Diffraction Limited X-Ray Source

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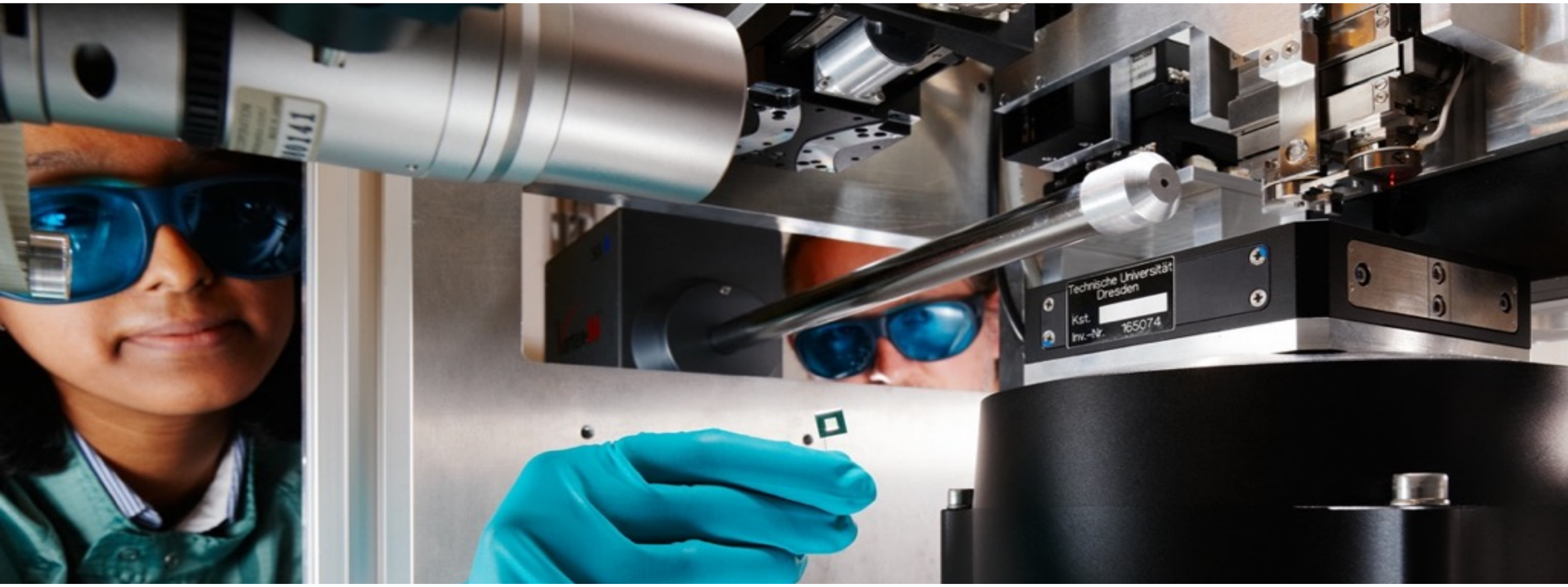

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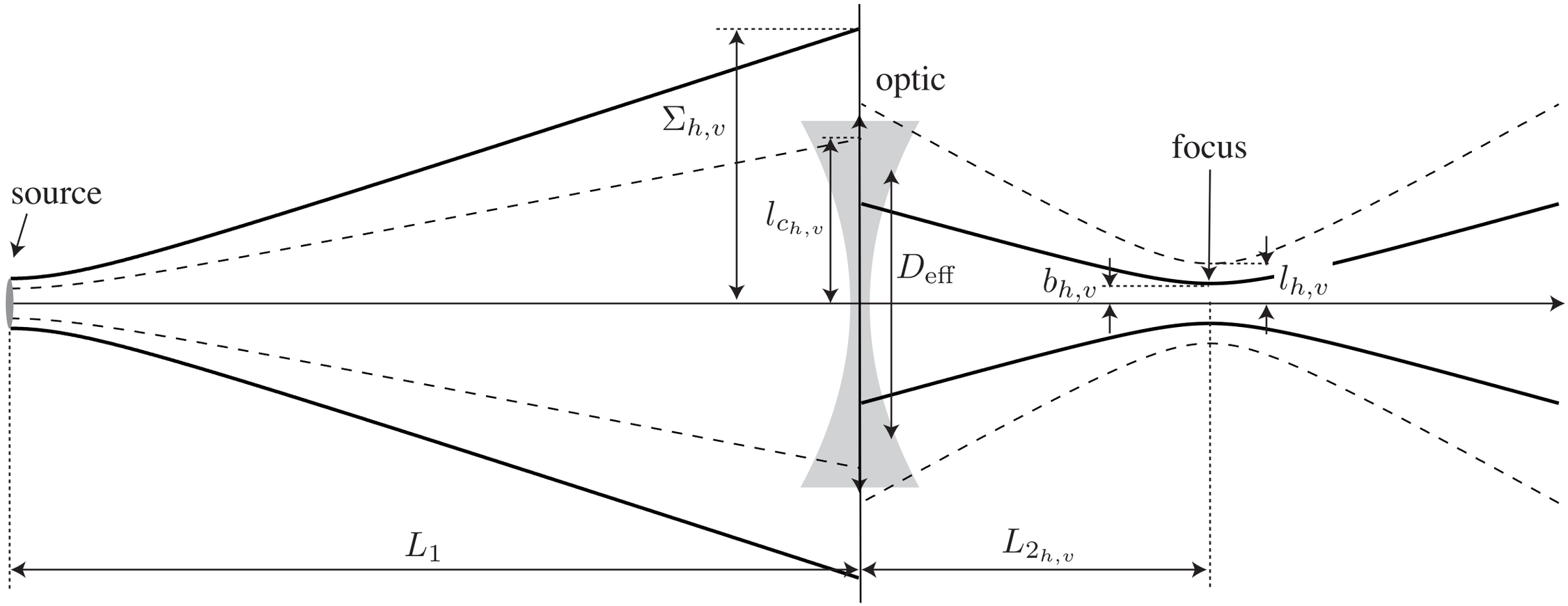
DESY

Low-Emittance Synchrotron Radiation Source

Example: simple nanofocusing geometry:



Optimal coherent fraction (equivalent to optimal brilliance):

- optimal flux in nanofocus
- optimal fluence for CXDI, ptychography, …
- optimal flux for XPCS

Schroer & Falkenberg, JSR **21**(5), 996 (2014).

Single Electron in Undulator

Single Electron in undulator:

Natural divergence

$$\sigma_\theta = \frac{1}{\gamma}\sqrt{\frac{1+\kappa^2/2}{2jN_u}}$$

implies diffraction limited source size

$$\sigma = \frac{1}{2k\sigma_\theta} \quad \text{with} \quad k = \frac{2\pi}{\lambda} \quad \text{and} \quad \sigma\cdot\sigma_\theta = \frac{\lambda}{4\pi} = \epsilon_0 = \frac{4\pi}{L_u}\sigma$$

σ and σ_θ measure the rms width of the intensities!

wavelength of *j*-th harmonic:

$$\lambda = \frac{\lambda_\mathrm{u}}{2\gamma^2 j}\left[1+\frac{\kappa^2}{2}+\gamma^2\vartheta^2\right]$$

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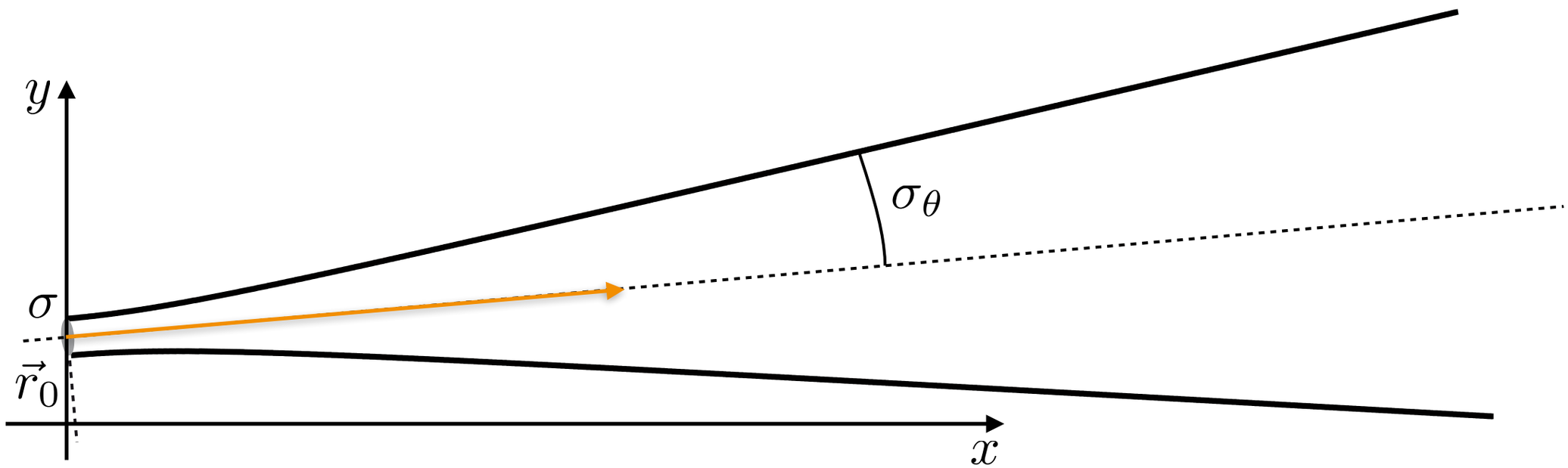
Gaussian Model of Wave Field Emitted by Single Electron

Electron at position $\vec{r}_0$ and with momentum in the direction of $(q_x/k, q_y/k, k)$

Oblique Gaussian limited plane wave at source:

$$\psi(\vec{r}) = A_0 \cdot \exp\left\{-\frac{1}{2}\frac{(\vec{r}-\vec{r}_0)^2}{2\sigma^2}\right\} \cdot \exp\left\{i\vec{q}\cdot\vec{r}\right\} \qquad (q_x/k, q_y/k)$$

Describes radiation field from single electron in the far field.

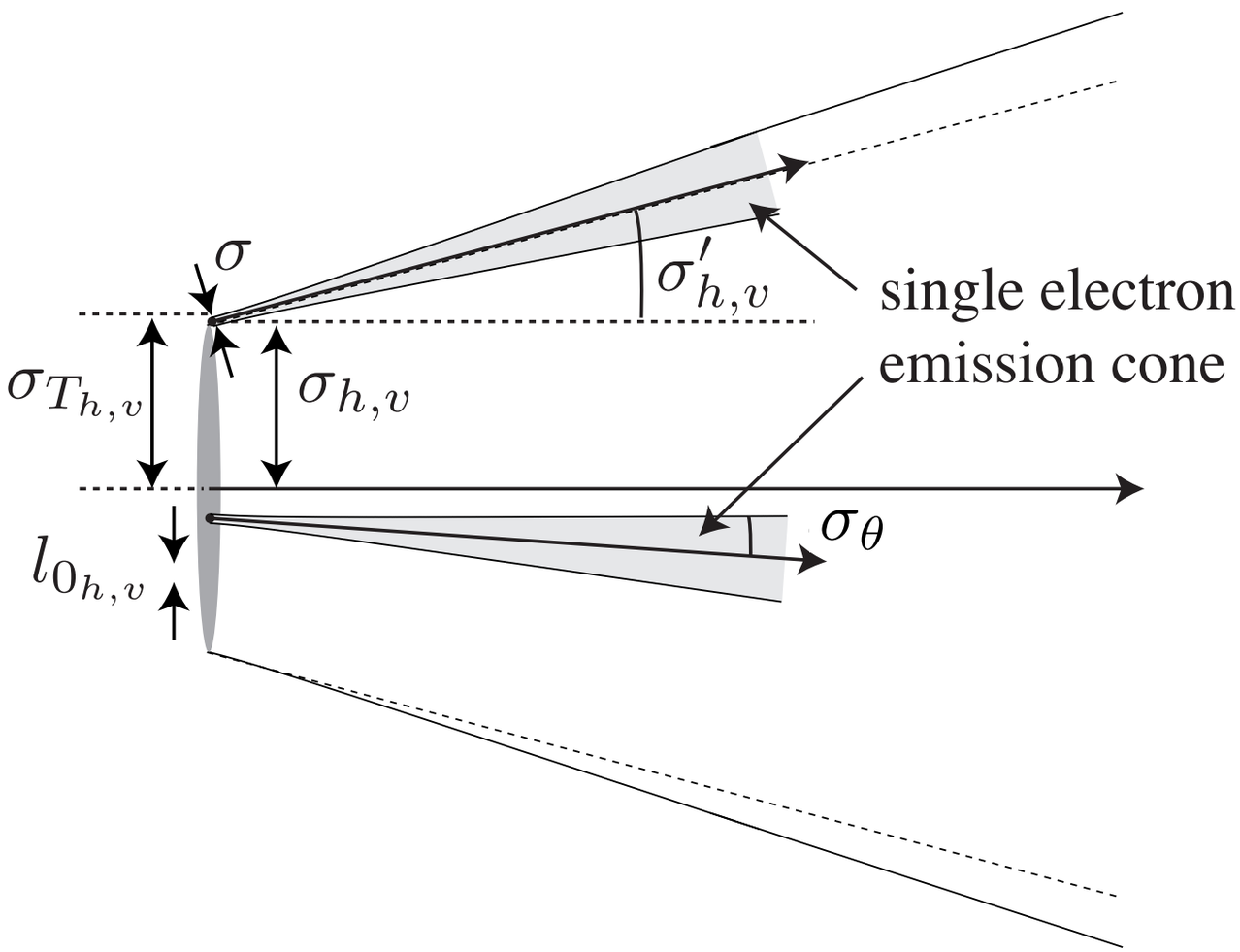


Schroer & Falkenberg, JSR **21**(5), 996 (2014).

Electron Bunch: Gaussian Ensemble in Phase Space

Phase-space density:

$$\rho(\vec{r},\vec{q}) = \rho_0 \cdot \exp\left\{-\frac{1}{2}\frac{x^2}{\sigma_h^2}\right\} \exp\left\{-\frac{1}{2}\frac{q_x^2}{k^2\sigma_h'^2}\right\} \cdot \exp\left\{-\frac{1}{2}\frac{y^2}{\sigma_v^2}\right\} \exp\left\{-\frac{1}{2}\frac{q_y^2}{k^2\sigma_v'^2}\right\}$$



$\sigma_{h,v}$ width of distribution in space

$\sigma'_{h,v}$ width of distribution in angle (transverse momentum)

emittance:

$$\epsilon_{h,v} = \sigma_{h,v} \cdot \sigma'_{h,v}$$

Gaussian Schell model: Singer & Vartanyants, JSR **21**(1), 5 (2014).

Schroer & Falkenberg, JSR **21**(5), 996 (2014).

Describe Statistical Ensemble by Mutual Intensity

Time-averaged field-field correlation function: mutual intensity

$$J_0(\vec{r}_1, \vec{r}_2) = \int \rho(\vec{r}_0, \vec{q}) \psi(\vec{r}_2) \psi^*(\vec{r}_2) \, dr_0 dq$$

Gaussian symmetry of model: separation in two transverse directions:

$$J_0(\vec{r}_1, \vec{r}_2) = J_{0_h}(x_1, x_2) \cdot J_{0_v}(y_1, y_2)$$

For the previously discussed model:

$$J_{0_h}(x_1, x_2) = \sqrt{I_0} \exp\left\{ -\frac{1}{2} \frac{x_1^2 + x_2^2}{2(\sigma^2 + \sigma_h^2)} \right\} \cdot \exp\left\{ -\frac{1}{2} \frac{1}{l_{0_h}^2} (x_1 - x_2)^2 \right\}$$

and similar for vertical direction.

Schroer & Falkenberg, JSR **21**(5), 996 (2014).

Interpretation of Mutual Intensity

$$J_{0_h}(x_1, x_2) = \sqrt{I_0} \exp\left\{-\frac{1}{2}\frac{x_1^2 + x_2^2}{2(\sigma^2 + \sigma_h^2)}\right\} \cdot \exp\left\{-\frac{1}{2}\frac{1}{l_{0_h}^2}(x_1 - x_2)^2\right\}$$

For $x_1 = x_2$ the mutual intensity corresponds to the local intensity at $x_1 = x_2$

→ source size $\sigma_{T_{h,v}} = \sqrt{\sigma^2 + \sigma_{h,v}^2}$ rms intensities!!!

This implies the definition of the diffraction limit as:

$$\epsilon_{h,v} = \sigma_{h,v} \cdot \sigma'_{h,v} \leq \sigma \cdot \sigma_\theta = \frac{\lambda}{4\pi}$$

(as opposed to 2π in the denominator if σ, σ_θ are interpreted as rms amplitudes.)

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Coherent Fraction

For given bandwidth: Coherent Flux

$$F_c = F_0 \cdot \frac{l_{c_h}}{2\Sigma_h} \cdot \frac{l_{c_v}}{2\Sigma_v} \cdot = F_0 \cdot \frac{\lambda^2}{(4\pi)^2 \sigma_{T_h} \sigma'_{T_h} \sigma_{T_v} \sigma'_{T_v}} = Br \cdot \left(\frac{\lambda}{2}\right)^2$$

1D coherent fraction:

$$\frac{\lambda}{(4\pi)\sigma_{T_{h,v}} \sigma'_{T_{h,v}}} = \frac{1}{\sqrt{1 + \frac{\epsilon_{h,v}}{\epsilon_0} \frac{4\pi\beta_{h,v}}{L_u}} \sqrt{1 + \frac{\epsilon_{h,v}}{\epsilon_0} \frac{L_u}{4\pi\beta_{h,v}}}}$$

$$\beta_{h,v} = \frac{\sigma^2_{h,v}}{\epsilon_{h,v}}$$

Natural units (dimensionless representation):

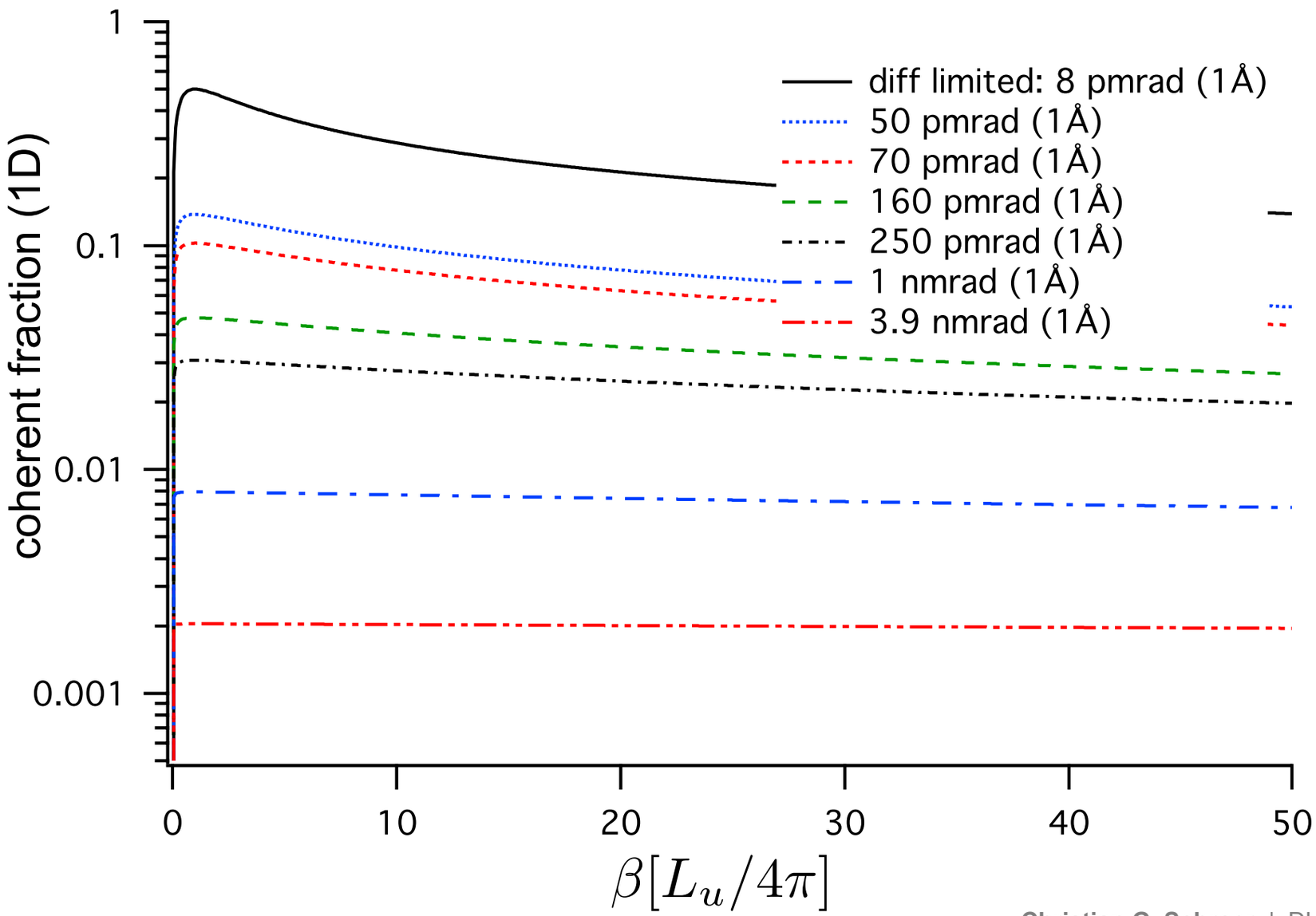
- emittance measured in natural single-electron emittance
- β function measured in units of (undulator length)/4π

R. Coïsson, Opt. Eng. **27**, 273250 (1988),
K. J. Kim, Nucl. Instrum. Meth. **246**, 71 (1986).

Coherent Fraction

1D coherent fraction:

$$\frac{\lambda}{(4\pi)\sigma_{T_{h,v}}\sigma'_{T_{h,v}}} = \frac{1}{\sqrt{1+\frac{\epsilon_{h,v}}{\epsilon_0}\frac{4\pi\beta_{h,v}}{L_u}}\sqrt{1+\frac{\epsilon_{h,v}}{\epsilon_0}\frac{L_u}{4\pi\beta_{h,v}}}}$$



maximum at:

$$\beta_{\max} = \frac{\sigma_{h,v}^2}{\epsilon_{h,v}} = \frac{L_u}{4\pi}$$

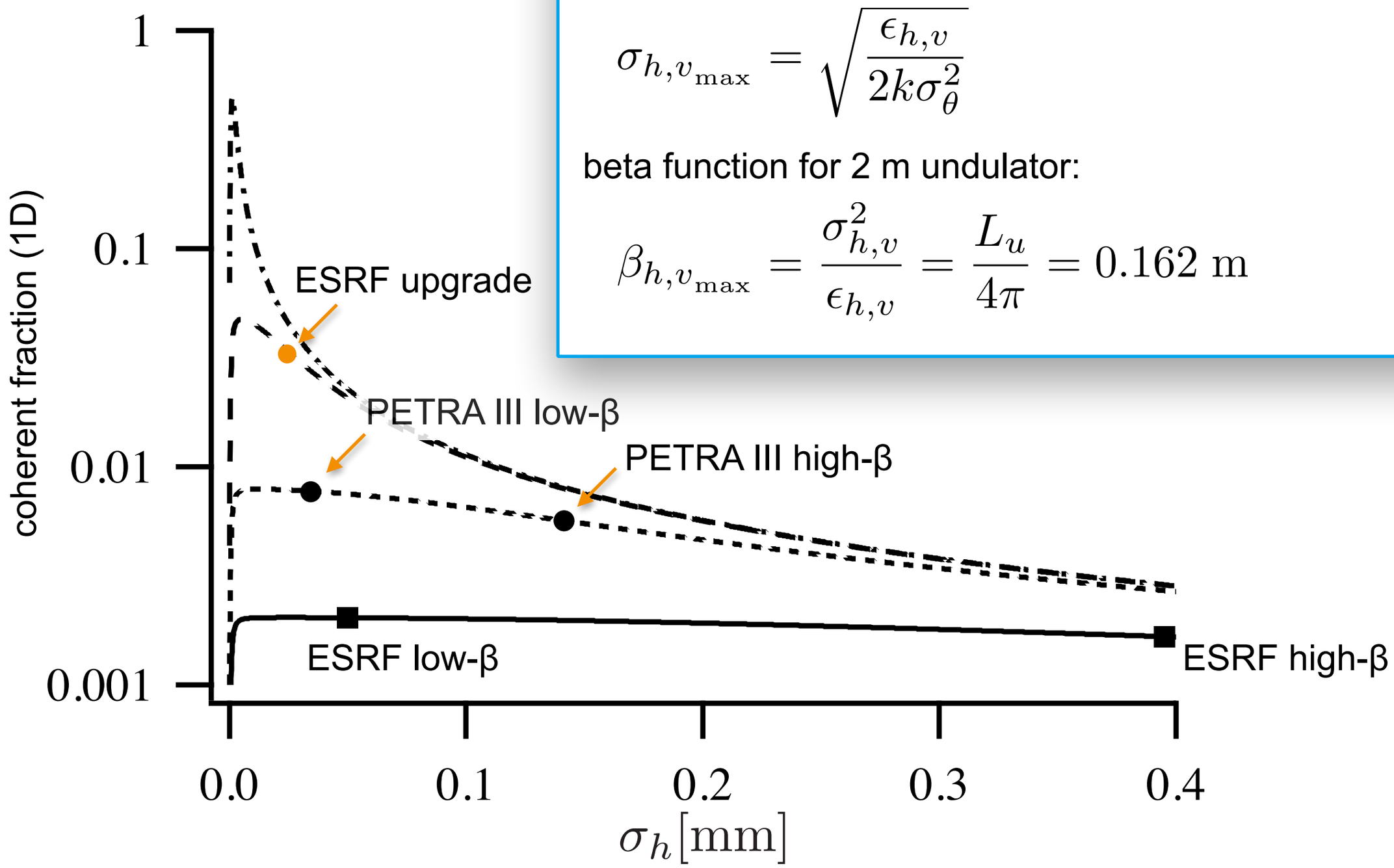
influence of

- source size
- divergence

are similar!

Coherent Fraction

Horizontal component:



maximum at (for given emittance):

$$\sigma_{h,v_{\max}} = \sqrt{\frac{\epsilon_{h,v}}{2k\sigma_\theta^2}}$$

beta function for 2 m undulator:

$$\beta_{h,v_{\max}} = \frac{\sigma_{h,v}^2}{\epsilon_{h,v}} = \frac{L_u}{4\pi} = 0.162 \text{ m}$$

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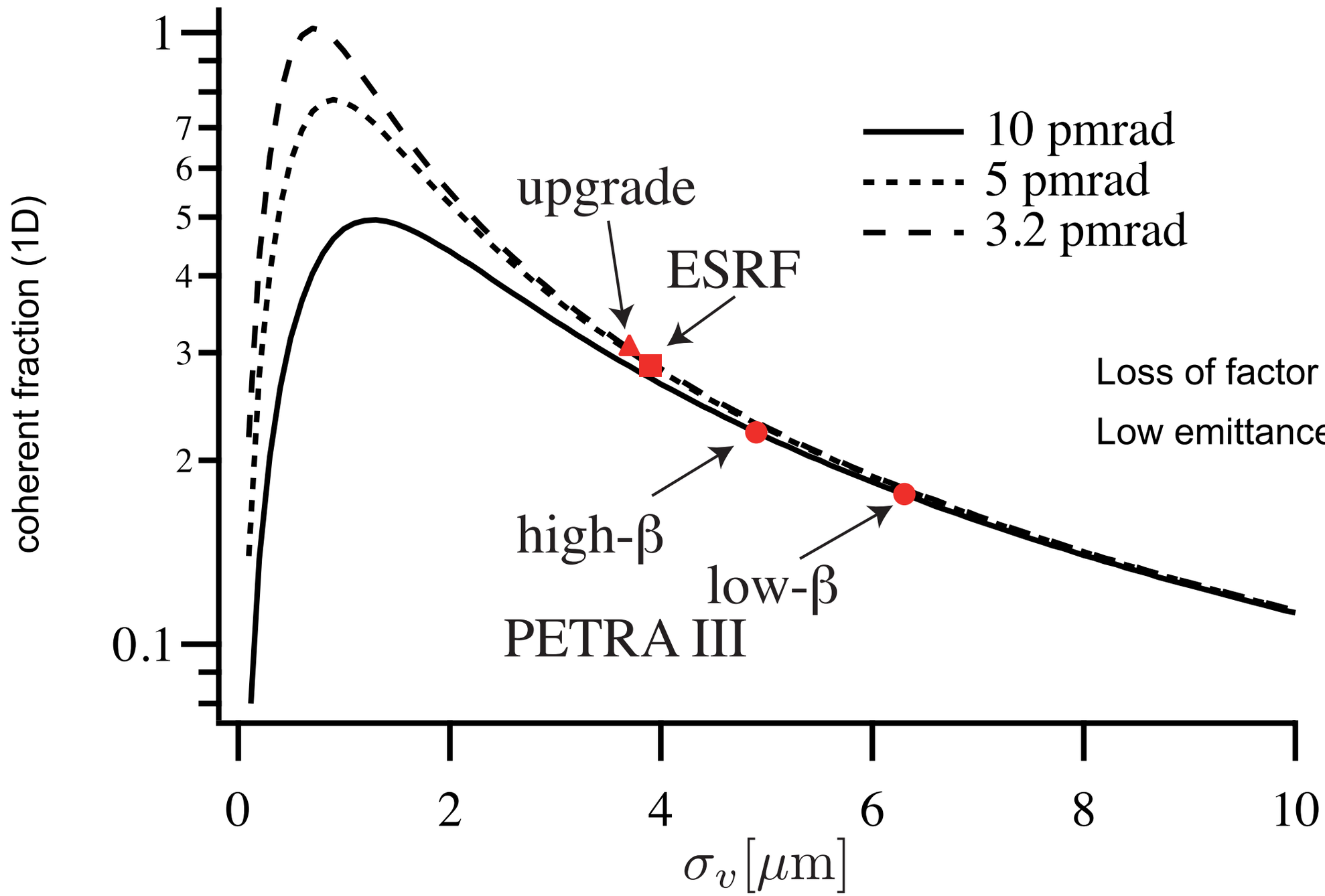
Coherent Fraction

Vertical component:



Loss of factor 2-3 in vertical

Low emittance does not help!

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Benefits of Low-Emittance Storage-Ring Source

- static
- homogeneous
- large samples
- large ensembles
- order
- …

- brightness
- numerical aperture of x-ray optics
- fast detectors
- computing power

- dynamic
- heterogeneous
- small samples
- individual details
- disorder
- …

Important: other enabling technologies:

- optics
- mechanics
- detectors
- data handling and computing
- …

in-situ and under working conditions!

But:

- low emittance alone is not sufficient!
- very stringent requirements on β-function, undulator, …!?