.aps.anl.gov/xfd/tech_bulletins/tb45.pdf

Wiggler Power Profile

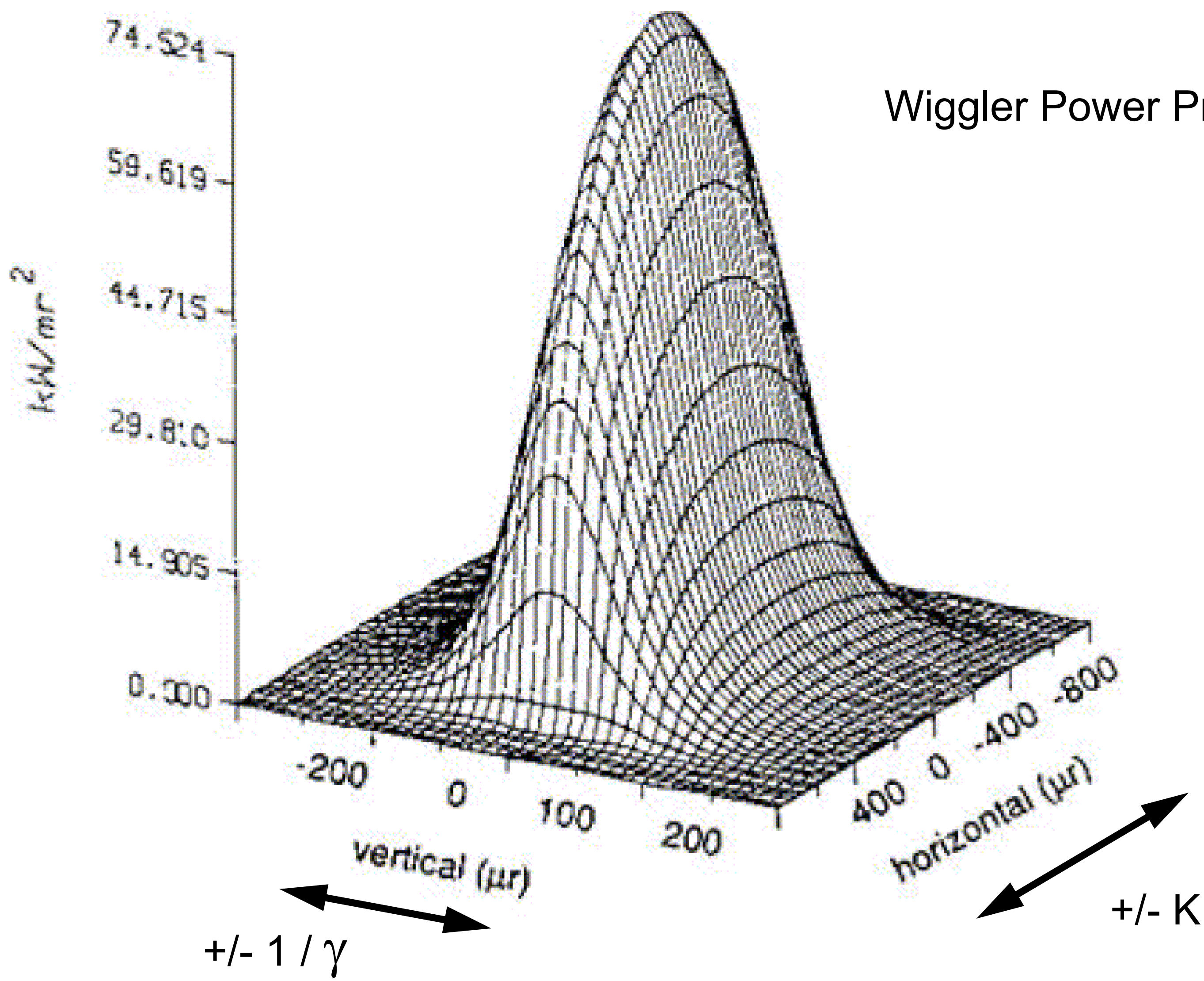

Fig. 11 Power profile of Wiggler A at 1.0 T field (Wiggler A [λ_u=8.5 cm], 7 GeV, 100 mA operation).

http://www.aps.anl.gov/xfd/tech_bulletins/TB-11.pdf