Orbit Feedback Dynamics

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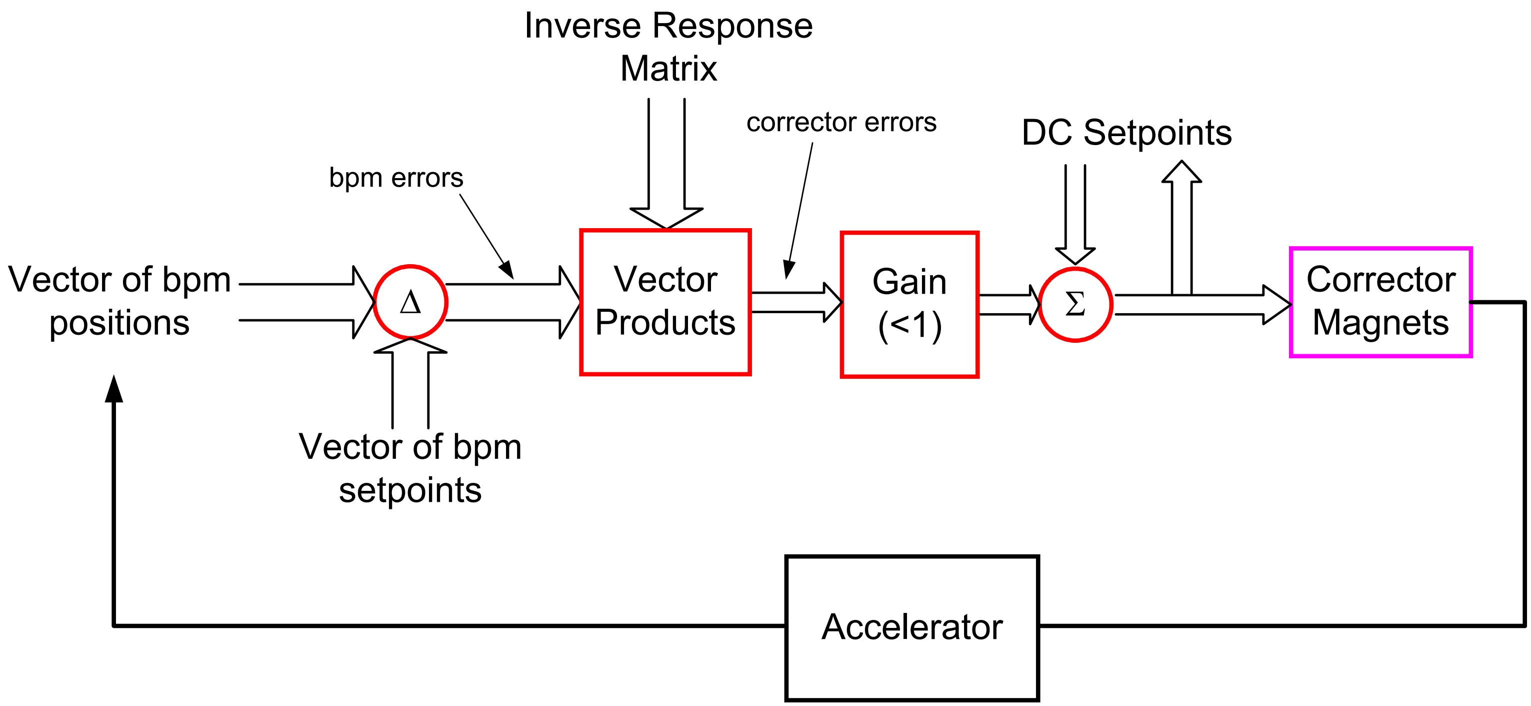
Outline

- Simple orbit correction algorithm as feedback loop.
- BPM filtering and processing
- Corrector dynamics and eddy current effects
- Regulator design matters
- Performance metrics for regulator design
- Orbit feedback system as a noise-shaping filter.
- Running separate DC and AC feedback systems
- Impact of different corrector dynamics (leading to a dilemma)

The global orbit correction process



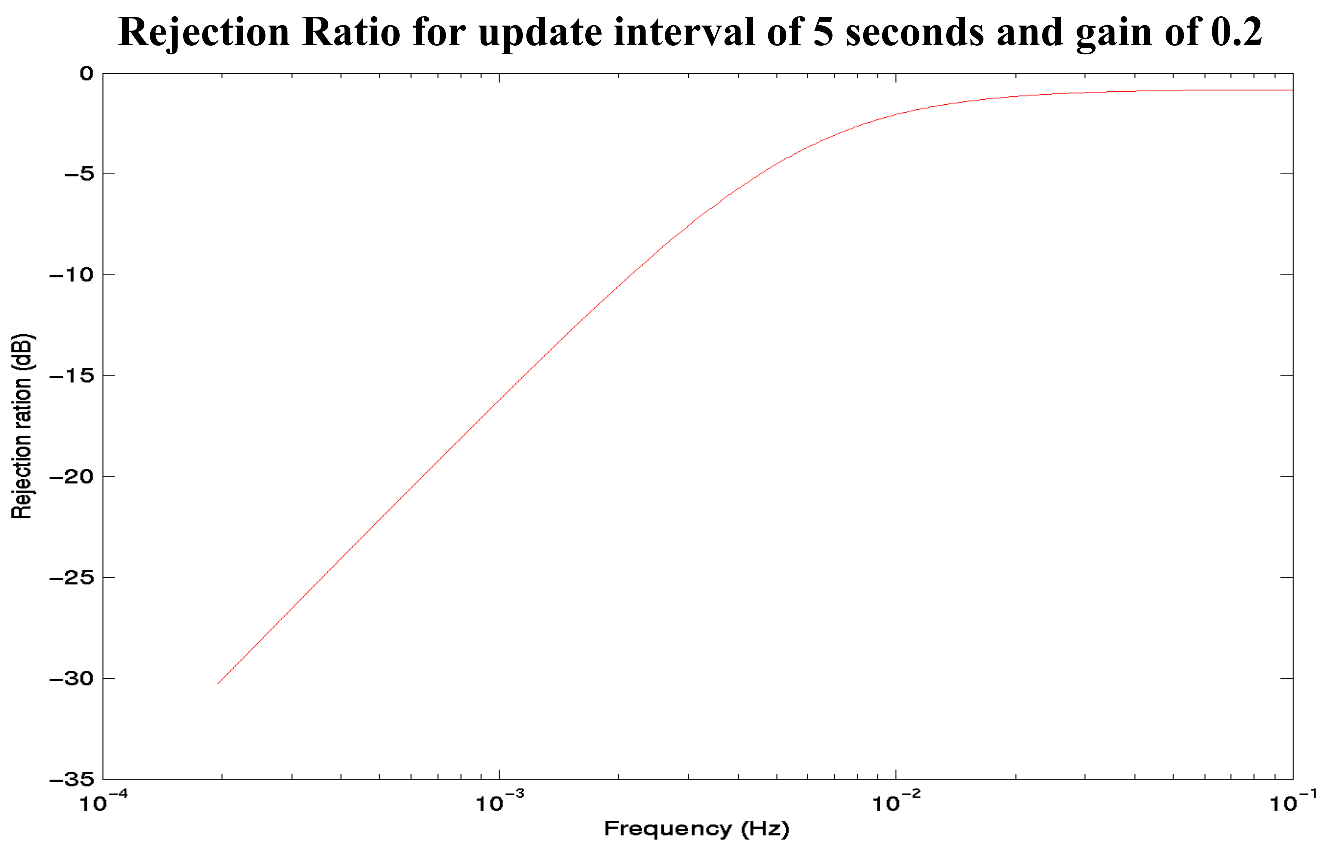
Simple regulator implementation

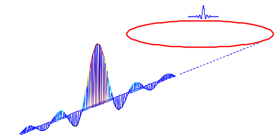
- At each orbit correction step
  - Measure the orbit error $\Delta \boldsymbol{x}$
  - Calculate corrector 'errors' from matrix multiplication $\Delta \boldsymbol{c} = \boldsymbol{R}^{-1} * \Delta \boldsymbol{x}$
  - Apply gain factor (<1) to $\Delta \boldsymbol{c}$
  - Add the resulting corrector deltas to the existing setpoints
  - Wait one tick
  - Repeat ad infinitum

- Resulting system is stable and well-behaved providing gain factor <<1, and provided the update rate is slow relative to system dynamics
  - Update rates and setting times of corrector power supplies.
  - Corrector dynamics (power supplies, magnets, eddy current effects).
  - BPM filtering.

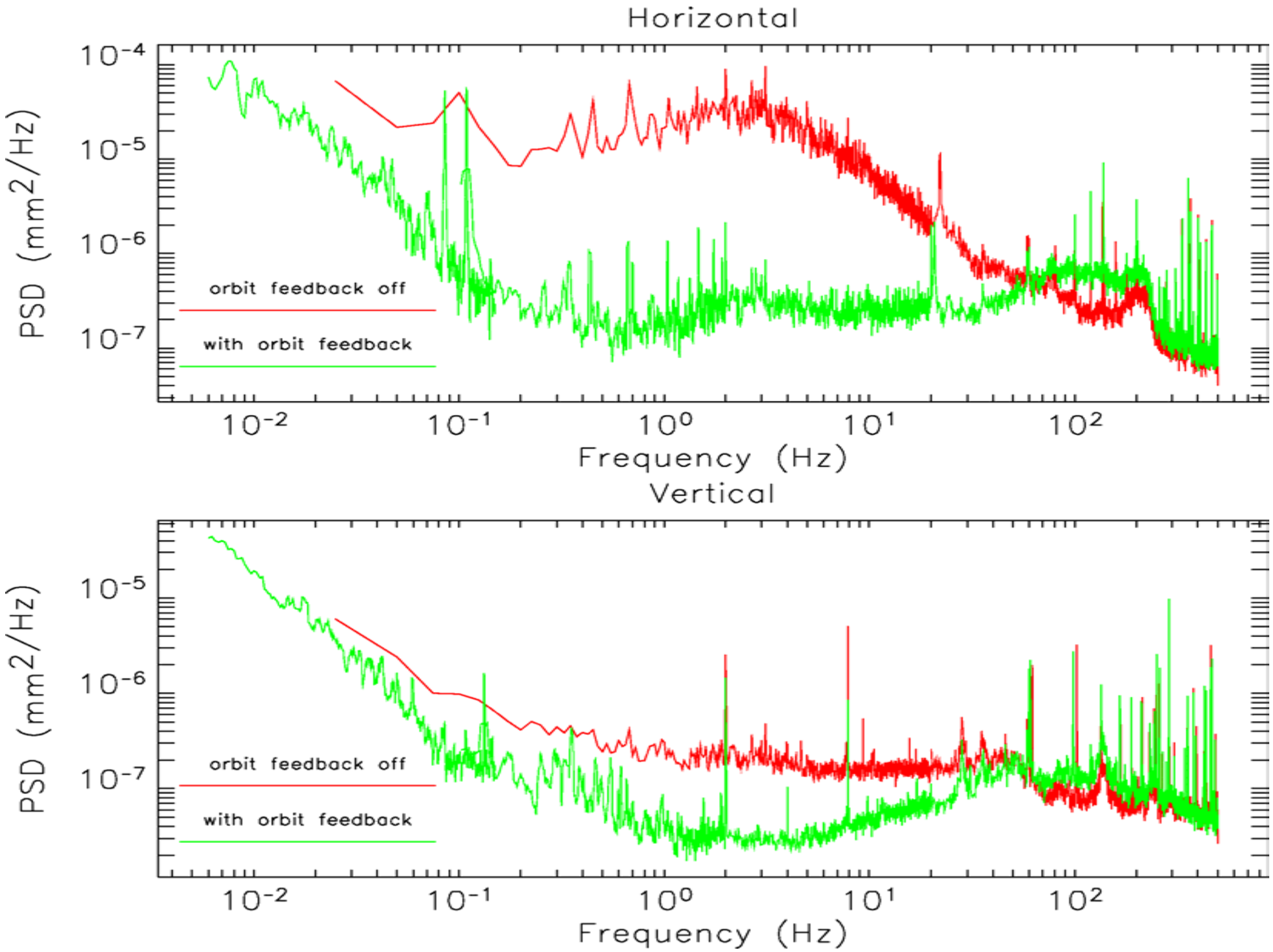
Frequency response of simple regulator

- Cumulating the errors from step to step has the same effect as a digital integrator.

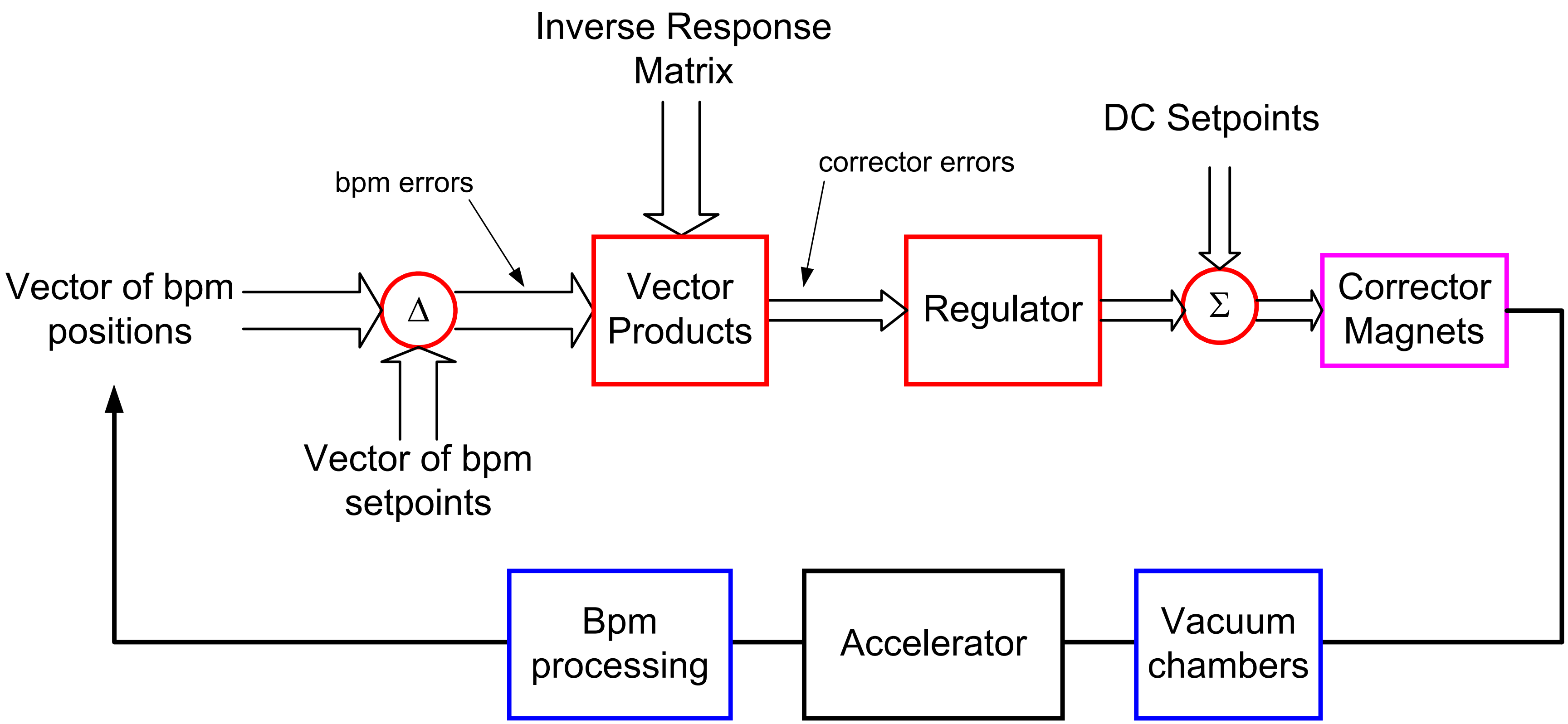




Orbit spectrum to be corrected



The dynamic global orbit feedback algorithm



One-dimensional (SISO) or multi-dimensional (MIMO) control problem?

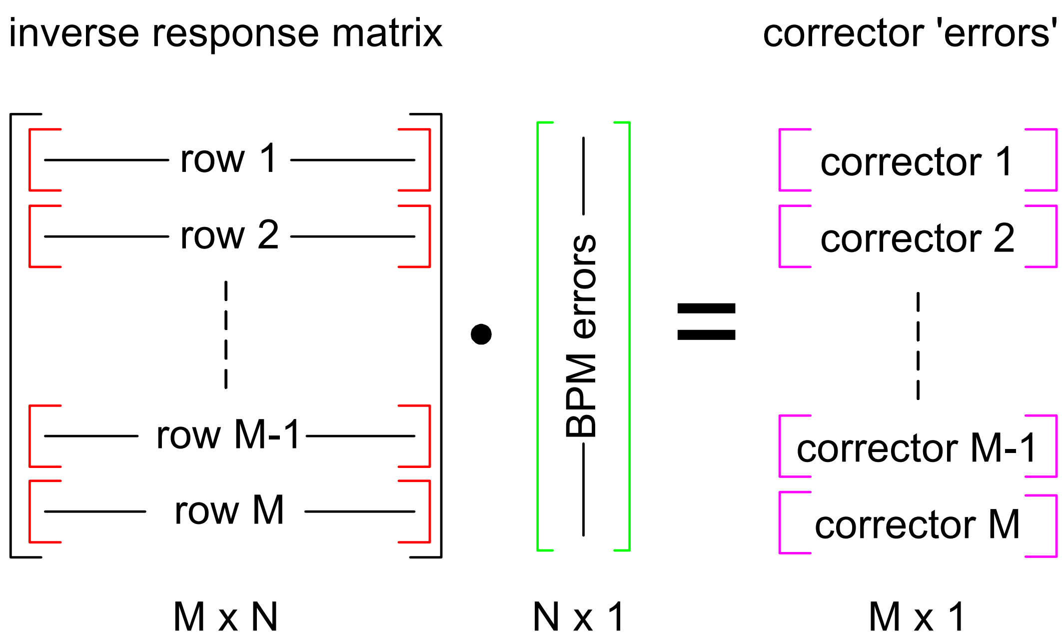
- Global RMS orbit feedback can be treated as a one-dimensional control problem
  - Relationship between bpms and corrector errors is static linear matrix.
  - Response matrix orthogonalizes the corrector control loops.
  - Design the regulator loop once, implement for every corrector.

- Even when corrector dynamics are different, the feedback control loops are still orthogonal.

- We will see later how different dynamics impact the overall correction effectiveness.

Implementing the orbit correction algorithm

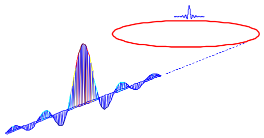
- Computation of corrector ‘errors’ is conveniently separated into a series of ***M*** vector dot-products, one for each of the ***M*** correctors



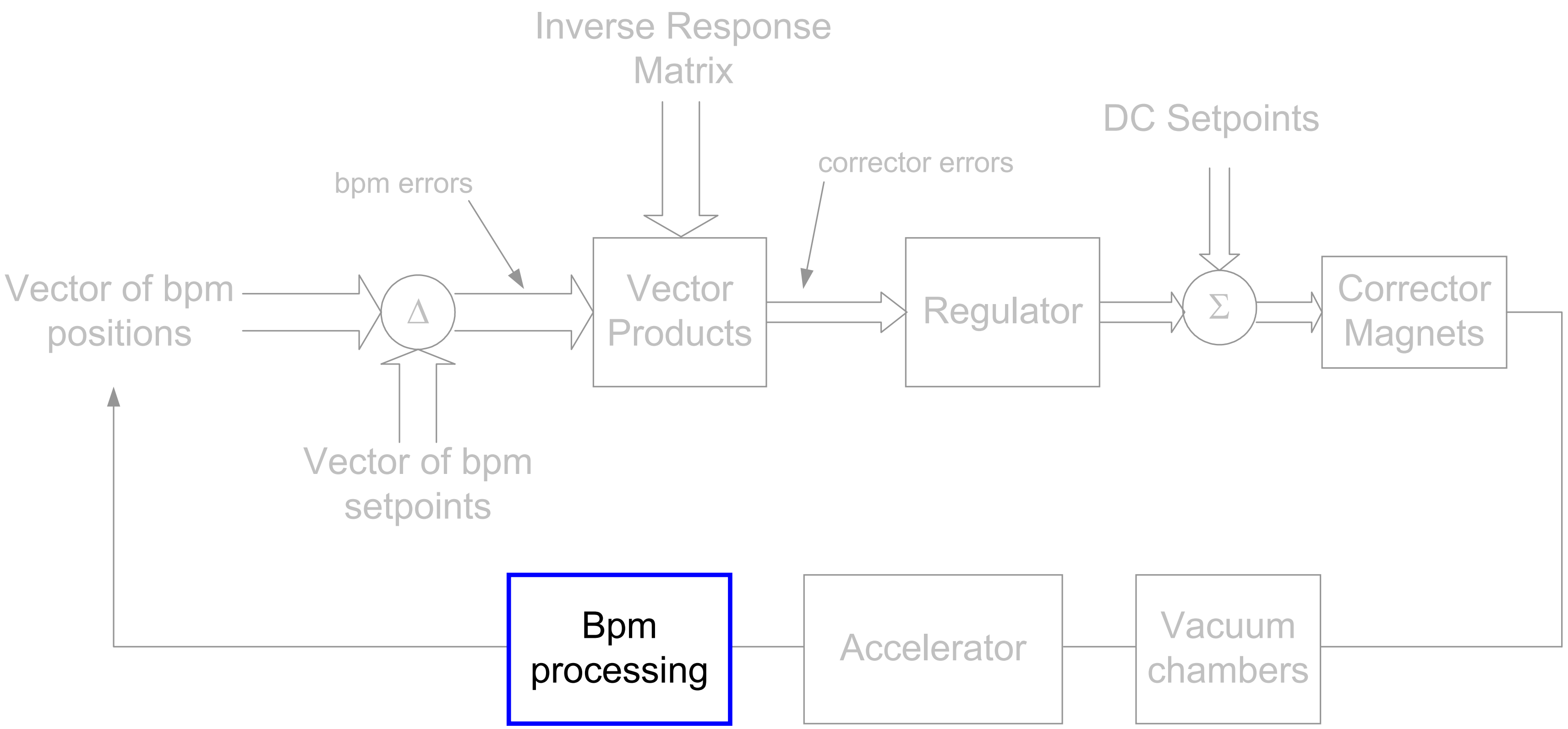
- Corrector ‘errors’ becomes the input to one of ***M*** feedback regulators.
- Provided the corrector and bpm dynamics are identical, the feedback loops are independent, and can be implemented as ***M*** single-input / single-output systems.

Issues

- Knowledge of the bpm dynamics.
- Knowledge of the corrector dynamics.
- Performance metrics for the closed-loop response.
- Designing the regulator.



BPM DYNAMICS



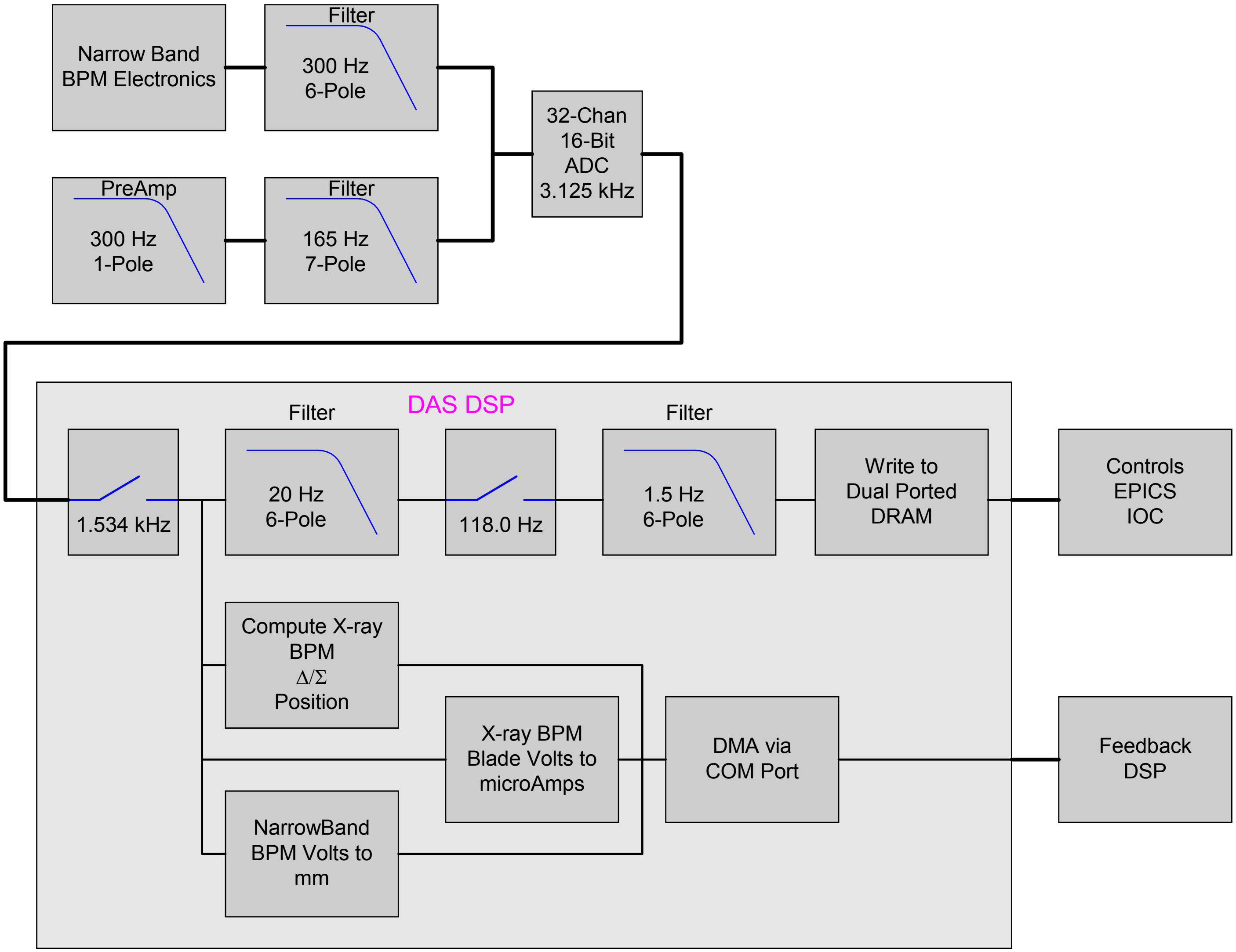
Different flavors of bpm processing at APS

- Narrow-band rf bpms
  - Analog signal processing with bandwidth in the 100's Hz.
  - 6-pole Butterworth anti-alias filter at 300Hz.
  - Digitized at feedback system sampling rate (1500Hz)

- Photon bpms
  - Analog signal processing with bandwidth in the 100's Hz.
  - 7-pole Butterworth anti-alias filter at 185 Hz.
  - Digitized at feedback system sampling rate (1500Hz)

- Turn-by-turn rf bpms
  - Sampled at ½ revolution frequency (135kHz)
  - 32-point boxcar averager at 135kHz rate.
  - Down-sampled to feedback system sampling rate (1500Hz)

Analog front-end of APS x-ray and Narrowband rf bpms



Butterworth Filters

- Butterworth filters are maximally-flat.
- There is no ripple in either the passband or stopband.
- The magnitude-response of an N^{th}-order filter rolls off at 20*N* dB/decade.
- The stopband phase delay of an N^{th}-order filter is -90N degrees.
- A Butterworth filter can be completely described by its -3dB cutoff frequency Ωc, and its order *N.*

Linear Magnitude, Linear Frequency

Magnitude in dB, Log Frequency

Transfer Functions of Normalized Butterworth Lowpass Filters

Filter Order	Coefficients for each power of s								
	S^8	S^7	S^6	S^5	S^4	S^3	S^2	S^1	S^0
1								1	1
2							1	1.4142	1
3						1	2	2	1
4					1	2.6131	3.4142	2.6131	1
5				1	3.2361	5.2361	5.2361	3.2361	1
6			1	3.8637	7.4641	9.1416	7.4641	3.8637	1
7		1	4.4940	10.0978	14.5918	14.5918	10.0978	4.4940	1
8	1	5.1258	13.1371	21.8462	25.6884	21.8462	13.1371	5.1258	1

- For example, the 4th order prototype Butterworth lowpass filter is described by the transfer function

$$H(s) = \frac{1}{s^4 + 2.6131s^3 + 3.4142s^2 + 2.6131s + 1}$$

- NB: these prototype filter transfer functions are normalized (ie their -3dB cutoff frequency is 1rad/s). To convert to a different cut-off frequency, replace each instance of **s** with $\mathbf{s/2\pi F_c}$, where $\mathbf{F_c}$ is the cut-off frequency in Hz.

Digital Filters

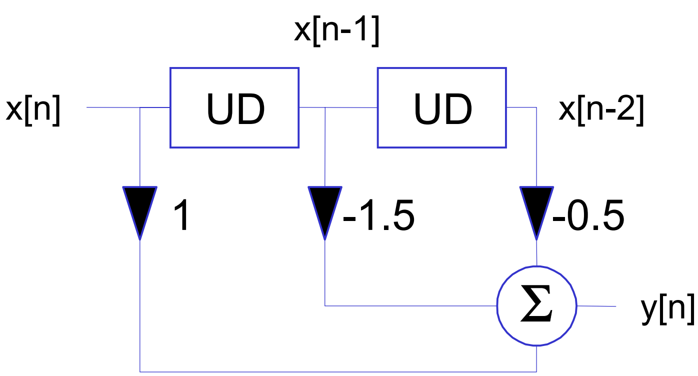
- Finite Impulse Response (FIR) filter
  - Common design basis: truncated impulse response of ideal brick-wall filter, possibly with windowing.

- Infinite Impulse Response (IIR) filter
  - Common design basis: analog prototype filter, converted to digital domain using impulse invariance or bilinear transform.

FIR Filter example

- Output is a linear combination of present and past inputs only.

Example:



Difference equation

$$y[n] = x[n] - 1.5x[n-1] - 0.5x[n-2]$$

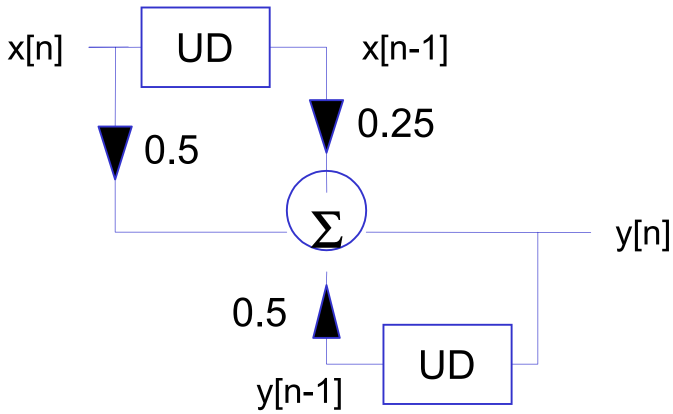
Z-transform of d.e. $\quad Y(z) = X(z) - 1.5z^{-1}X(z) - 0.5z^{-2}X(z)$

Transfer function $\quad \dfrac{Y(z)}{X(z)} = 1 - 1.5z^{-1} - 0.5z^{-2}$

IIR (recursive) filter

- Output is a linear combination of present & past inputs and past outputs.

Example:



Difference equation

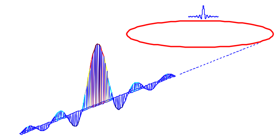
$$y[n] = 0.5x[n] + 0.25x[n-1] + 0.5y[n-1]$$

Z-transform of d.e.

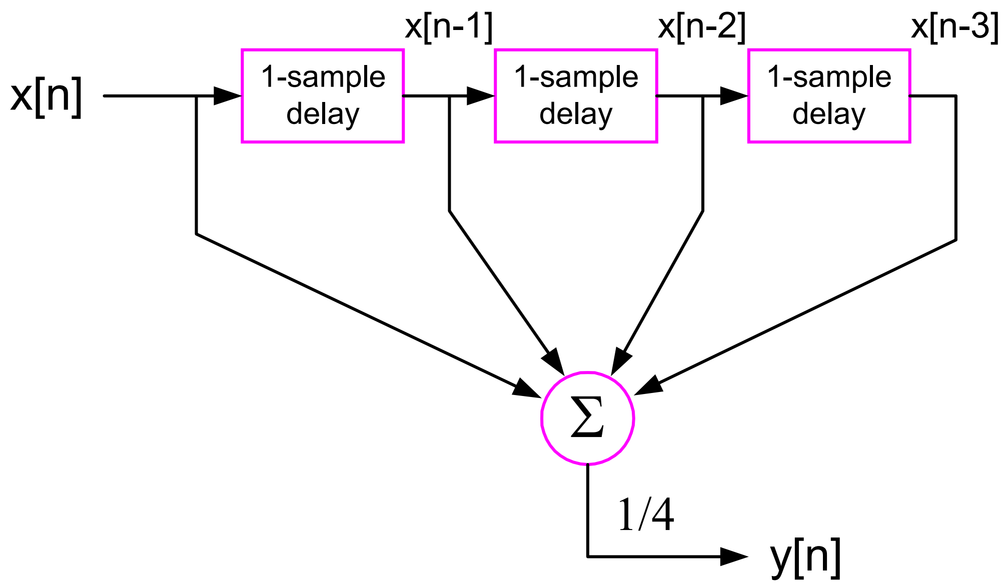
$$Y(z) = 0.5X(z) + 0.25z^{-1}X(z) + 0.5z^{-1}Y(z)$$

Transfer function

$$\frac{Y(z)}{X(z)} = \frac{0.5 + 0.25z^{-1}}{1 - 0.5z^{-1}}$$



4-Point FIR averager



- This can be described with the following *difference equation*

$$y[n] = \frac{1}{4}\left(x[n] + x[n-1] + x[n-2] + x[n-3]\right)$$

- Or with the following z-transform transfer function

$$H_{lp}(z) = \frac{Y(z)}{X(z)} = \frac{1}{4}(1 + z^{-1} + z^{-2} + z^{-3})$$

4-Point FIR averager frequency response

- 4-point moving average z-transfer function

$$H_{lp}(z) = \frac{Y(z)}{X(z)} = \frac{1}{4}(1 + z^{-1} + z^{-2} + z^{-3})$$

- Evaluate frequency response by setting

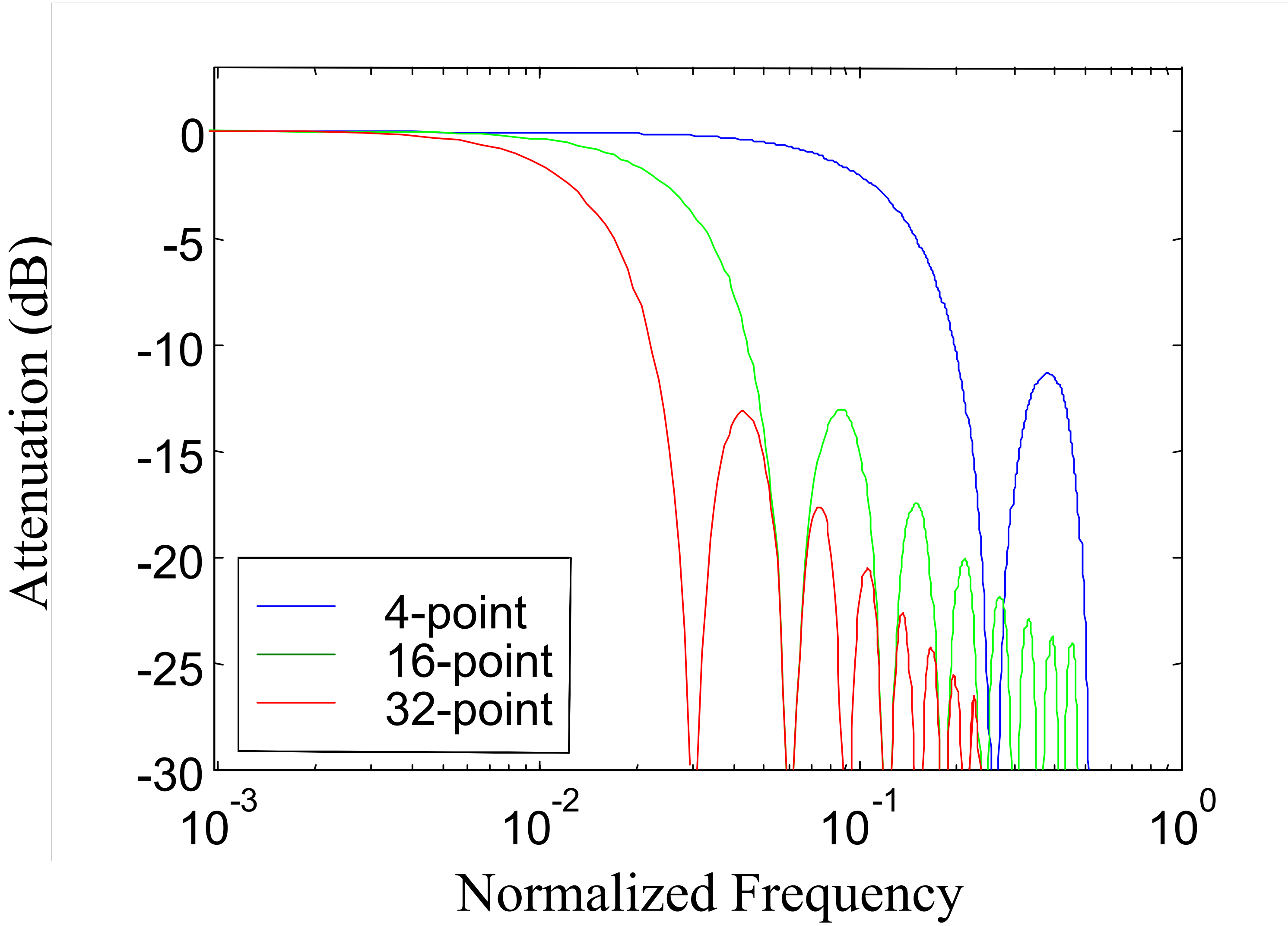
$$z^{-1} = e^{-jw}$$

Giving (by Euler):

$$H_{lp}(e^{j\omega}) = e^{-j3\omega/2}\frac{1}{2}\cdot\left[\cos\frac{3\omega}{2} + \cos\frac{\omega}{2}\right]$$

Magnitude Response **Phase Response**

Averagers with Different Number of Points



Using averaging to get more effective resolution

- Single-sample (turn-by-turn) resolution of APS bpms is nominally 12-bits.
- Residual noise in the analog front-end provides an opportunity to get more resolution by averaging data samples
  - Assuming Gaussian noise, we improve the resolution by a factor 2 (one additional bit) by averaging four samples.
  - The APS bpm processing system uses a 1024-sample boxcar averager to improve the resolution by a factor 32, giving effectively 17-bit resolution.
  - In principle we can increase the resolution ad infinitum, provided we are willing to wait long enough to collect the requisite number of samples.

When does this breakdown?

- Averaging will always work when dealing with Gaussian noise, but at some point, other non-Gaussian processes start to dominate, limiting the performance
  - Front-end amplifier non-linearity.
  - Digitizer quantization errors (integral and differential non-linearity).
  - Word-length effects in the digital processing circuits.
  - Drift.
- Usually digitizers with 12-bit performance do not have 17-bit systematics.

32-Tap Averager vs 32-Tap FIR Filter

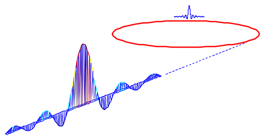
- About the only benefit of a boxcar averager is that it's easy to implement, but does not provide the optimum level of filtering for the number of terms used.

Averager Coefficients

Attenuation (dB)

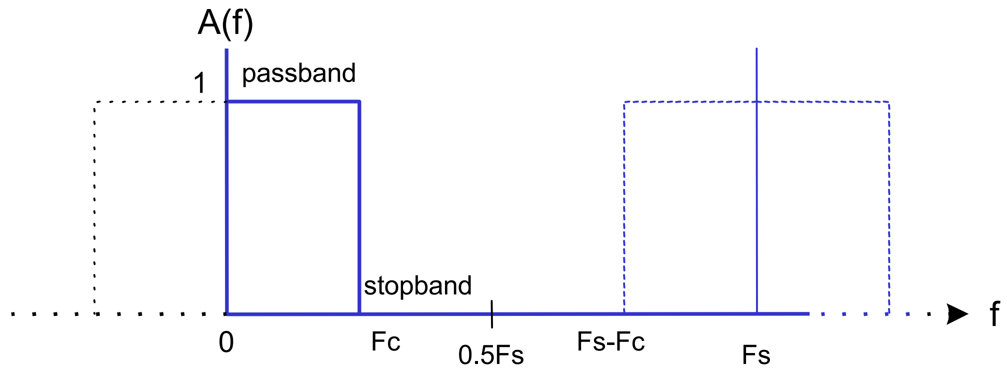
FIR Filter Coefficients

Normalized Frequency

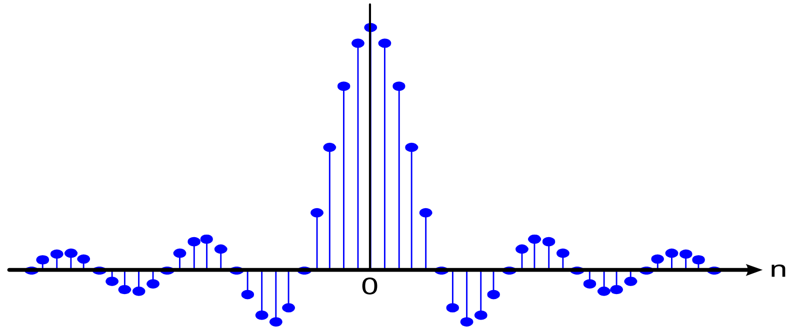


Ideal Frequency-Selective Digital Filters

- The frequency response of an ideal frequency-selective lowpass filter has a passband with constant magnitude, an infinitely sharp transition between passband and stopband, and infinite attenuation in the stopband. The phase delay is zero for all frequencies.



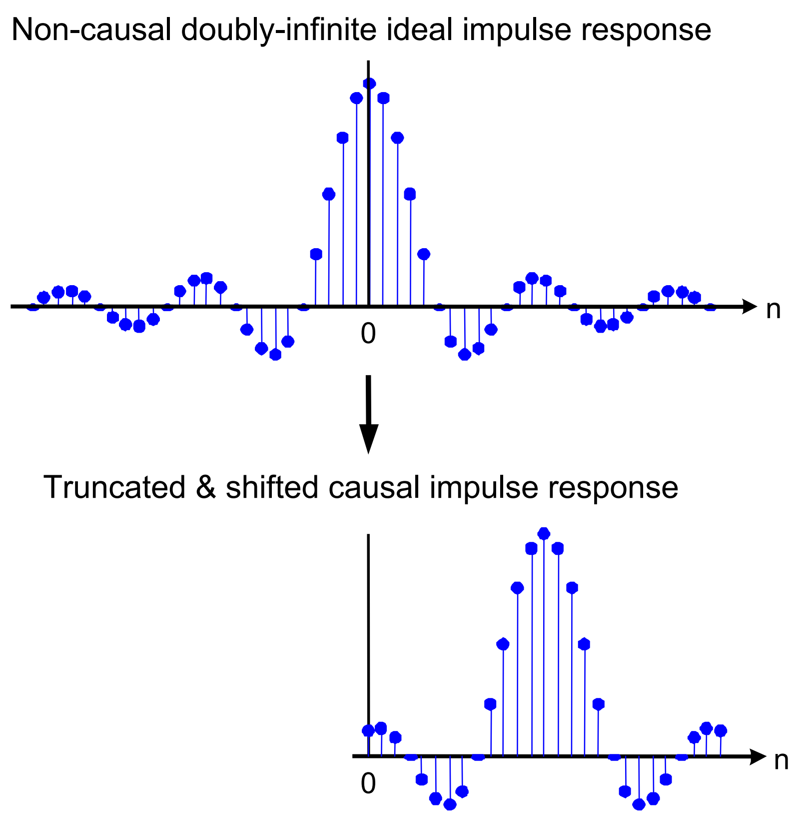
- As we have discussed before, the impulse response of this ideal filter is a doubly-infinite sin(*x*)/*x* function that cannot be implemented in practice



$$h[n]=\begin{cases} 2f_c & n=0 \\ 2f_c\cdot\dfrac{\sin(2\pi\cdot f_c\cdot n)}{2\pi\cdot f_c\cdot n} & n\neq 0 \end{cases}$$

FIR Filter Design by Impulse Response Truncation (IRT)

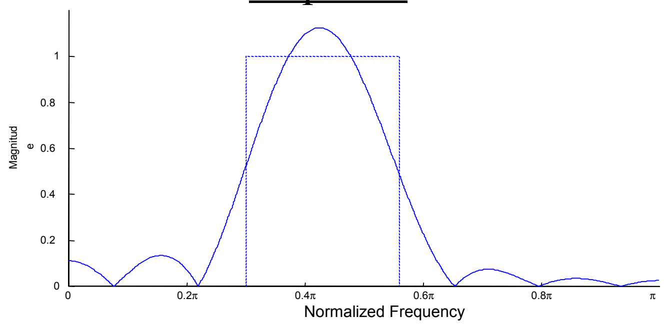
- In the IRT method of designing an FIR filter, we take the impulse response of the idealized impulse response, truncate it to (say) *2M+1* samples, and shift it by *M* samples to make the impulse response causal.



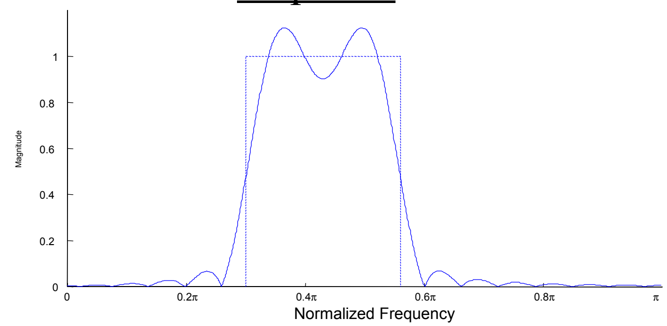
Frequency Response vs Length of Truncated Impulse Response

- More points (coefficients) give a better approximation to the ideal frequency response

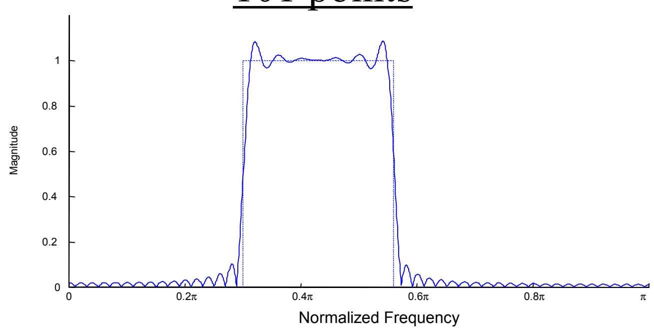
11 points



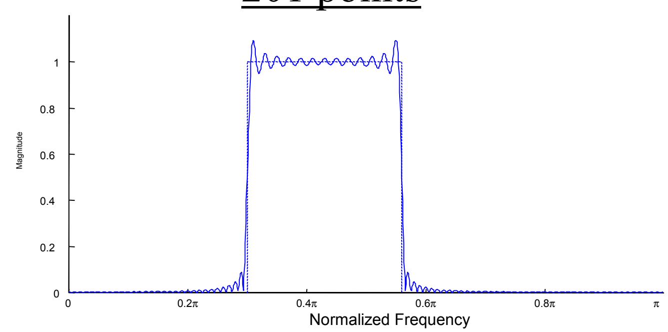
31 points



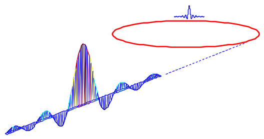
101 points



201 points



…but as the number of points increases, there is no change in the amplitude of the passband or stopband ripple.



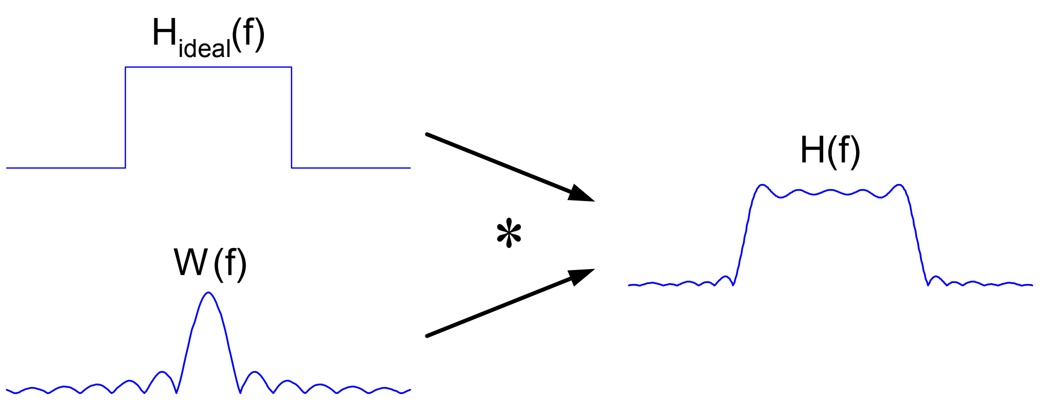
Gibbs Effect and the Impulse Response Truncation Method

- The truncation process is in effect multiplication of the ideal impulse response by a rectangular window (c.f. windowing in the DFT).

$$h[n] = h_{ideal}[n] \cdot w[n]$$

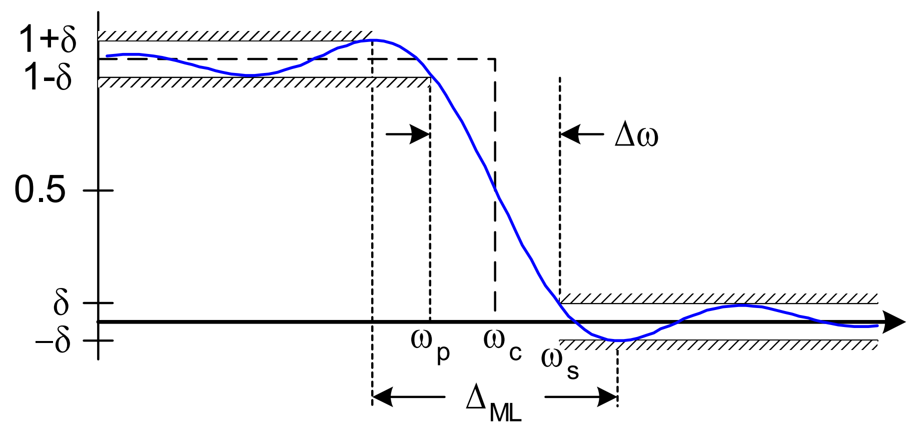
- In the frequency domain, this means the actual frequency response is the convolution of the ideal response and the frequency response of the window function

$$H[\omega] = H_{ideal}[\omega] * W[\omega]$$



FIR Filter Design by Windowing

- Window functions other than rectangular (truncation) can be used to change the response of the filter. These are the same window functions used with the DFT.
- Windows used for FIR filter design include *Hann, Hamming,* and *Blackman*.
- Characteristics of filters designed with these windows are shown below

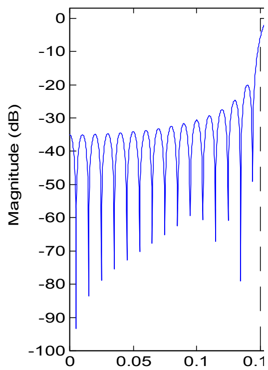


Window	Main-lobe width (Δ_{ML})	Transition width ($\Delta\omega$)	δ	Passband Ripple (dB)	Stopband Ripple (dB)
Rectangular	$4\pi/(2M+1)$	$0.92\pi/M$	0.09	0.75	-21
Hanning	$8\pi/(2M+1)$	$3.11\pi/M$	0.0063	0.055	-44
Hamming	$8\pi/(2M+1)$	$3.32\pi/M$	0.0022	0.019	-53
Blackman	$12\pi/(2M+1)$	$5.56\pi/M$	0.0002	0.0017	-74

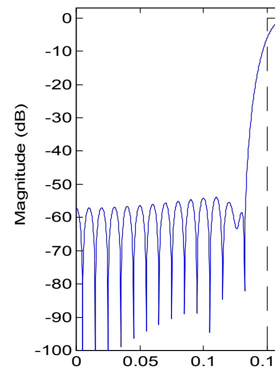
Effect of Windowing on Bandpass Filter Example

- Magnitude responses of bandpass filters with length 101 for different window functions (band edges at 0.15*Fs* and 0.28*Fs*)

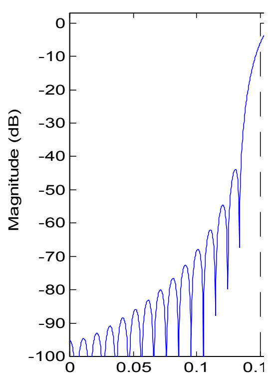
Rectangular Window



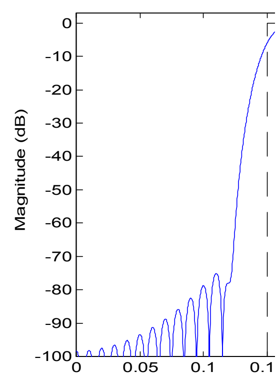
Hamming Window



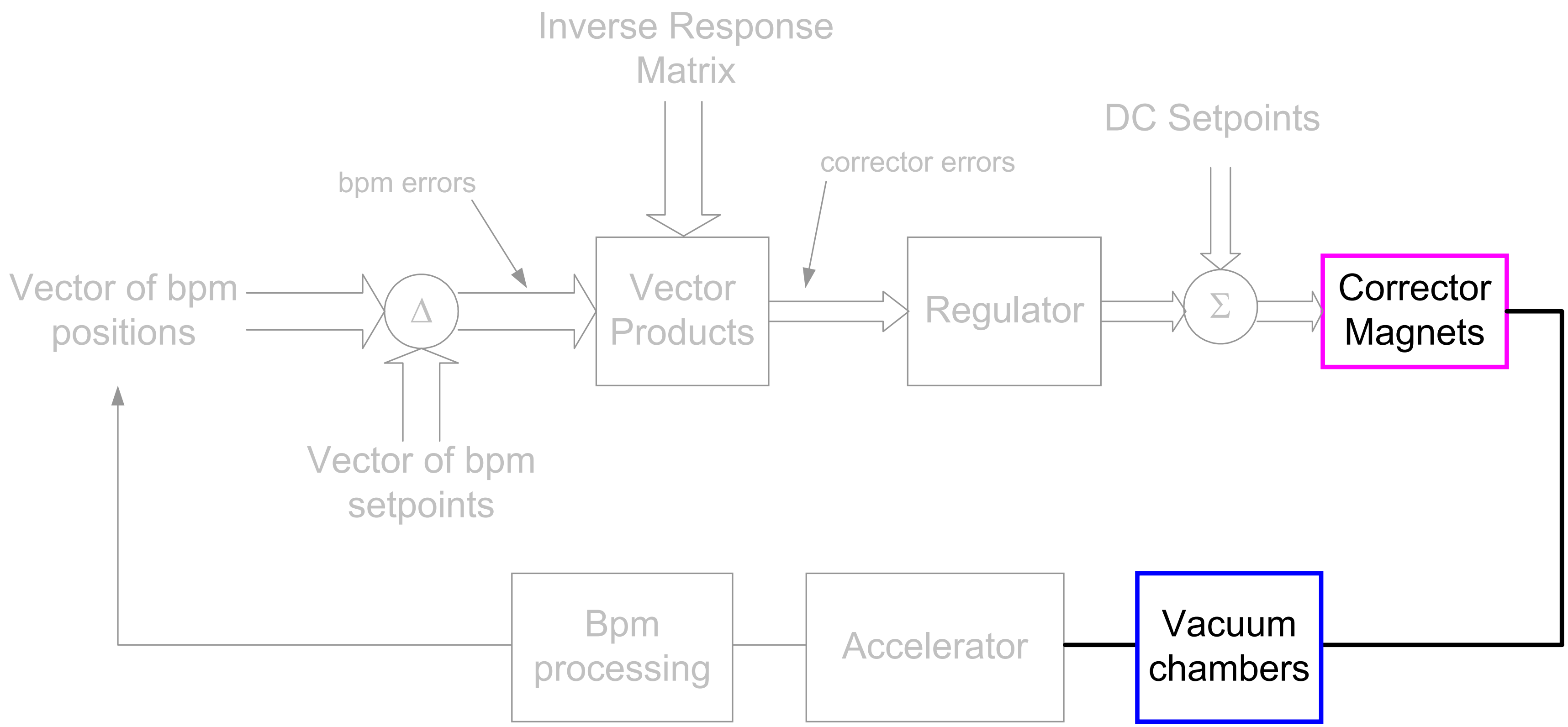
Hanning Window



Blackman Window



CORRECTOR DYNAMICS

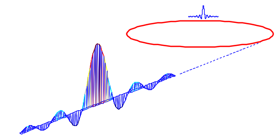


Measurement of corrector dynamics

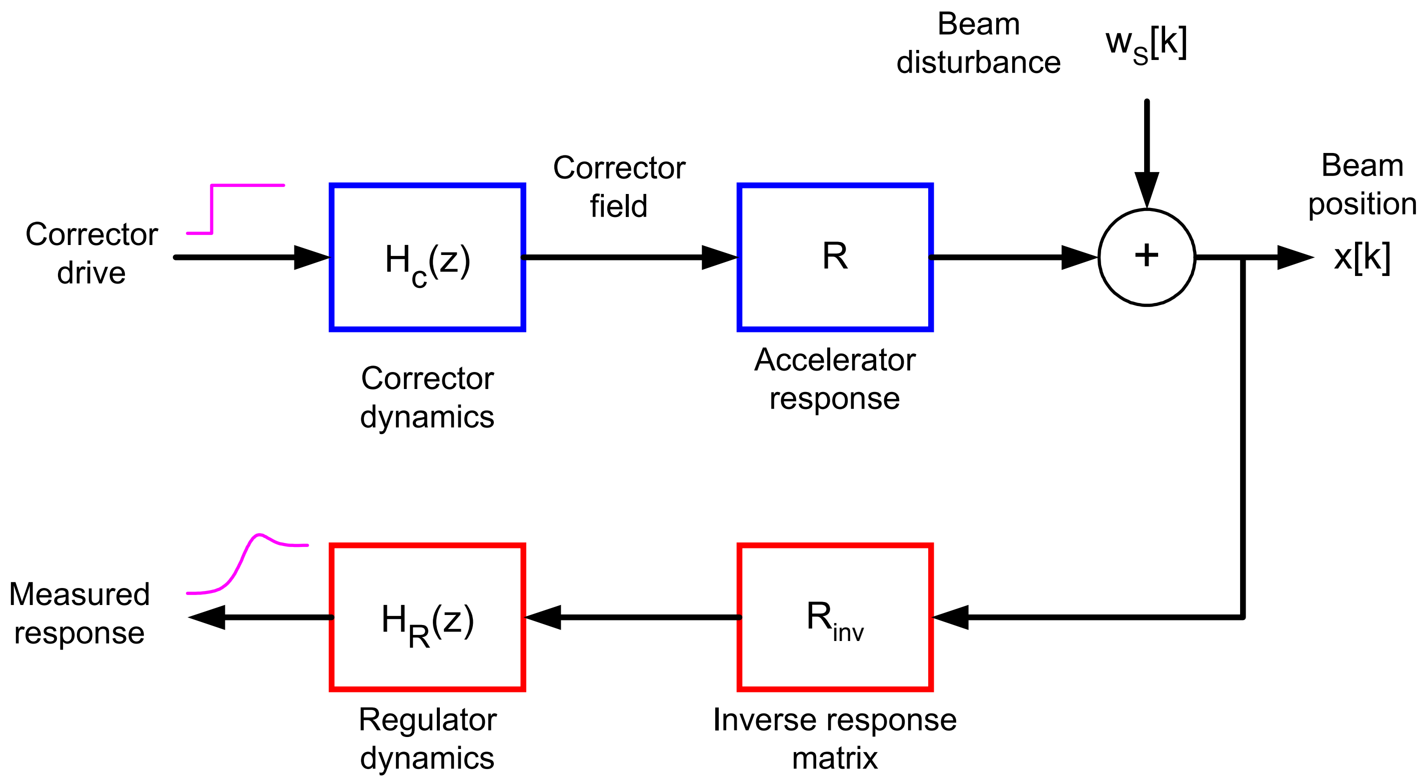
- Measuring the response of a magnet / power supply is often straight-forward
  - Swept sine measurements
  - Transfer function analysis

- But we need the response of the global orbit to changes in corrector strength that include magnet and eddy-current effects.
- The only real means to determine system dynamics is to measure the responses in-situ using beam-based measurements.

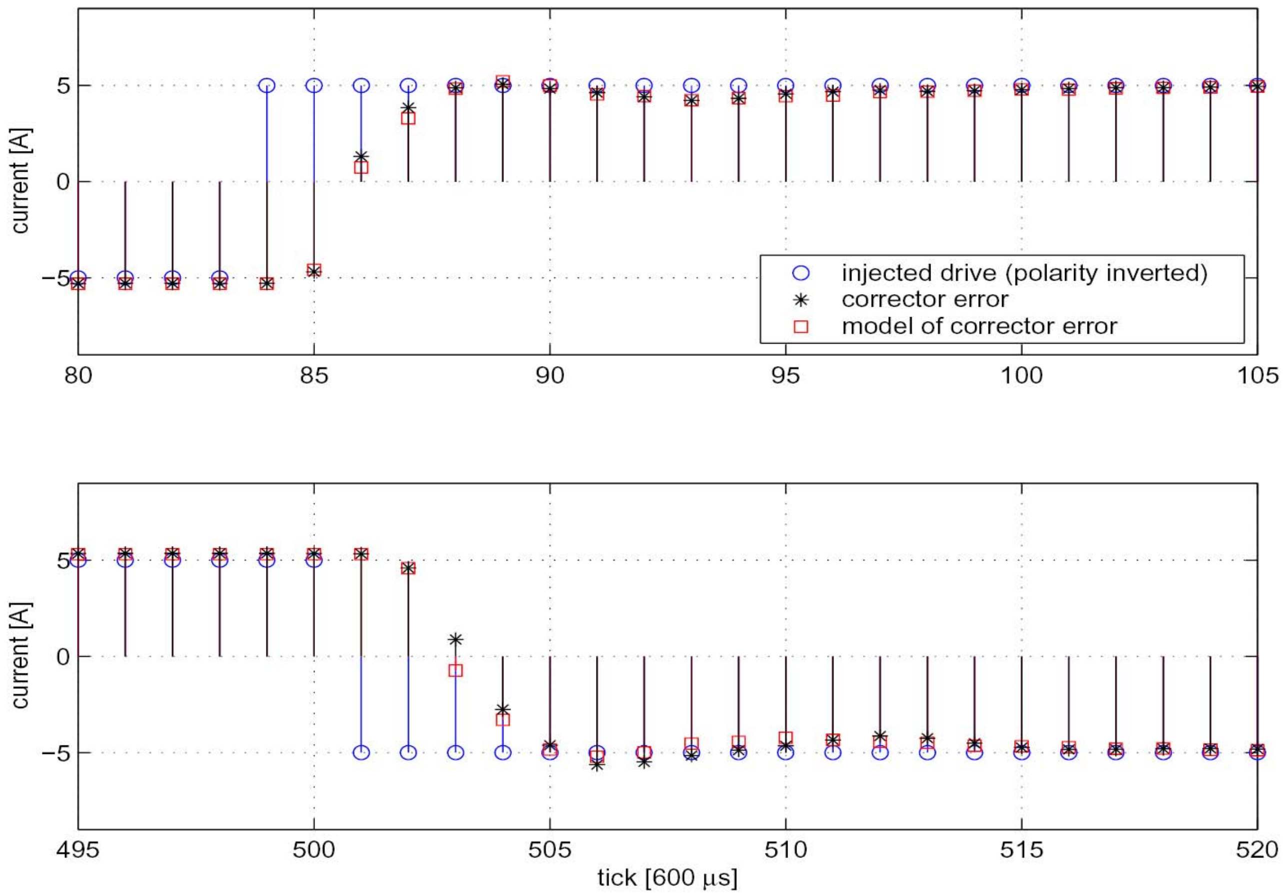
- The procedure is to drive a single corrector at a time, and measure global orbit changes caused by the corrector drive.
  - Both frequency-domain and frequency-domain methods can be used.
  - At APS, time-domain measurements have resulted in more robust parametric models.



Beam-based corrector response measurement

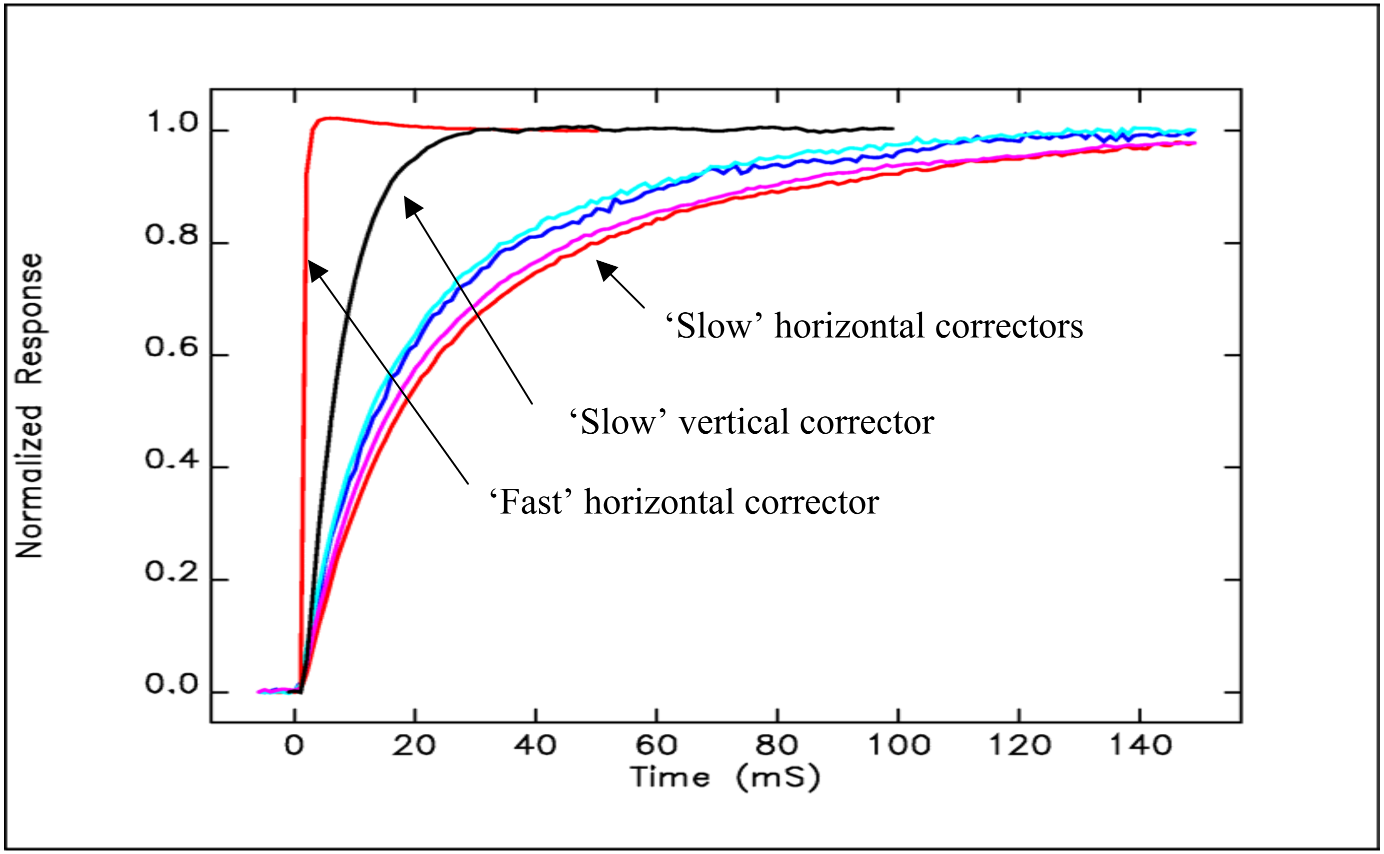


Corrector response example



Measured step responses of APS corrector magnet fields

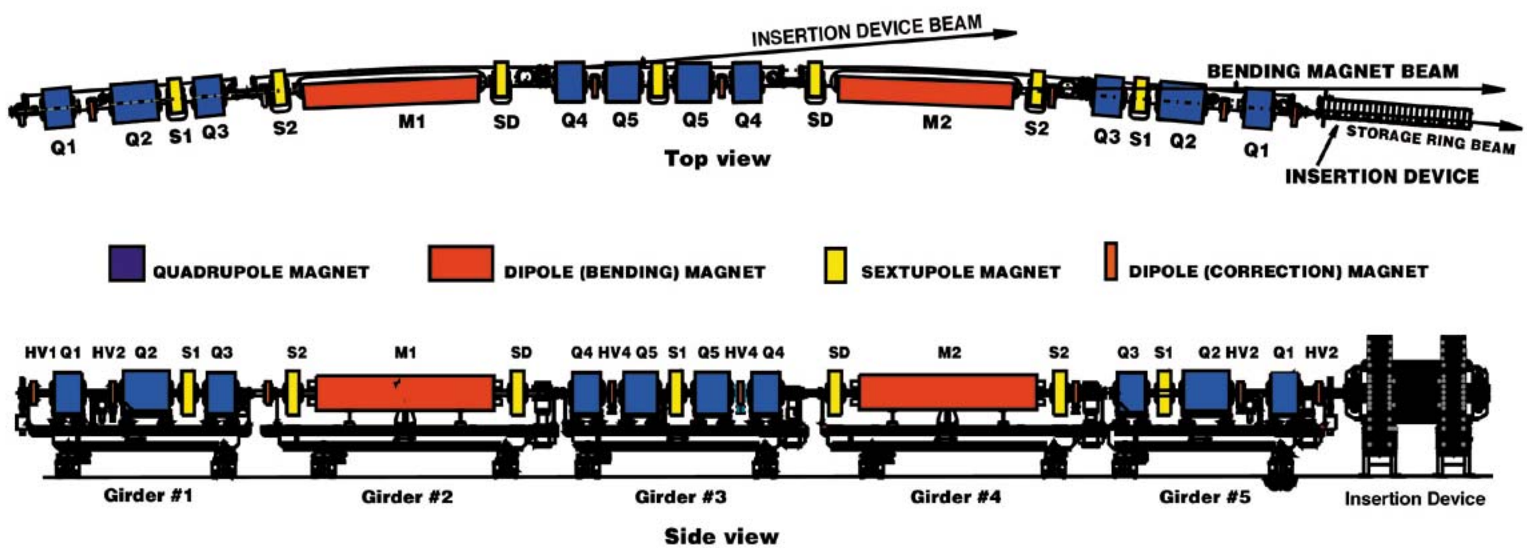
- Step responses of the power supplies and magnets are the same in all cases.
- Overall responses of each corrector family are dominated by eddy-current effects in the vacuum chambers. Effects are different for each family.



Why do different correctors families have different dynamics?

- Location, location, location…

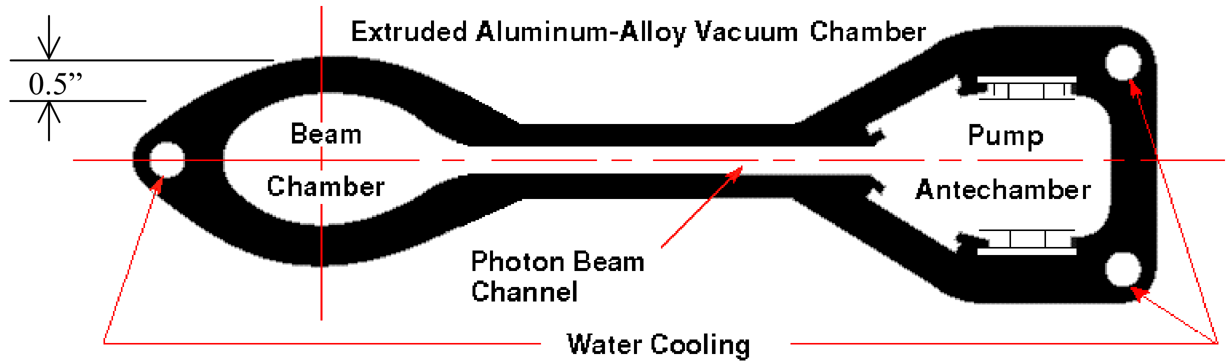
One Sector of the Advanced Photon Source Storage Ring



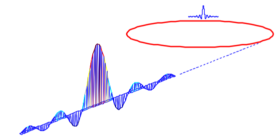
APS storage ring corrector location S1A:H1

Effect of vacuum chamber eddy currents

- Magnet fields have to penetrate ½” of aluminum in the APS vacuum chamber



- This has no impact on DC fields, and therefore on ‘slow’ orbit correction, but significantly impacts dynamic orbit correction bandwidth for those magnets surrounding the aluminum chamber.



Eddy current wall model

- Skin depth:

$$\partial = \sqrt{\frac{2}{\mu \cdot \omega \cdot \sigma}}$$

- Frequency domain (from Jackson):

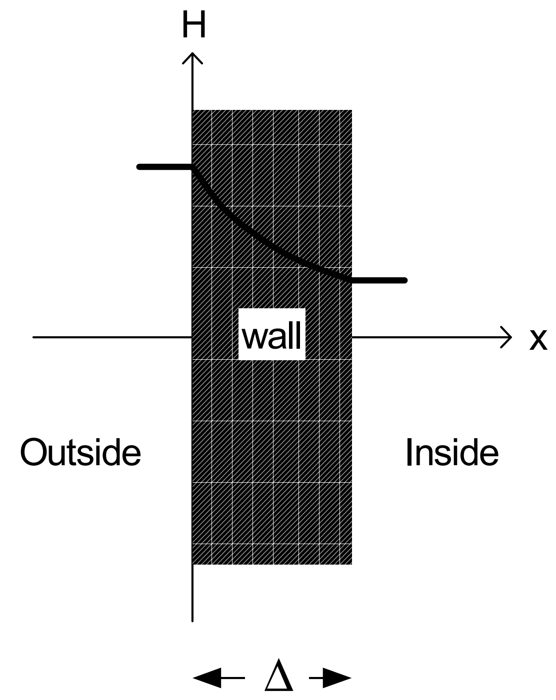
$$H(w) = e^{-\frac{x}{\delta}(1+j)}$$

- Laplace domain

$$H(s) = e^{-k\sqrt{s}} \qquad \text{where} \qquad k = \Delta\sqrt{\mu \cdot \sigma}$$

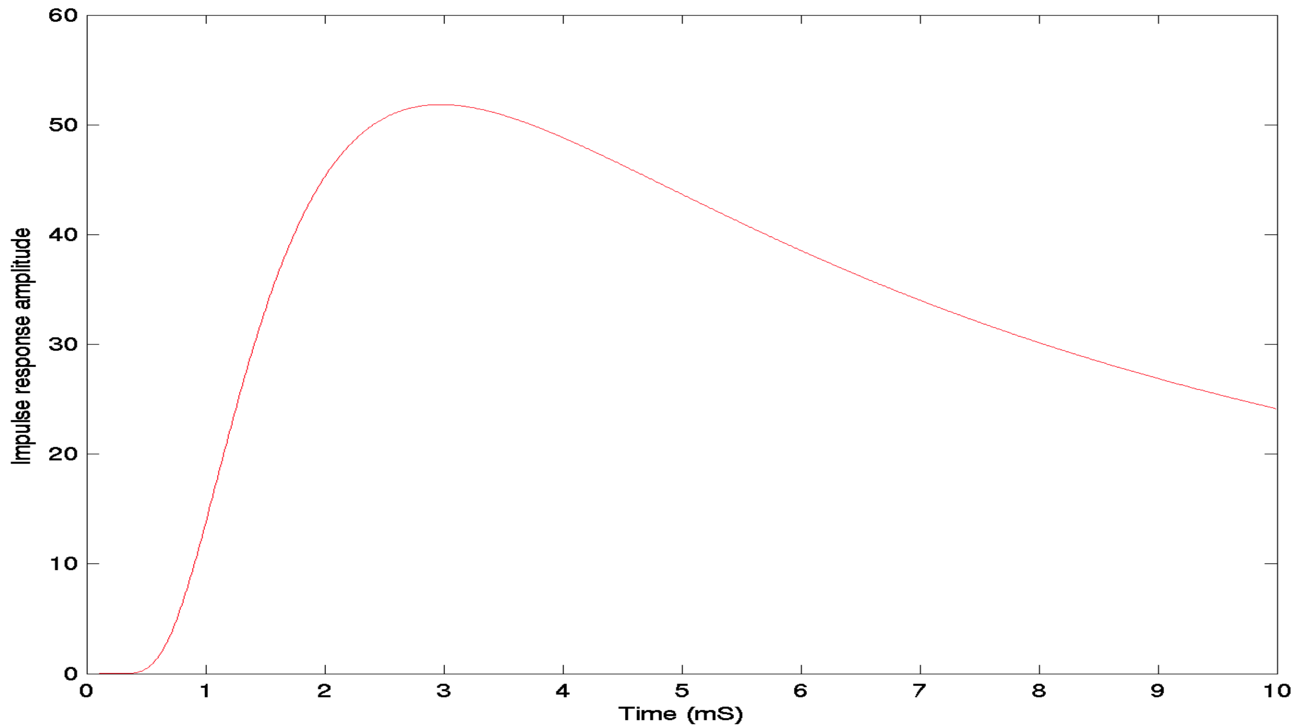
- Time domain:

$$F(t) = \frac{k}{2\sqrt{\pi \cdot t^3}} e^{-\frac{k^2}{4t}} \qquad \text{where} \qquad k = \Delta\sqrt{\mu \cdot \sigma}$$



K. Evans, 5/96

Impulse response of 2cm aluminum wall



- The 0.5mS time delay significantly impacts the maximum correction bandwidth because regulator phase advance will not speed up the response.

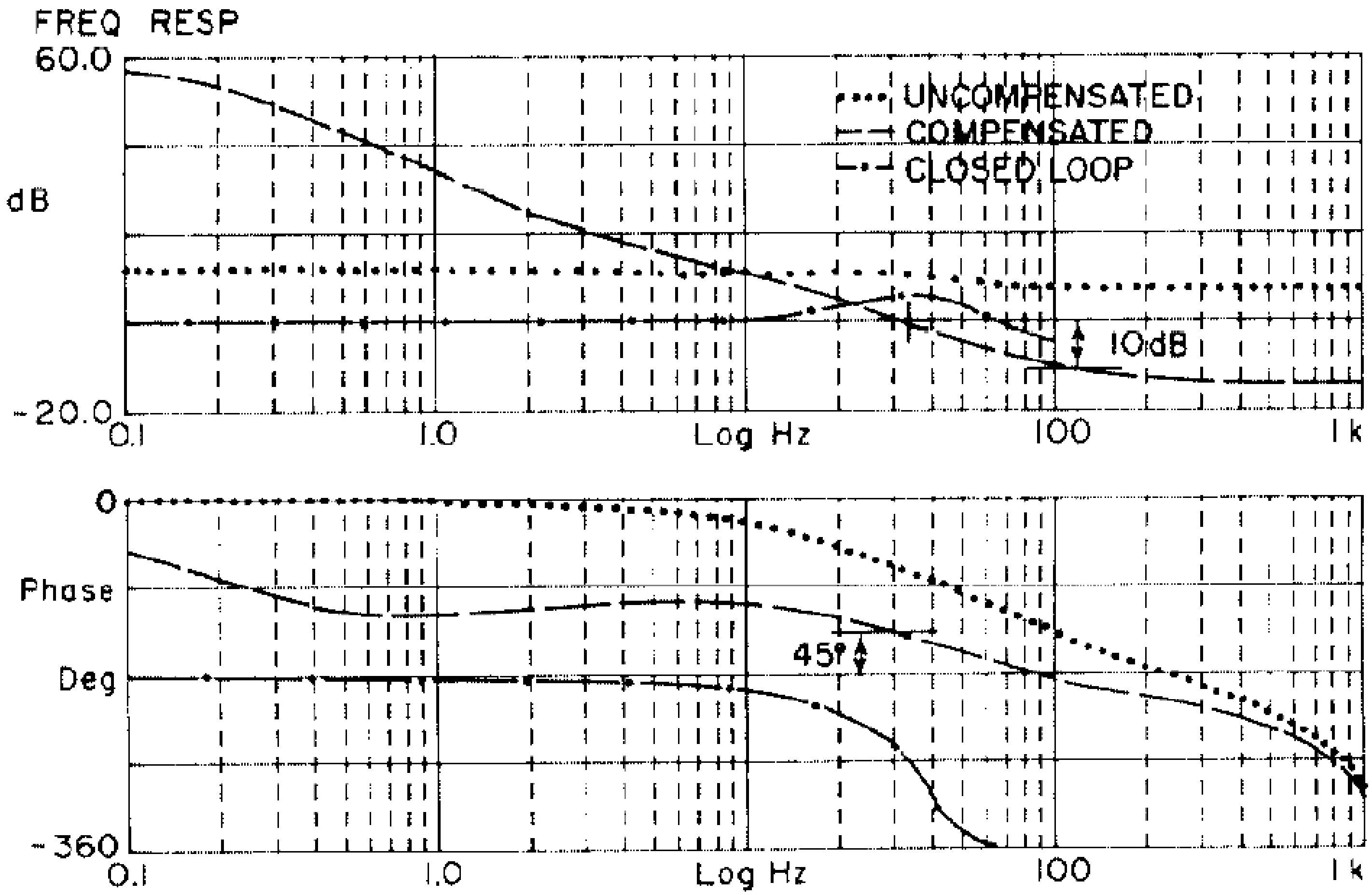
Corrector equalization

- Eddy current effects are two-fold
  - The chamber acts as a low-pass filter.
  - It takes a finite amount of time for the field to penetrate the chamber.
- We can compensate for the low-pass filter, but not for the time delay.

- Provided the time delay is small compared with system dynamics and correction rates, reasonable equalization can be accomplished using frequency-domain techniques (eg lead/lag compensators)
  - NSLS used frequency-based equalization filters for the VUV local bumps.
  - Time domain techniques based on measured step responses provide more robust models when the time delay is significant.

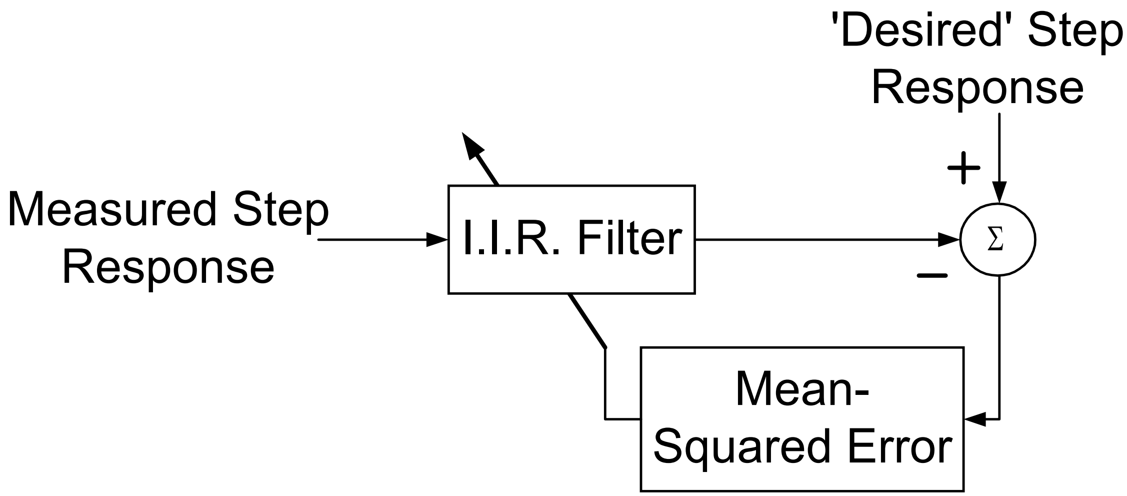
- Appreciable eddy current time delays will ultimately become the limiting factor in optimizing the closed loop performance because time-delay = linearly increasing phase with frequency.

Impact of eddy currents on NSLS local correction frequency & phase responses



Step Response Equalization using optimal filters

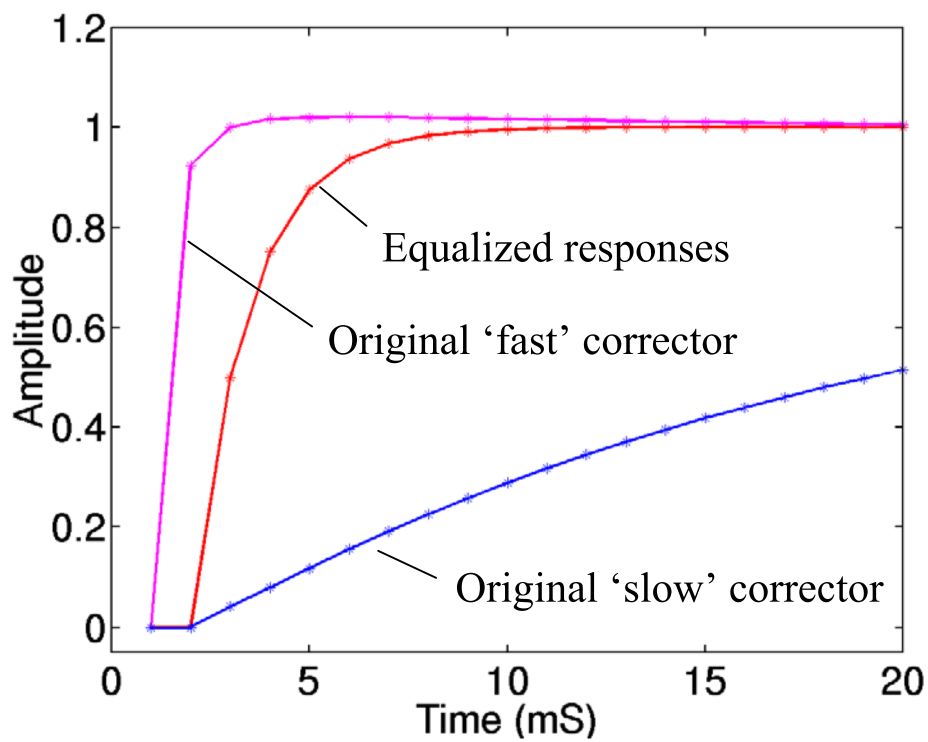
- Optimal least-squares filter design based on measured and desired time-domain sequences offers a more robust alternative to frequency-based methods of modeling step responses.



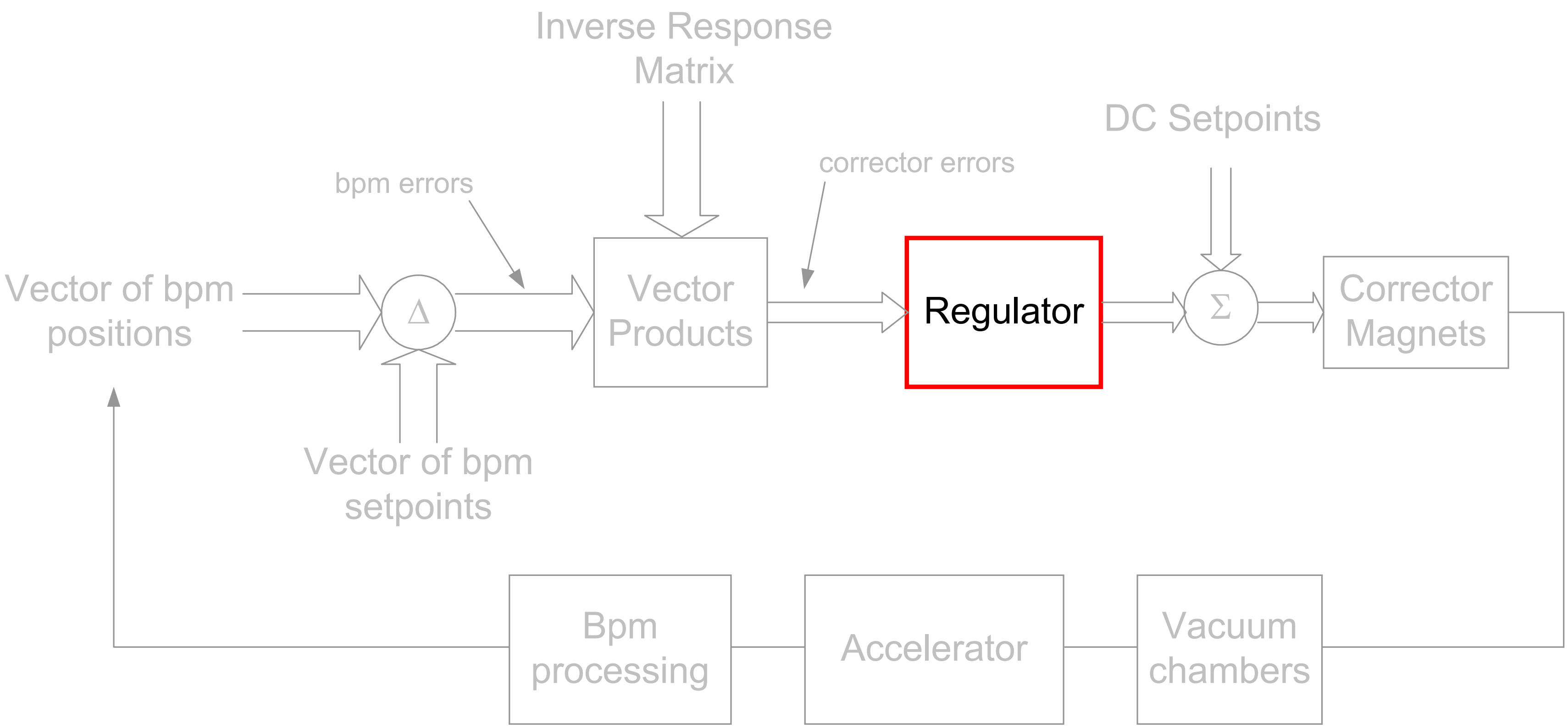
- Digital filter coefficients are computed from the “Normal” equations that use auto-and cross-correlation functions between desired and measured responses.
- The resultant all-point IIR (‘auto-recursive’) filter represents the transfer function of the inverse filter.

Results of least-squares filter equalization

- Desired' response was chosen to be a simple low-pass filter with a one-sample delay.
- Reasonable results were obtained with a 3-pole, 3-zero IIR filter.

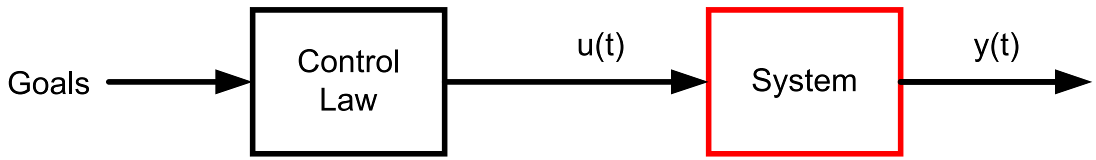


REGULATOR DESIGN



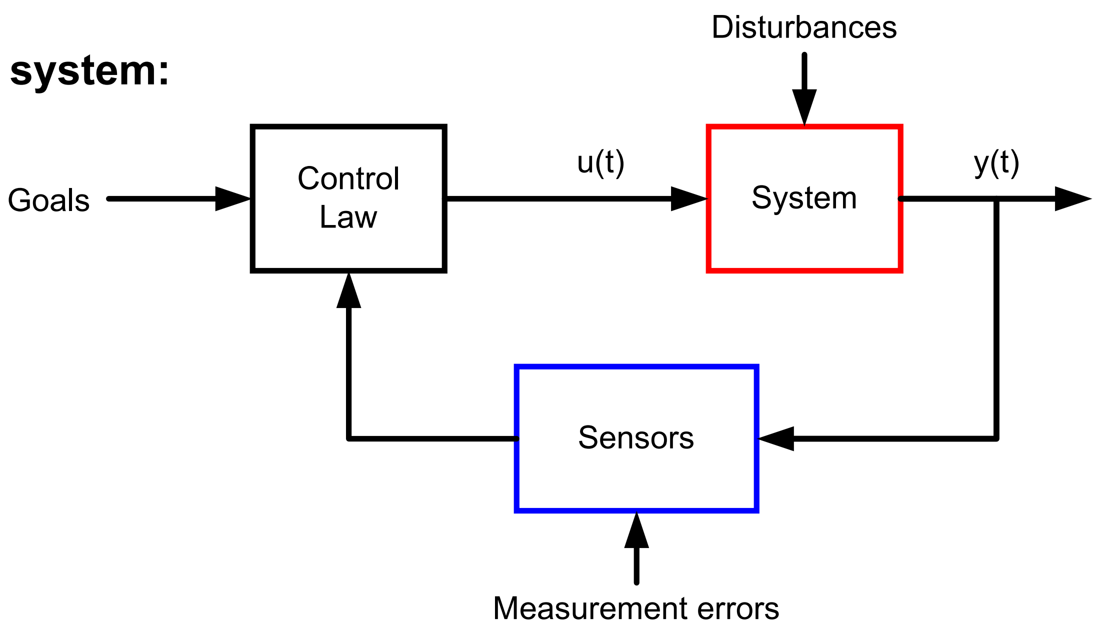
Simple plant/regulator models

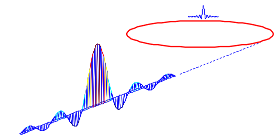
Open-loop system:



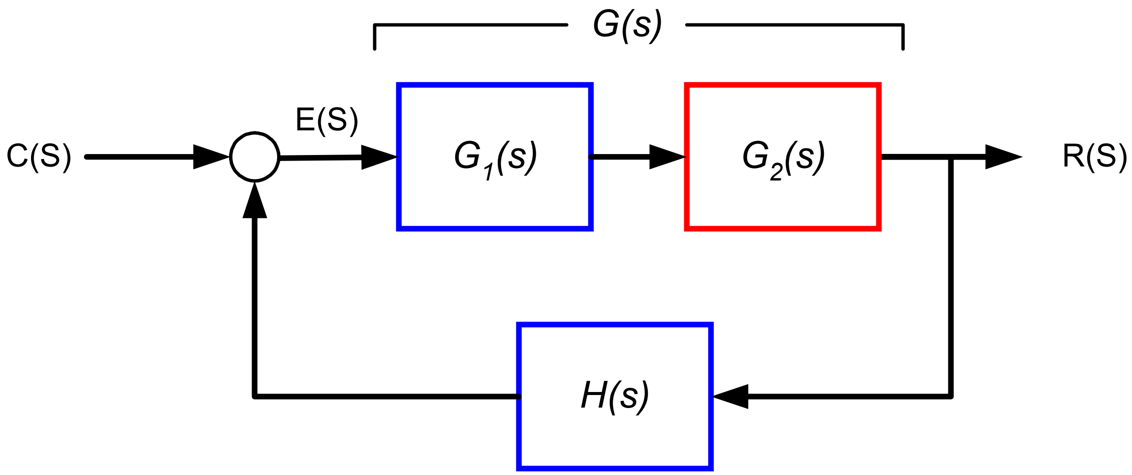
- Works fine, provided there are no disturbances to the system, and the system itself does not change over time.

Closed-loop system:





Elementary feedback control system



- Black's formula: $$TF = \frac{\text{forward gain}}{1 + \sum \text{loop gains}}$$

$$C(s) = \frac{G(s)}{1 + G(s) \cdot H(s)} \cdot R(s)$$

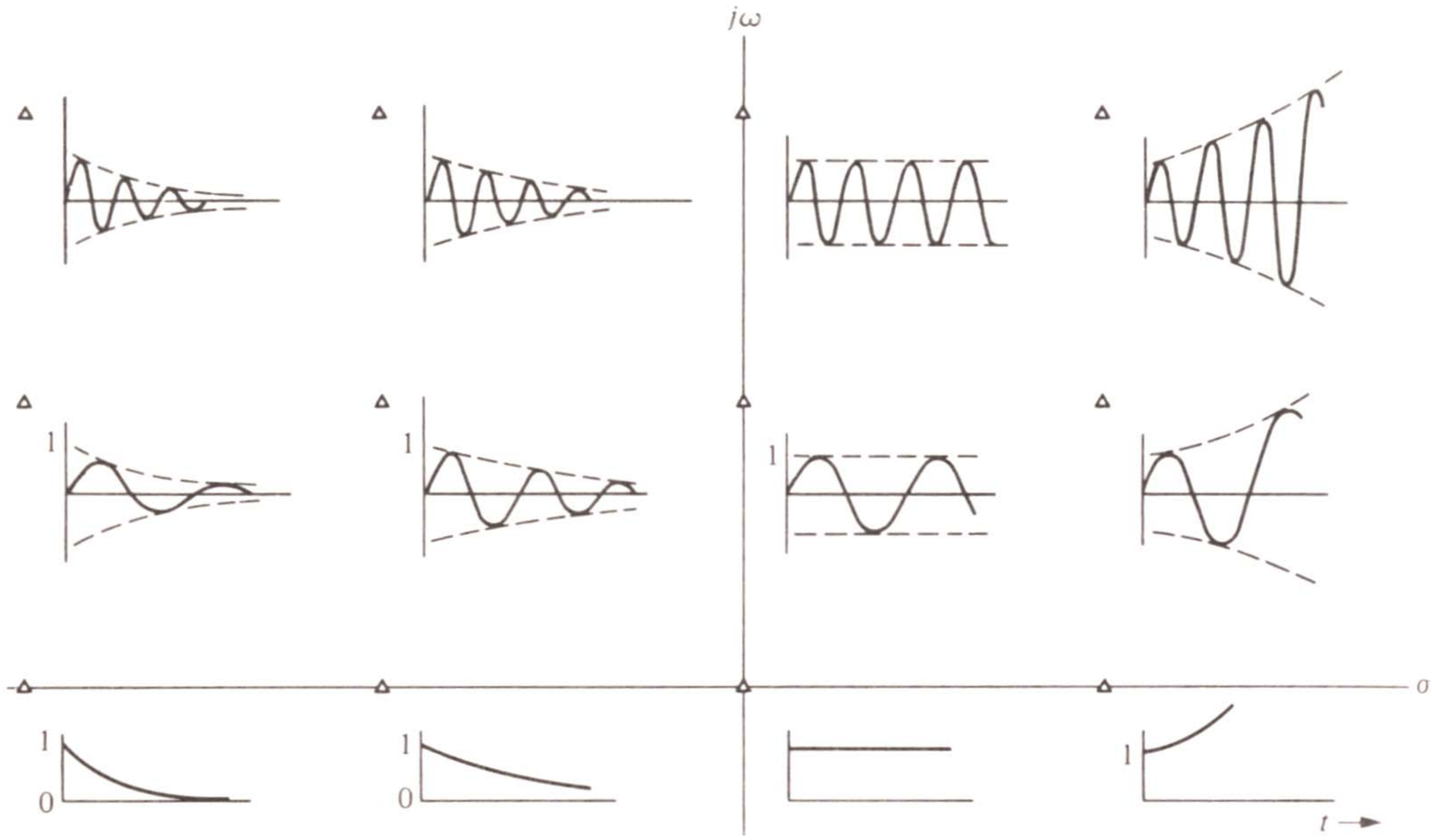
- For a stable system, all roots of the denominator (poles) of the closed loop transfer function must be in the left half of the complex plane.

Elementary Laplace Transform Pairs

f(t)	*F(s)*
$\delta(t)$	1
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$t \cdot e^{-at}$	$\frac{1}{(s+a)^2}$
$t^n \cdot e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$

f(t)	*F(s)*
$\sin(\omega \cdot t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega \cdot t)$	$\frac{s}{s^2+\omega^2}$
$t \cdot \sin(\omega \cdot t)$	$\frac{2 \cdot \omega \cdot s}{(s^2+\omega^2)^2}$
$t \cdot \cos(\omega \cdot t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$e^{-at} \cdot \sin(\omega \cdot t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cdot \cos(\omega \cdot t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

Impulse responses vs transfer function pole locations

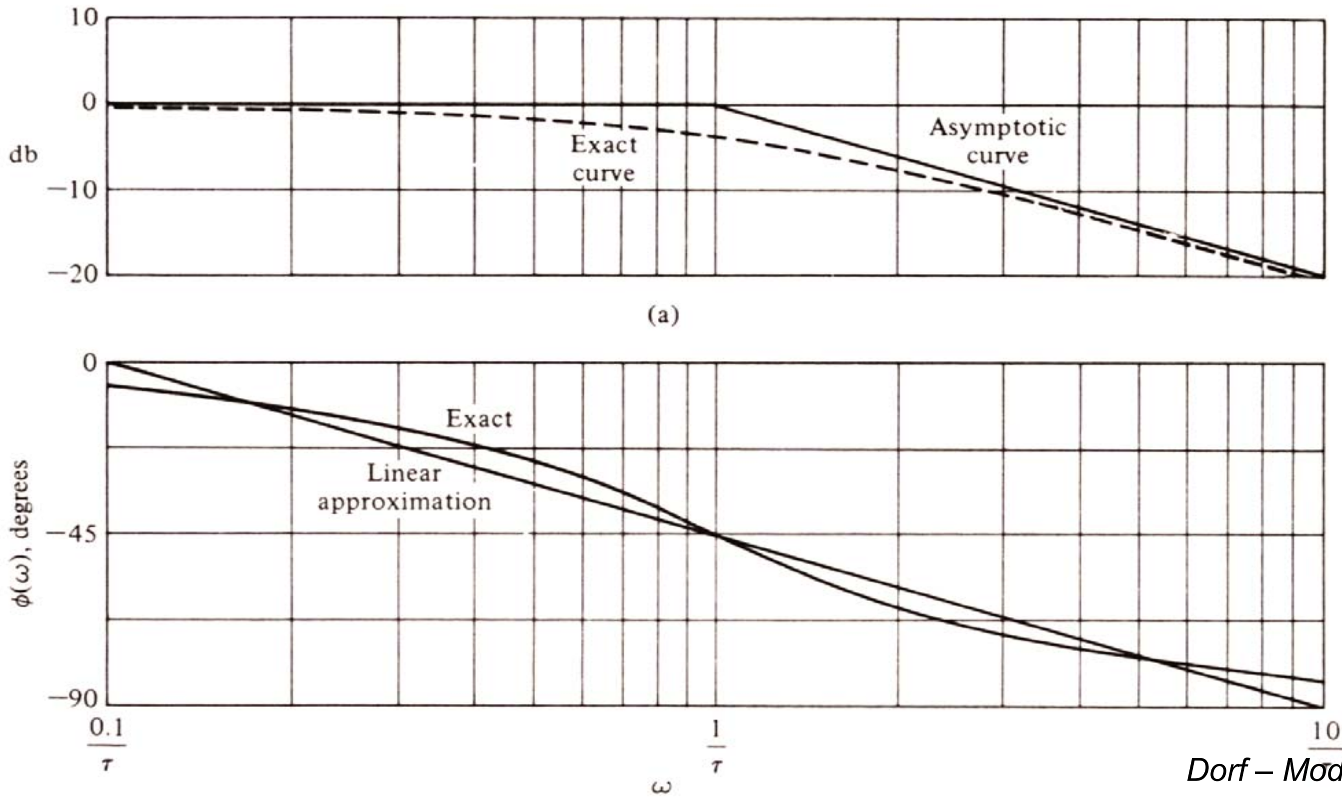


Dorf – Modern Control Systems

Bode diagram

- Provides a convenient graphical method of evaluating frequency and phase responses using asymptotic approximations

Example (single pole):



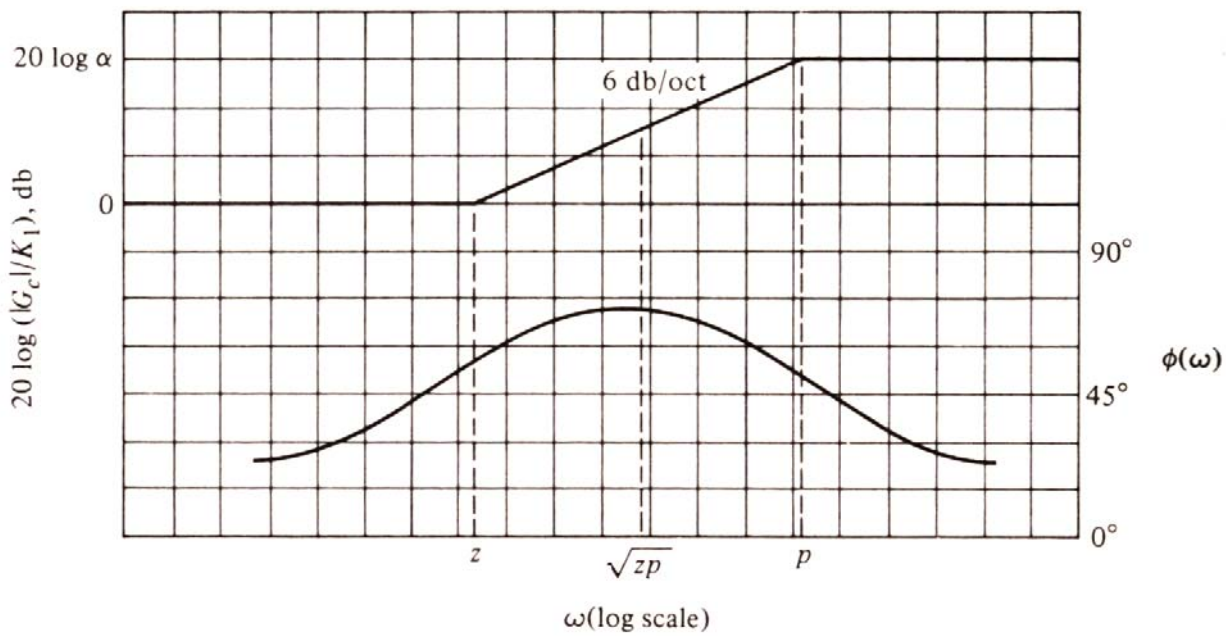
Dorf – Modern Control Systems

Phase lead compensators

- Phase lead compensator
  - Increases gain at high frequencies, while low frequency gain is unchanged.
  - Improves stability margins at expense of high frequency gain.
  - State-space notation (time-domain design) (still to do)

$$TF = \frac{1+\alpha\tau \cdot s}{\alpha\left(1+\alpha \cdot s\right)}$$

$$\sin\phi_{max} = \frac{\alpha - 1}{\alpha + 1}$$



Dorf – Modern Control Systems

State-space notation

- Time-domain representation of system dynamics in terms of first-order difference equations (digital control), or differential equations (analog control)
- State equation describes the dynamics of internal states of the system.
- Output equation describes the dynamics of the system output.

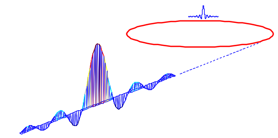
Continuous-time

$$x[n+1] = Ax[n] + Bu[n]$$
$$y[n] = Cx[n] + Du[n]$$

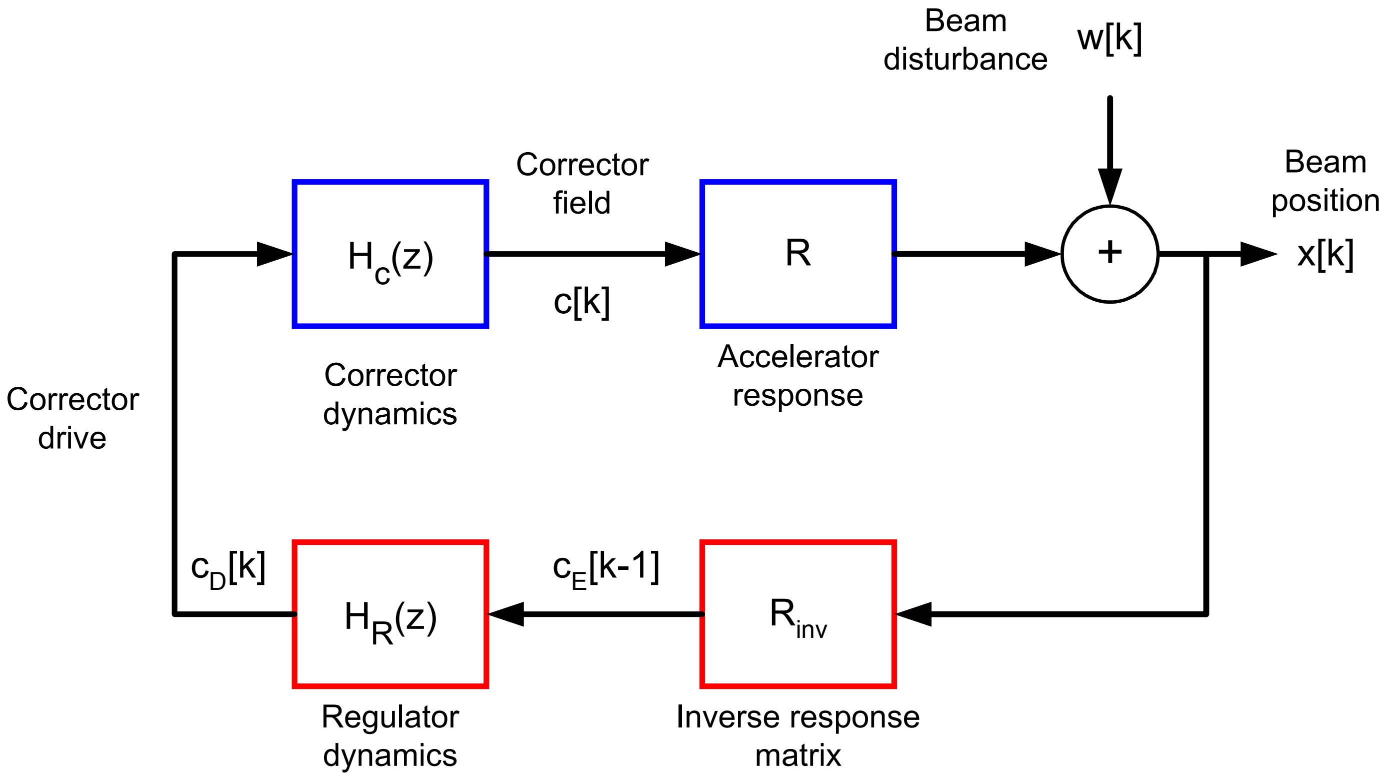
Discrete-time

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

- More versatile than classical frequency domain representation
  - Easily extended to multi-input/multi-output systems.
  - Applicable to LTI and non-LTI systems.
  - Greater variety of control design techniques, eg Optimal control techniques allow computation of regulator design based on cost functions, eg rms actuator power, rms tracking error, etc



Mathematical model of orbit feedback system



$$x[k] = R \cdot c[k] + w[k]$$

$$c_E[k] = -R_{inv} \cdot x[k-1]$$

$$c(z) = z^{-1} H_c(z) \cdot H_R(z) c_E(z)$$

System Transfer Function

$$X(z) = R \cdot C(z) + W(z) \qquad \text{and} \qquad C_E(z) = -R_{inv} \cdot X(z)$$

$$C(z) = z^{-1} H_c(z) \cdot H_R(z) \cdot C_E(z)$$

$$\therefore X(z) = z^{-1} R \cdot H_c(z) \cdot H_R(z) \cdot C_E(z) + W(z)$$

$$= -z^{-1} R \cdot H_c(z) \cdot H_R(z) \cdot R_{inv} \cdot X(z) + W(z)$$

Transfer Function: $$G(z) = \left[I + R \cdot H(z) \cdot R_{inv}\right]^{-1}$$

where $$H(z) = z^{-1} H_R(z) \cdot H_c(z)$$

- The system transfer function quantifies the effect of the system in rejecting orbit disturbances.

REGULATOR PERFORMANCE METRICS

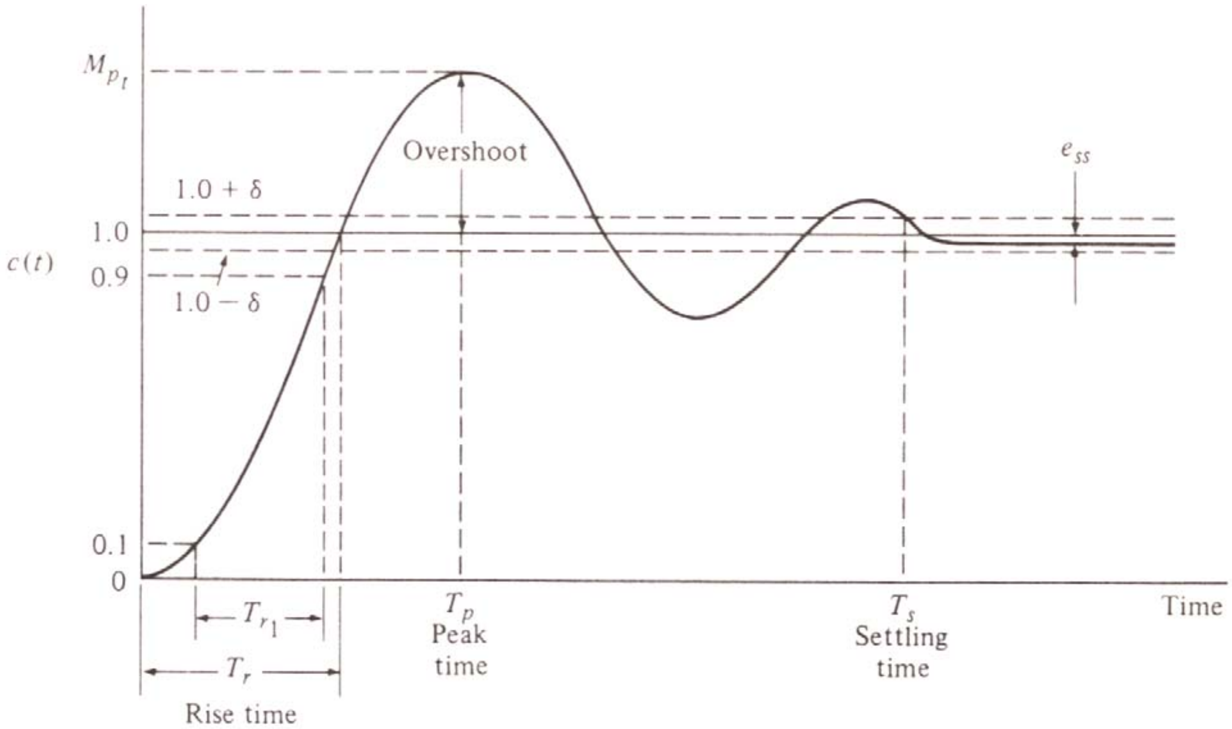
Control system performance metrics

- Elementary performance criteria
  - The plant didn't blow up.
  - The process measurements stay close enough to the setpoint.
  - They say it's OK and you can go home now.

- Classical (deterministic) performance criteria
  - Closed loop bandwidth.
  - Gain margin, phase margin.
  - Maximum disturbance rejection.
  - Steady-state error.
  - Minimum overshoot, minimum setting time.

- "Modern/optimal' performance criteria
  - Usually based on some form of cost function
    - Integral of absolute value of error or error squared
    - Integral of time x absolute value of error or error squared
    - Actuator effort (peak and/or rms)
    - Stochastic metrics (eg error/spectral shaping).

Step response performance measures



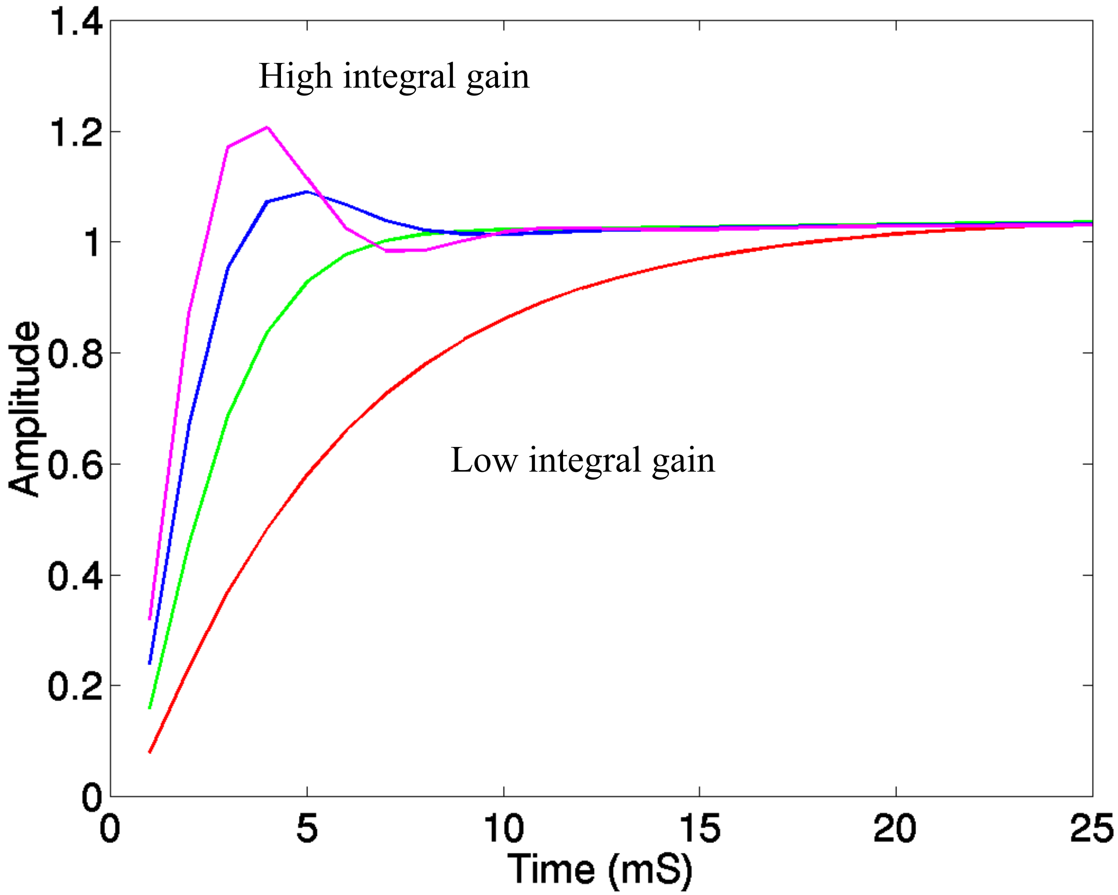
Dorf – Modern Control Systems

Statistical control options

- The control objective of an orbit feedback system is to minimize the rms orbit motion (most likely weighted as a function of frequency).

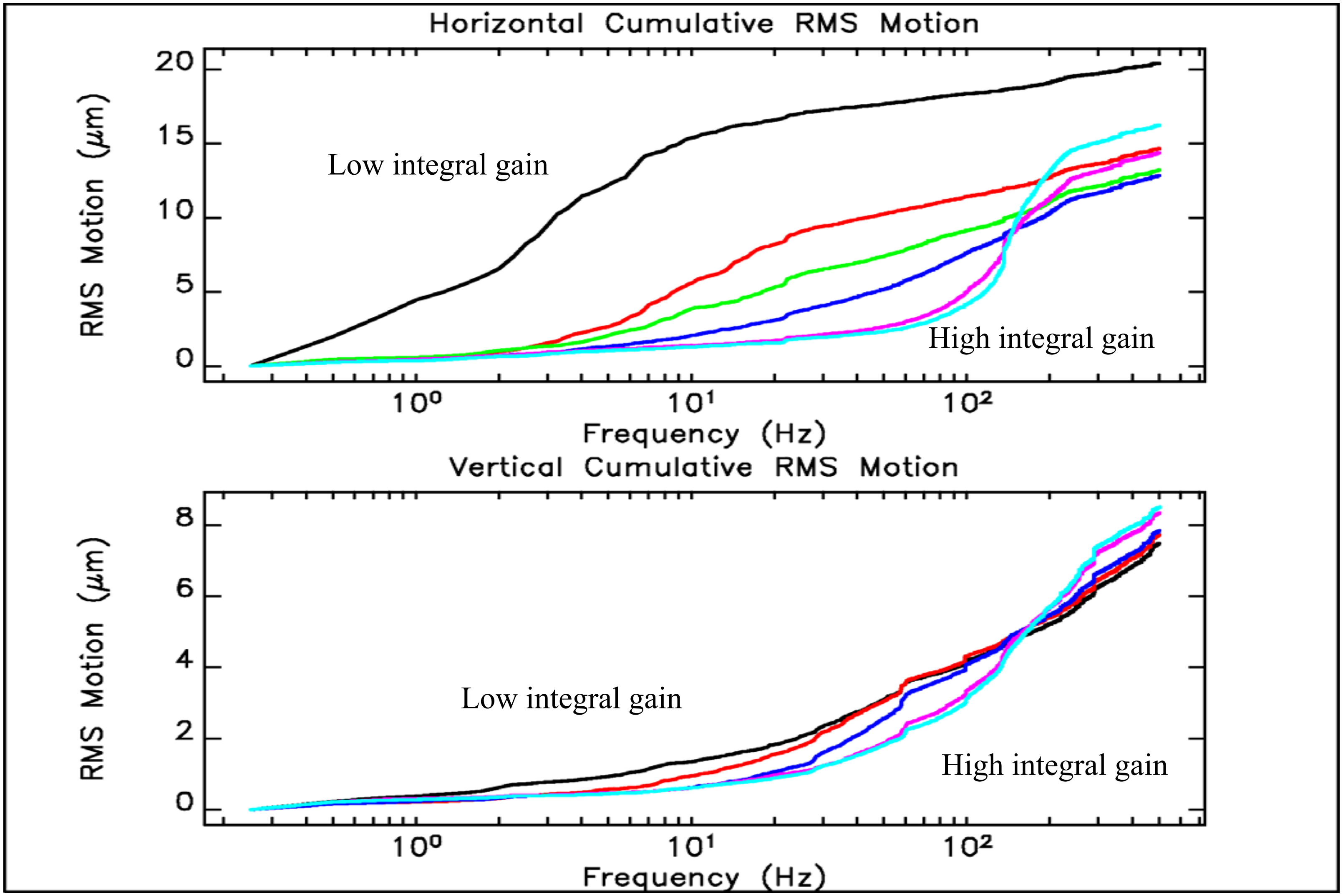
- Options for accomplishing objective of minimizing rms over wide band
  - Use conventional regulator (eg PID), and tune empirically to minimize rms motion over selected frequency band
  - Include measured or modeled orbit spectrum as a cost function in the regulator design (‘optimal control’ techniques).
  - Design closed loop response as a noise shaping filter (requires knowledge of both actual orbit motion spectrum and desired spectrum)

Effect of PID regulator tuning on closed-loop step response



- Which is better: fast rise-time with overshoot; or slower rise-time with no overshoot?

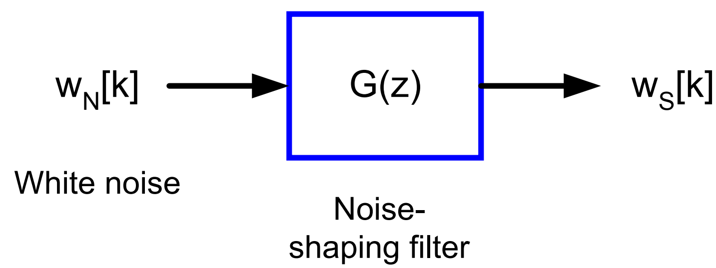
Effect of PID regulator tuning on rms orbit motion

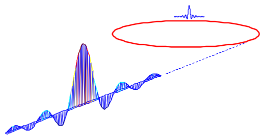


Stochastic vs deterministic orbit motion

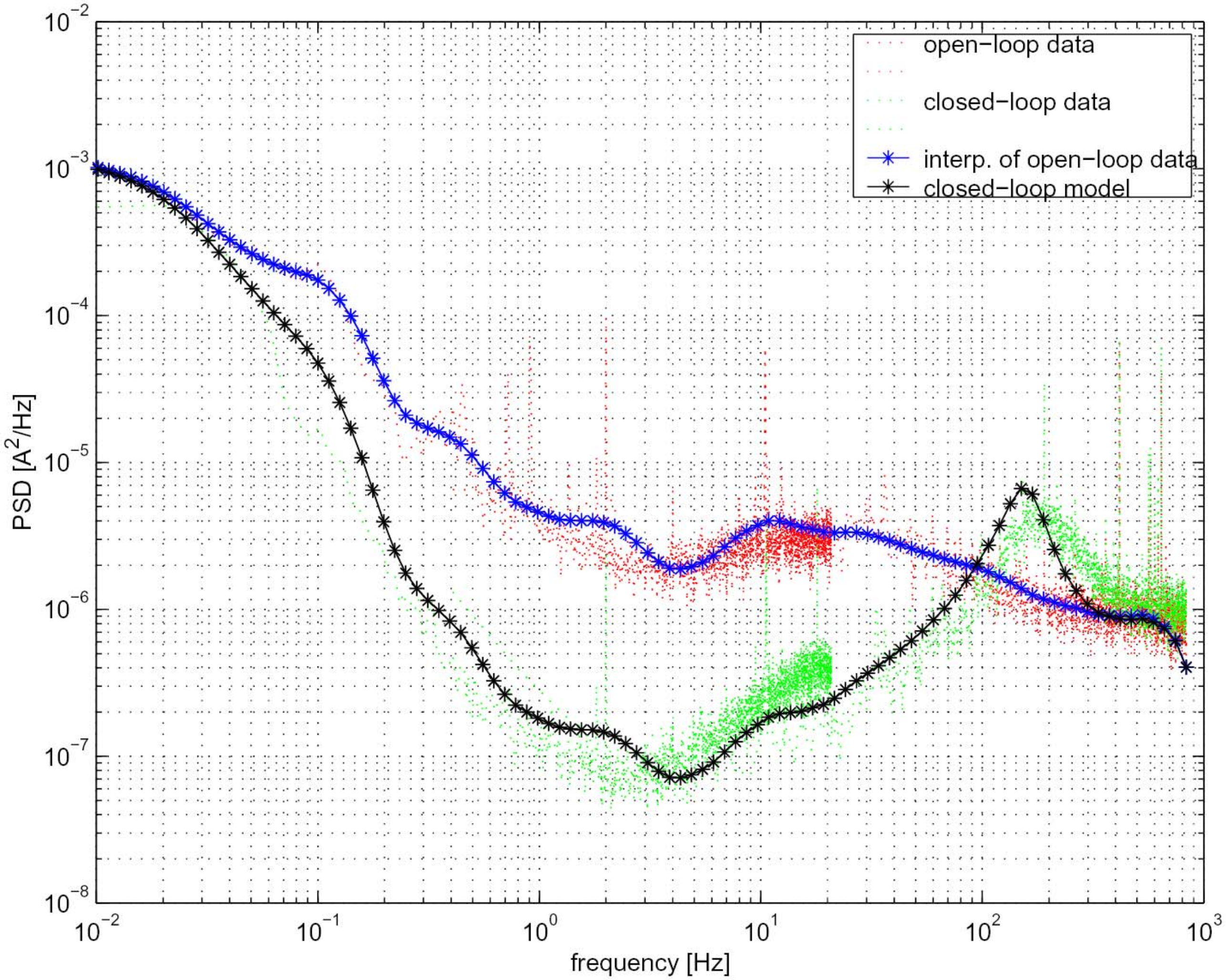
- The orbit motion spectrum contains both deterministic and stochastic (random) time-domain components.
  - We can correct deterministic components within the correction bandwidth.
  - we cannot change the ***rms*** of the stochastic component, but we can shape its frequency spectrum.

- Assuming stationary Gaussian statistics, the orbit motion spectrum can be modeled as a spectrally-shaped white noise source

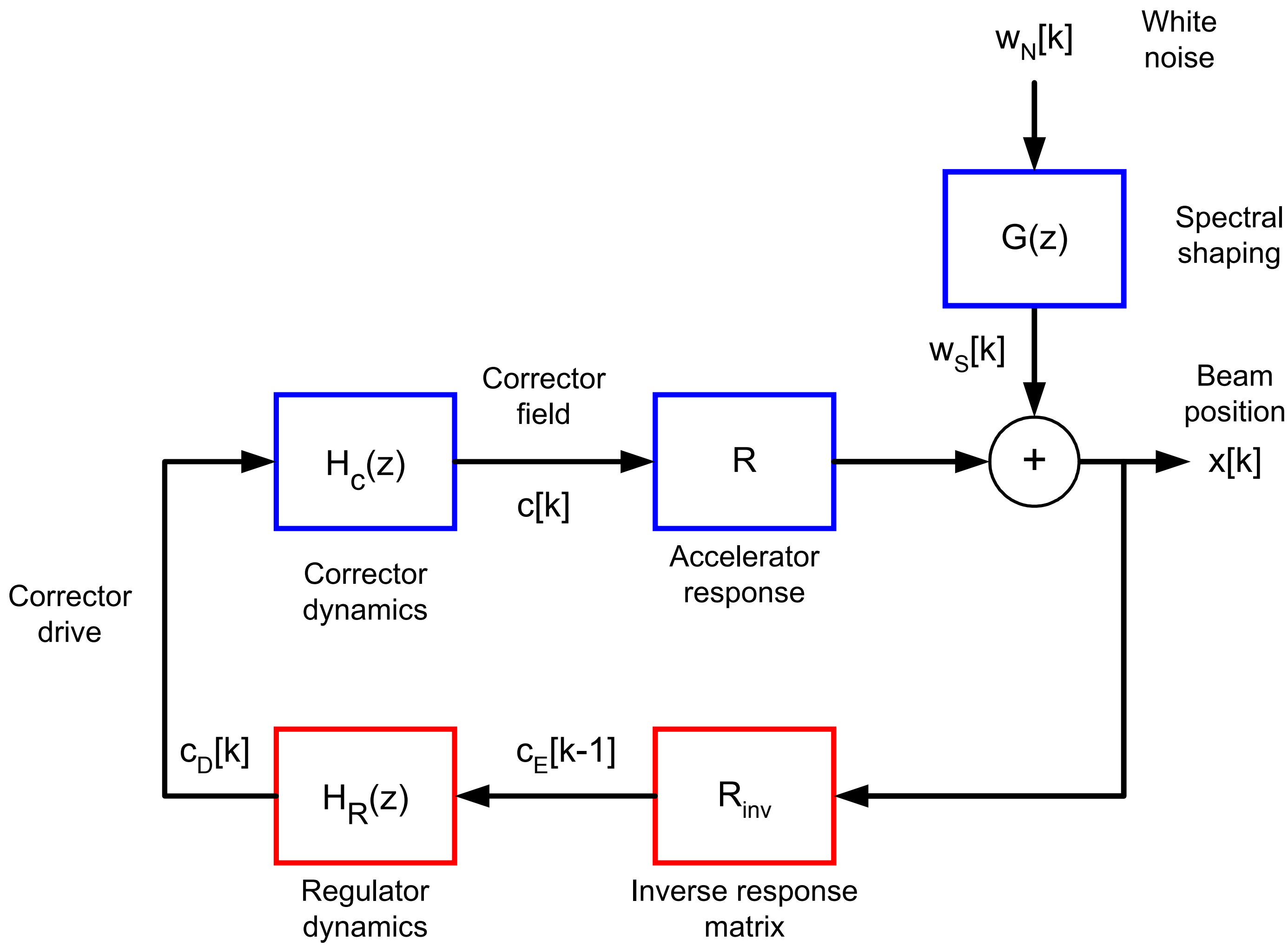




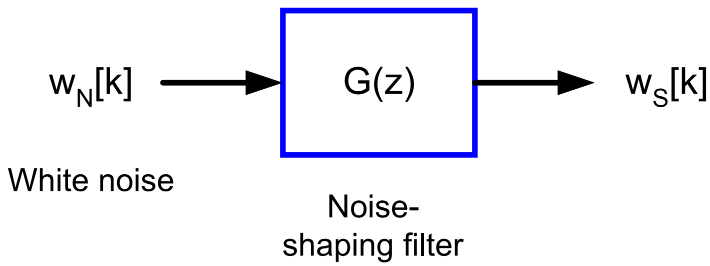
Closed-loop verification of model for S4A:V3 corrector

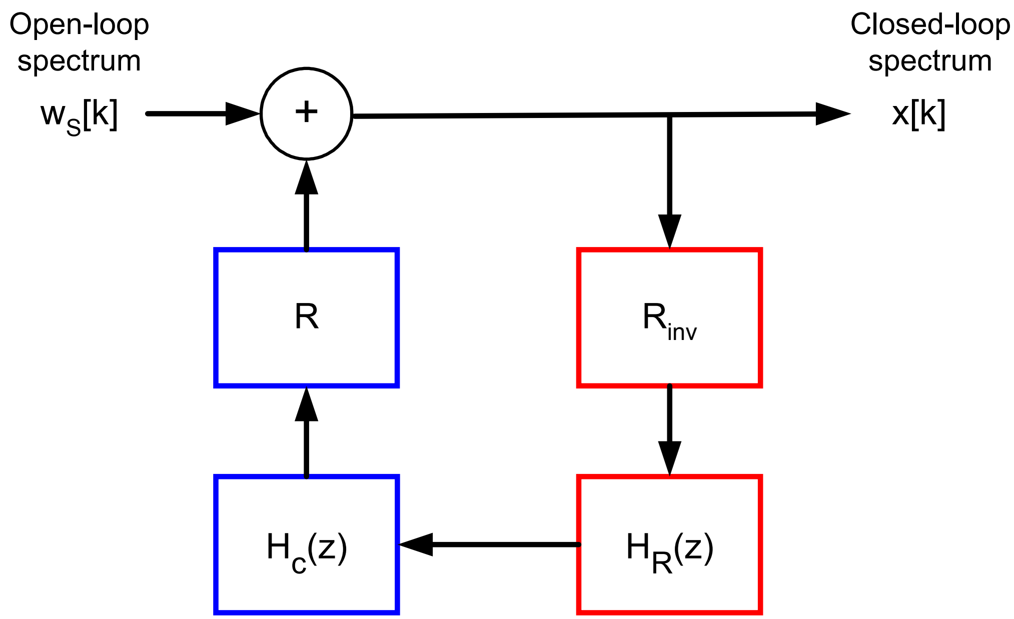


Orbit feedback system with noise model



Orbit feedback system re-drawn as a noise-shaping filter





USING SEPARATE DC AND AC SYSTEMS

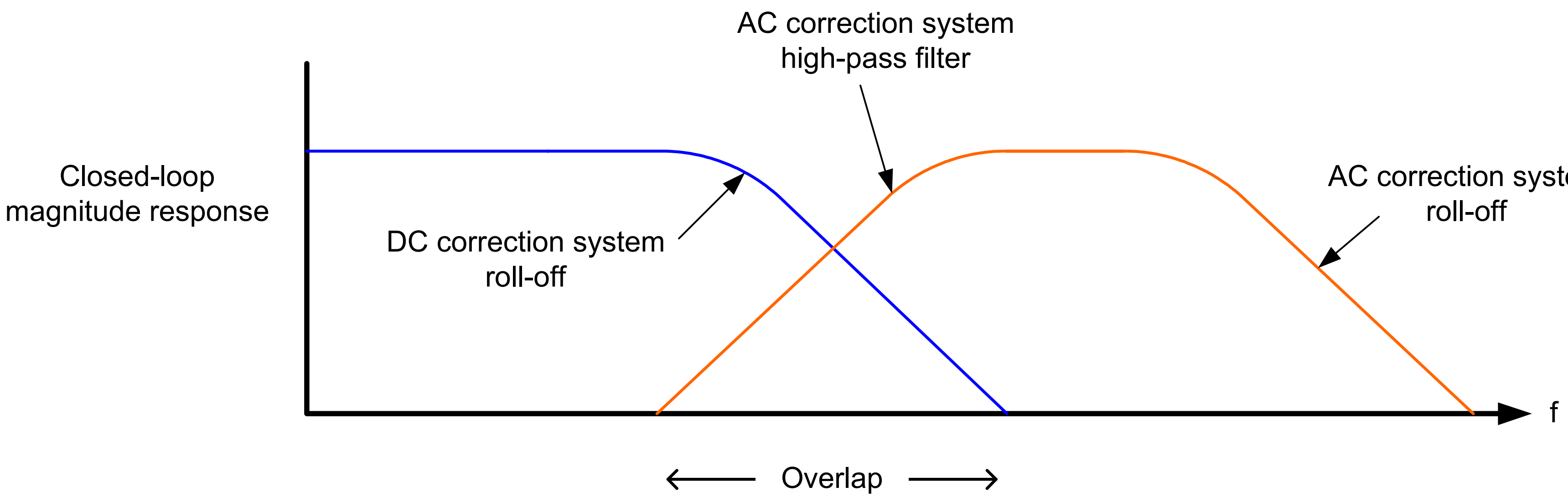
Using separate DC and AC systems

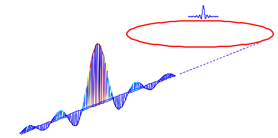
- Slow correction system (up to 25Hz update rate)
  - Many bpms and correctors to get good DC correction.
  - Include polynomial correction for bpm pin-cushions
  - Include bpm outlier removal

- Fast correction system (1500Hz)
  - Fewer bpms and correctors to reduce computational load.
  - Wider bandwidth due to faster sampling rate.

DC and AC correction system frequency responses

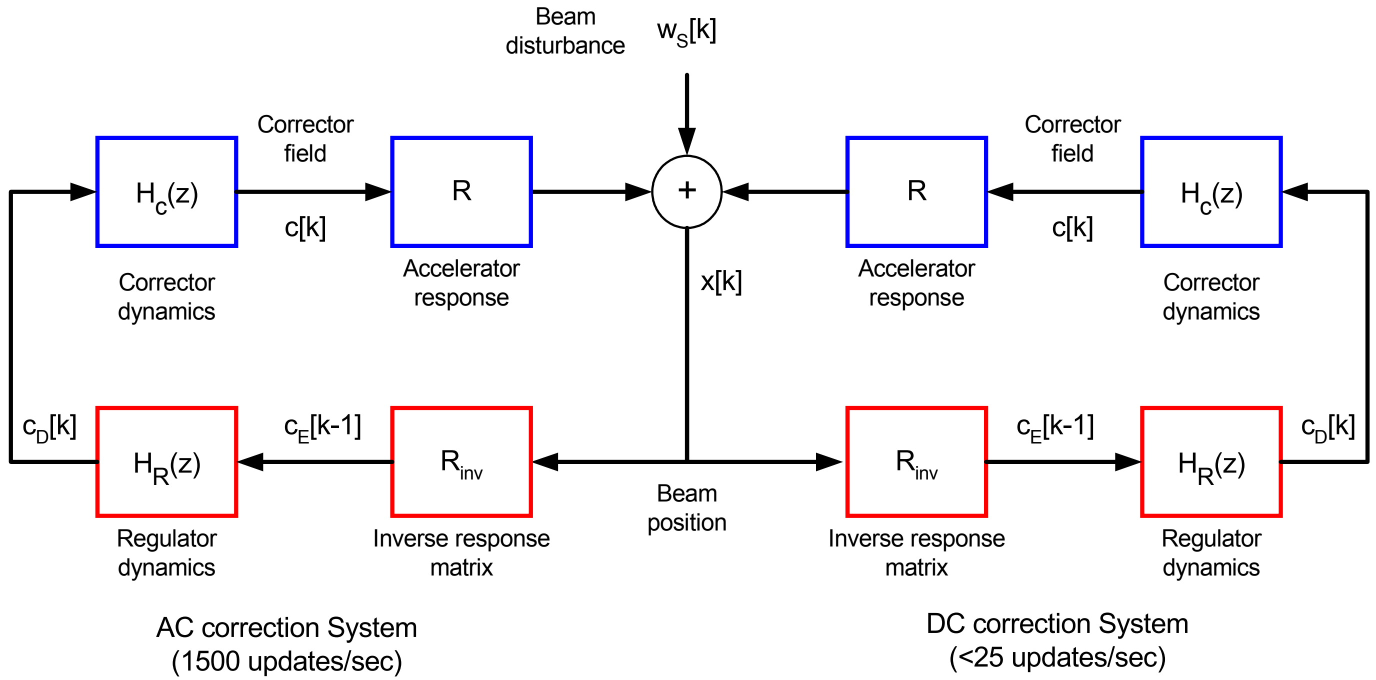
- The two systems will fight if there is too much frequency overlap.





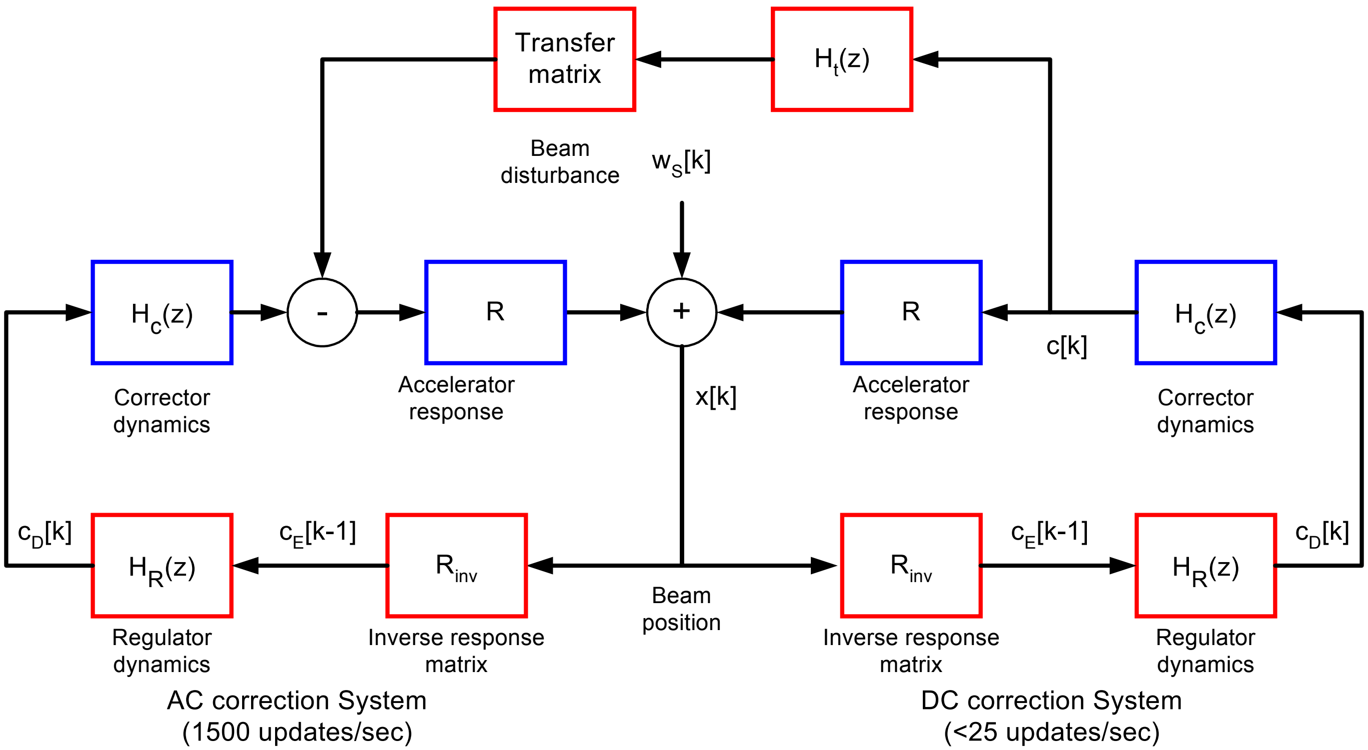
Block diagram of combined DC and AC systems (no overlap compensation)

- Both systems attempt to correct the same orbit motion within the frequency band where they are both effective.



Block diagram of combined DC and AC systems (with overlap compensation)

- Overlap compensation prevents fighting by introducing a correction factor into one system derived from the action of the other system. Since the two systems use different correctors & bpms and have different dynamics, mappings are required between the two.
- A scheme similar to this is in use at the APS.

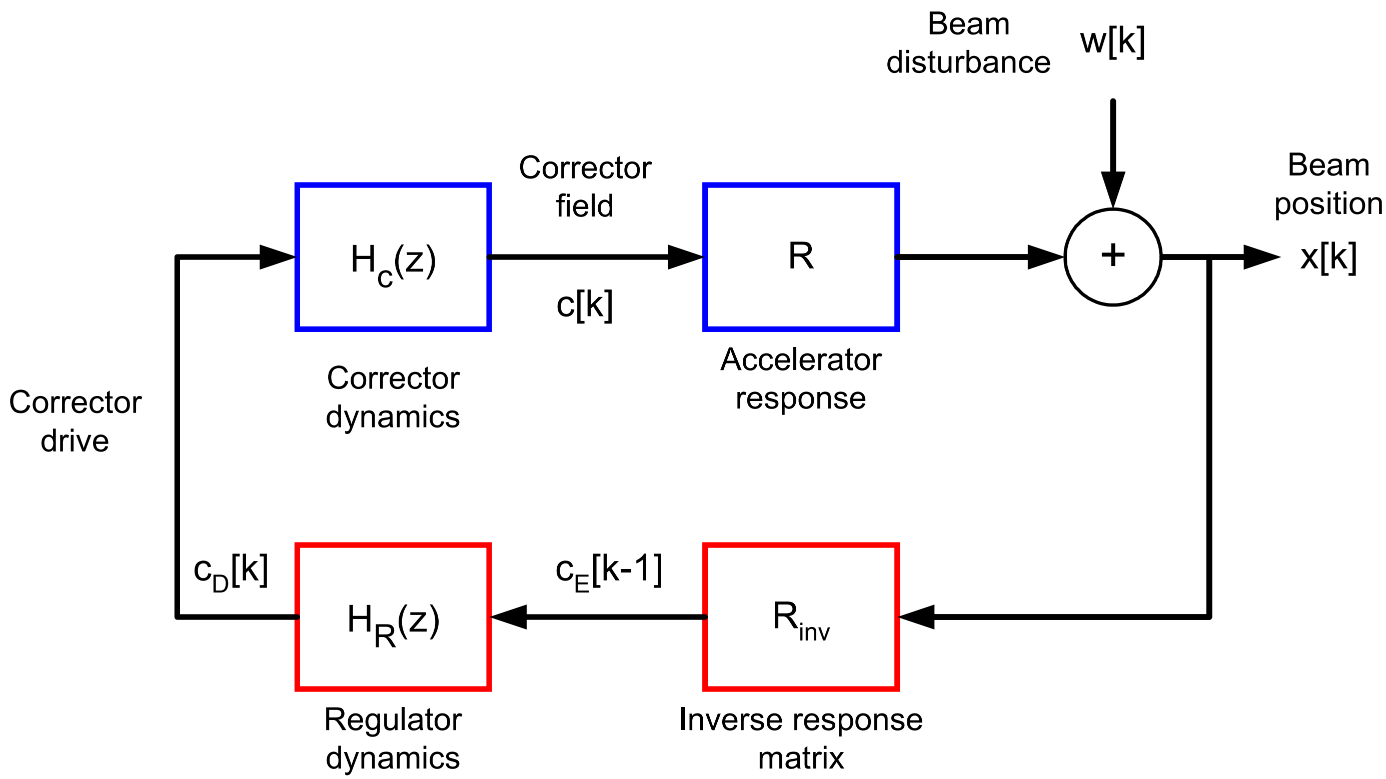


IMPACT OF DIFFERENT CORRECTOR DYNAMICS ON ORBIT CORRECTION PERFORMANCE

Impact of different corrector dynamics on control loop

- To this point, we have modeled the system assuming that all corrector dynamics are identical (or at least close).
- Each corrector control loop is decoupled from all others, therefore each can be independently controlled and stabilized without regard to other loops.
- To first order, the correction algorithm requires the corrector (and bpm) dynamics to be identical.
- What happens if they are not?

Orbit feedback model revisited (corrector drive to corrector error)



Corrector error in terms of corrector drive…

$$C_E(z) = R_{inv} \cdot R \cdot H_C(z) \cdot C_D(z)$$

Corrector regulator loop decoupling

- Expand to show diagonal nature of $\boldsymbol{H_C(z)}$

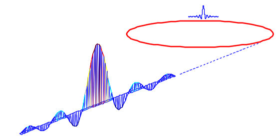
$$\begin{bmatrix} c_{e1}(z) \\ c_{e2}(z) \\ c_{e3}(z) \end{bmatrix} = R_{inv} \cdot R \cdot \begin{bmatrix} h_{c1}(z) & 0 & 0 \\ 0 & h_{c2}(z) & 0 \\ 0 & 0 & h_{c3}(z) \end{bmatrix} \cdot \begin{bmatrix} c_{d1}(z) \\ c_{d2}(z) \\ c_{d3}(z) \end{bmatrix}$$

But to first order, $R_{inv} \cdot R = I$

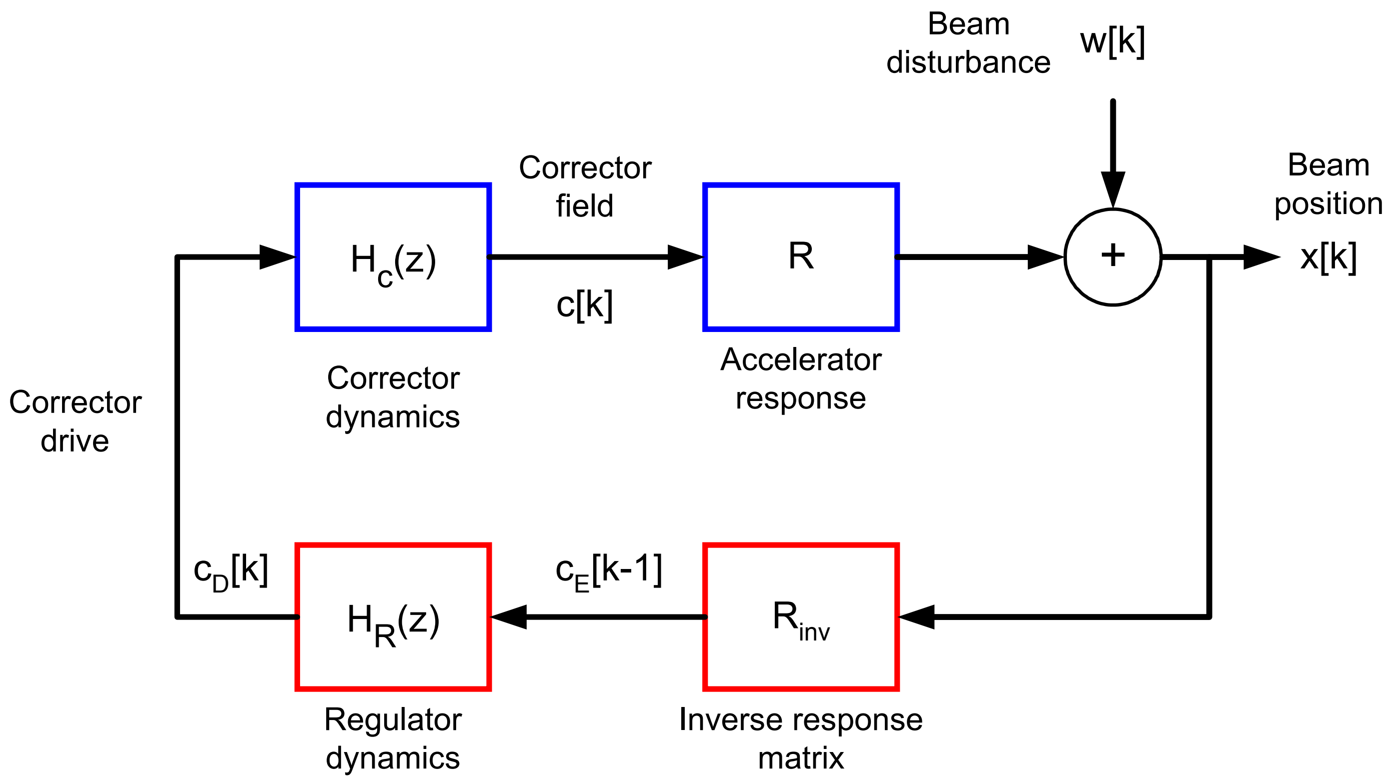
So…

$$\begin{bmatrix} c_{e1}(z) \\ c_{e2}(z) \\ c_{e3}(z) \end{bmatrix} = \begin{bmatrix} h_{c1}(z) & 0 & 0 \\ 0 & h_{c2}(z) & 0 \\ 0 & 0 & h_{c3}(z) \end{bmatrix} \cdot \begin{bmatrix} c_{d1}(z) \\ c_{d2}(z) \\ c_{d3}(z) \end{bmatrix}$$

- Therefore the loops are decoupled, irrespective of individual corrector dynamics



Orbit feedback model revisited
(orbit correction to orbit error)



Orbit correction in terms of orbit error…

$$z \cdot X(z) = R \cdot H(z) \cdot R_{inv} \cdot X(z)$$

Orbit correction to orbit error

- Expand to show diagonal nature of $\boldsymbol{H_C(z)}$

$$\begin{bmatrix} x_1(z) \\ x_2(z) \\ x_3(z) \end{bmatrix} = Z^{-1} \cdot R \cdot \begin{bmatrix} h_{c1}(z) & 0 & 0 \\ 0 & h_{c2}(z) & 0 \\ 0 & 0 & h_{c3}(z) \end{bmatrix} \cdot R_{inv} \cdot \begin{bmatrix} x_1(z) \\ x_2(z) \\ x_3(z) \end{bmatrix} + W(z)$$

Simplest case, no dynamics, and unity gain per corrector channel…

$$\begin{bmatrix} x_1(z) \\ x_2(z) \\ x_3(z) \end{bmatrix} = Z^{-1} \cdot R \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot R_{inv} \cdot \begin{bmatrix} x_1(z) \\ x_2(z) \\ x_3(z) \end{bmatrix} + W(z)$$

so $$R \cdot H(z) \cdot R_{inv} = I$$

Orbit correction to orbit error with different regulator gains

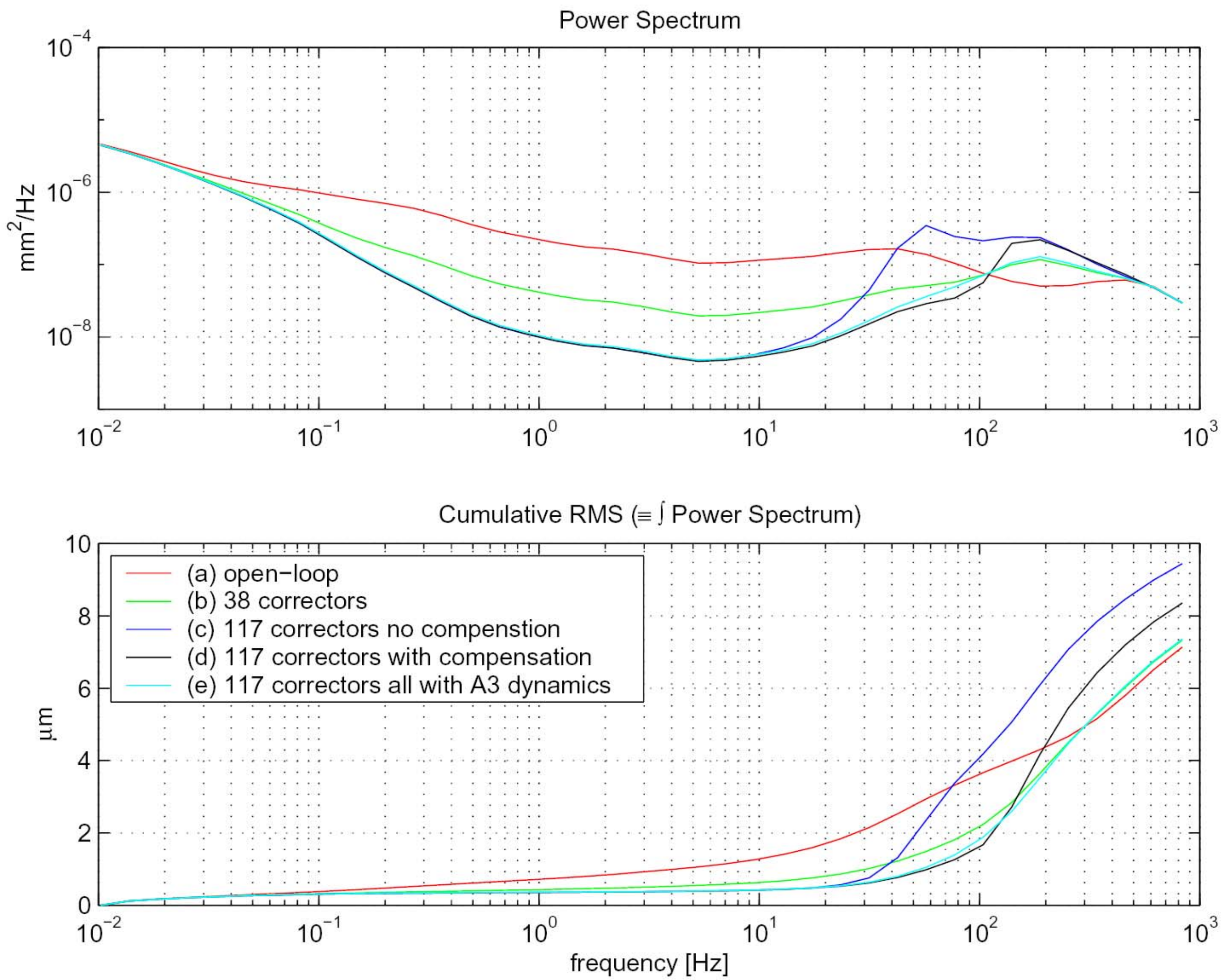
- Different regulator gains…

$$\begin{bmatrix} x_1(z) \\ x_2(z) \\ x_3(z) \end{bmatrix} = Z^{-1} \cdot R \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot R_{inv} \cdot \begin{bmatrix} x_1(z) \\ x_2(z) \\ x_3(z) \end{bmatrix} + W(z)$$

So, in this case $$R \cdot H(z) \cdot R_{inv} \neq I$$

- This means that the correction algorithm is rendered partially ineffective when each corrector channel has different dynamics.
- Seen another way, the orbit error is decomposed into ***M*** error vectors (one for each of ***M*** correctors). Only if all error vectors are treated identically does the correction algorithm hold.

Modeled vertical beam motion for various configurations



An orbit feedback configuration dilemma

- The APS accelerator contains 317 correctors per plane
  - 38 correctors have ‘fast’ response (100’s Hz)
  - 279 correctors have varying degrees of ‘slow’ response (10’s Hz or worse).

- **Option #1**
  - Use only the 38 fast correctors.
  - Algorithm will hold throughout the closed loop bandwidth.

- **Option #2**
  - Use 38 fast correctors + additional slow correctors (eg 117 total)
  - Algorithm will break down at above the cut-off frequency of the slow correctors.
  - DC orbit correction is improved over 38-corrector case.

Which is better?