20 keV Minimum Gap Undulator at 6 GeV

S. H. Kim

Argonne National Laboratory

1. Introduction

One important goal of the proposed 6 GeV Synchrotron Light Source is to provide a photon beam of high flux and brilliance at the photon energy of about 20 keV (0.62A) in the fundamental harmonic. In this note, restrictions of the undulator parameters are discussed on the basis of the adoption of a rare-earth cobolt (REC) or REC-steel hybrid magnet design.

2. Parameters

For an undulator of REC, usually $SmCo_5$, permanent magnet material, the peak field B_o on the axis is given by[(1)]:

$$B_o = B_r \frac{2 \sin \epsilon\pi/M}{\pi/M} [1 - e^{-2\pi h/\lambda_u}] e^{-g/\lambda_u}, \quad (1)$$

where B_r = remanent field of the material,

ϵ = filling factor,

M = number of blocks per period,

h = height of the blocks,

λ_u = length of the undulator period, and

g = undulator gap.

Typically $M = 4$, $h/\lambda_u = 1/4$ and $Br = 0.9 \sim 0.95T$ are used for the design of undulators. Assuming $\epsilon = 1$, one may obtain

$$B_o = (1.3 \sim 1.6) e^{-\pi g/\lambda_u}. \quad (2)$$

In the case of hybrid magnet design, Halbach obtained a semi-empirical relation[2]

$$B_o = 3.33\, e^{-\frac{g}{\lambda_u}\left(5.47 - 1.8\, g/\lambda_u\right)}, \tag{3}$$

which is valid for $0.07 < g/\lambda_u < 0.7$. Equation (3) is an optimized relation assuming a REC material with $B_r = 0.9T$ and a high grade iron-cobolt-ronadium pole plate material with a saturation magnetization of 2.4T.

Equations (2) and (3) are plotted in Fig. 1. In the practical range of $g/\lambda_u \gtrsim 0.5$, one obtins almost the same values of B_o from the two expressions. For a fixed magnet gap g, the undulator deflection parameter K, defined as

$$K = 0.934\, B_o\, (T)\, \lambda_u(cm), \tag{4}$$

is a function of only λ_u. Alternatively, for a fixed λ_u, K is a function of only g. Figures 2 and 3 show the variations of K for the both cases.

The fundamental wavelength of the radiated photons from the undulator in the forward direction is given by

$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right), \tag{5}$$

where $\gamma = 1.957 \times 10^3$ E, and E is the electron energy in GeV. Here the wavelength is a function of only λ_u and g for a given electron energy. For the case of E = 6 GeV, the variation of the fundamental wavelength is shown in Fig, 4 as a function of the undulator gap.

To achieve the photon energy of 20 keV (0.62A) at the fundamental harmonic, it is seen from Fig. 4 that the gap and the undulator period could be chosen in the range of 0.8 ~ 1.2 cm and 1.55 ~ 1.75 cm, respectively. It is seen from Fig. 3, however, that the deflection parameter K is reduced significantly by increasing the gap for the range of the undulator period. In Table 1, at the election energy of 6 GeV numerical values of λ_u, g, B_o, K, λ_1, and the corresponding photon energy E_1 are listed.

The photon flux, $f_\nu(\omega)$, in the forward direction of the electron beam current I is

$$f_\nu(\omega) = (4.56 \times 10^{16})\ \pi N I \frac{\Delta\omega}{\omega}\ Q_\nu(K)\ \left(\frac{\sin N\pi\nu}{2N \cos \frac{\pi\nu}{2}}\right)^2, \qquad (6)$$

where $\nu = \omega/\omega_1$ and ω_1 is the fundamental harmonic. The photon flux or the brilliance depends on the function

$$Q_\nu(K) = 4q\ [J_{\frac{\nu+1}{2}}(q) - J_{\frac{\nu-1}{2}}(q)]^2,$$

$$\left(q = \frac{\nu K^2/4}{1+K^2/2}\right) \qquad (7)$$

which is plotted in Fig. 5. For small values of K, $Q_\nu(K)$ is monotonically increasing function of K. Especially for $K < 1$, the fundamental harmonic dominates over all the higher harmonics.

Figure 6 is the spectrum of the photon brilliance with the minimum undulator gap of 0.8 cm. The undulator period is chosen 1.55 cm to have the photon energy of 20 keV in the fundamental harmonic. The variation of the brilliance of the fundamental harmonic as a function of the electron beam coupling constant is shown in Fig. 7.

Figure Captions

Fig. 1. Peak fields B_o in the midplanes of pure REC and hybrid undulators (h = magnet height, λ_u = undulator period, g = undulator gap).

Fig. 2. Undulator deflection parameter K vs λ_u for various values of the undulator gap.

Fig. 3. Undulator deflection parameter vs undulator gap for various values of λ_u.

Fig. 4. Wavelength of the fundamental harmonic λ_1 vs undulator gap for various λ_u at an electron energy of 6 GeV. It is seen that the tunable range near λ_1 = 0.62A (20 keV) is limited.

Fig. 5. The function $Q_\nu(K)$ vs K for small values of K.

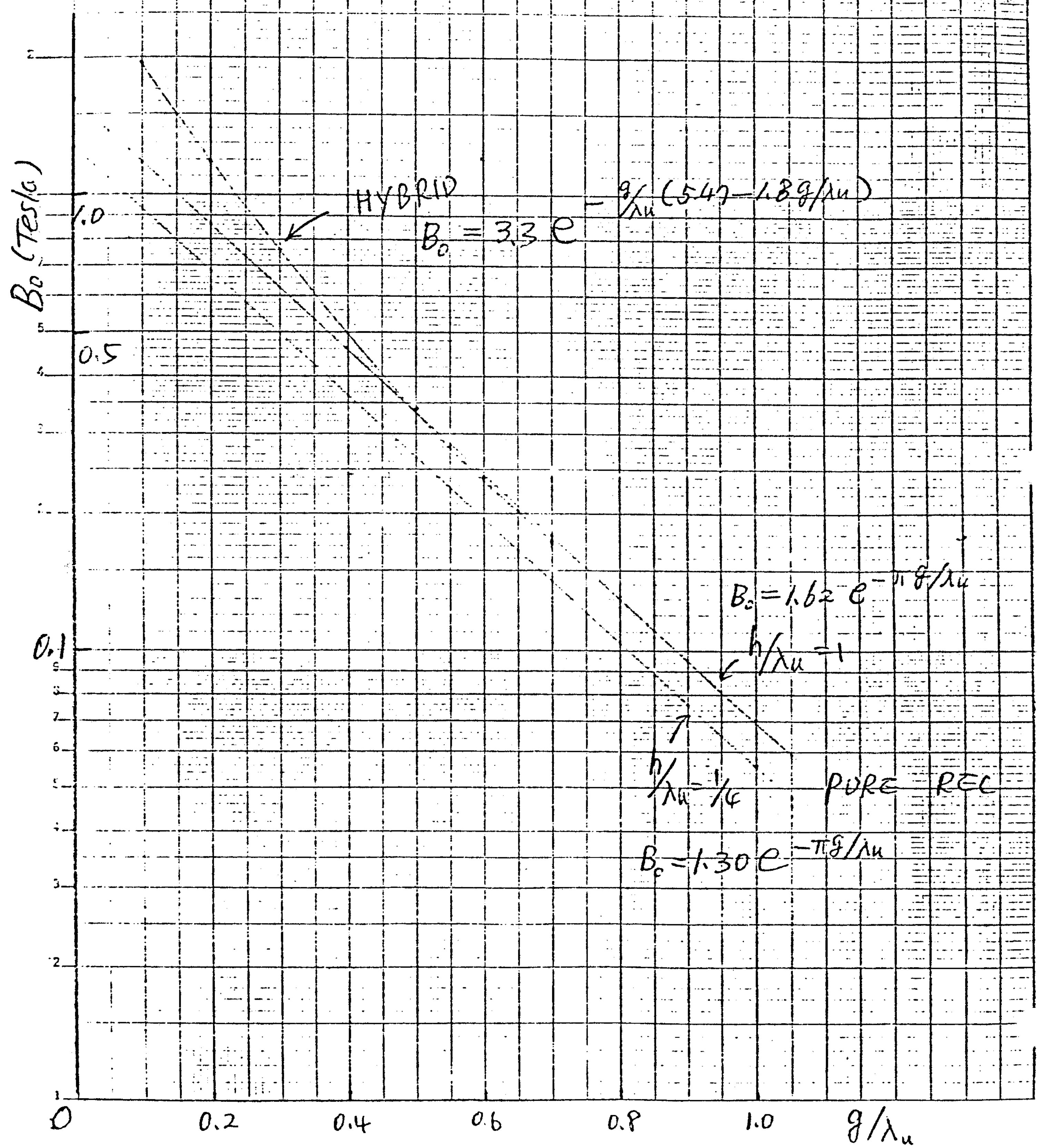
Fig. 6. Spectrum of brilliance as a function of photon energy Ep with an undulator gap of 0.8 cm. The first three peaks correspond to ν = 1.0, 1.9, and 3.0, respectively. The electron beam parameters are from the current design studies at ANL.

Fig. 7. Photon brilliance of the fundamental harmonic vs electron beam coupling constant.

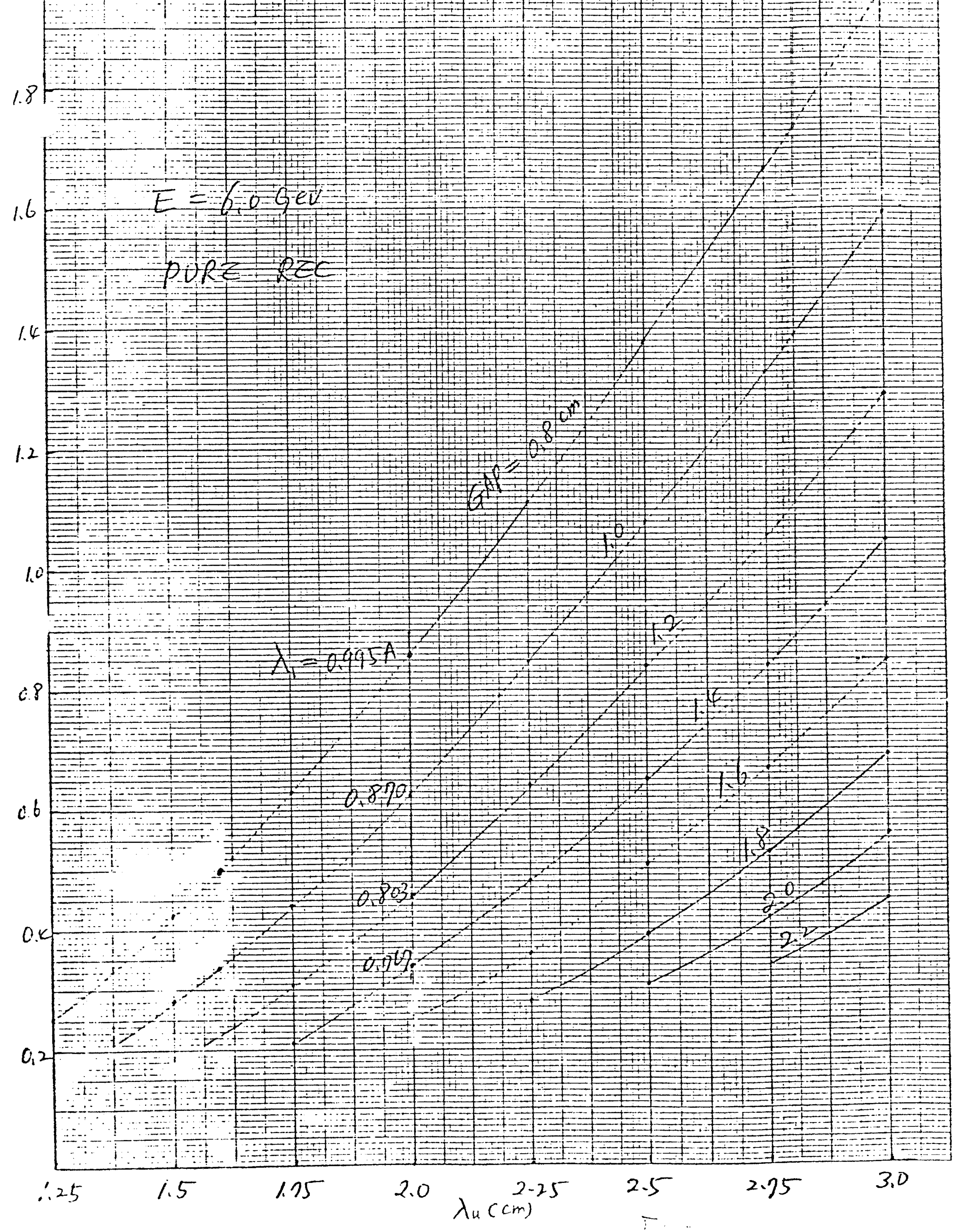
References

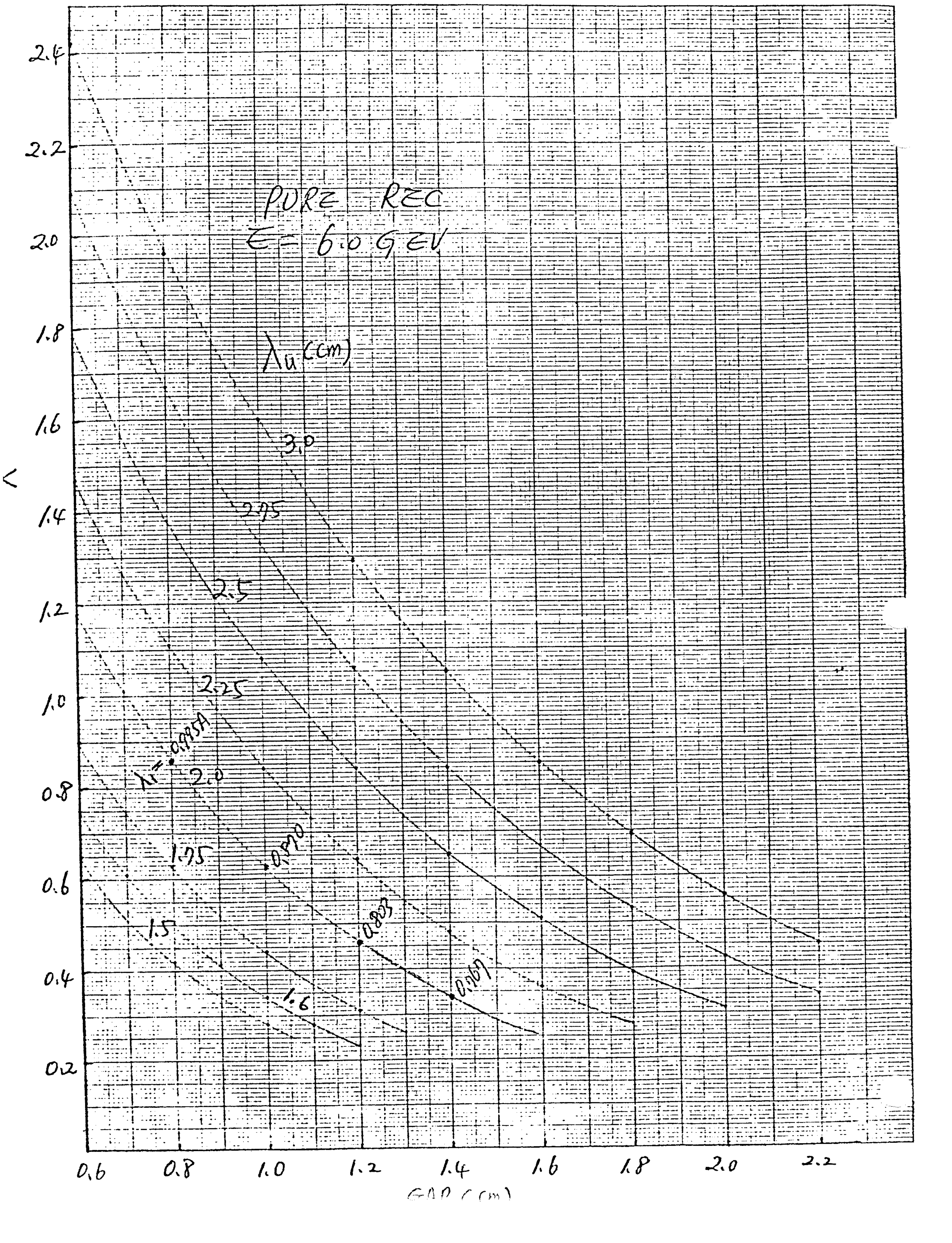
1. M.W. Poole and R.P. Walker, IEEE Trans. Magnetics Mag-17 (1981) 1978.

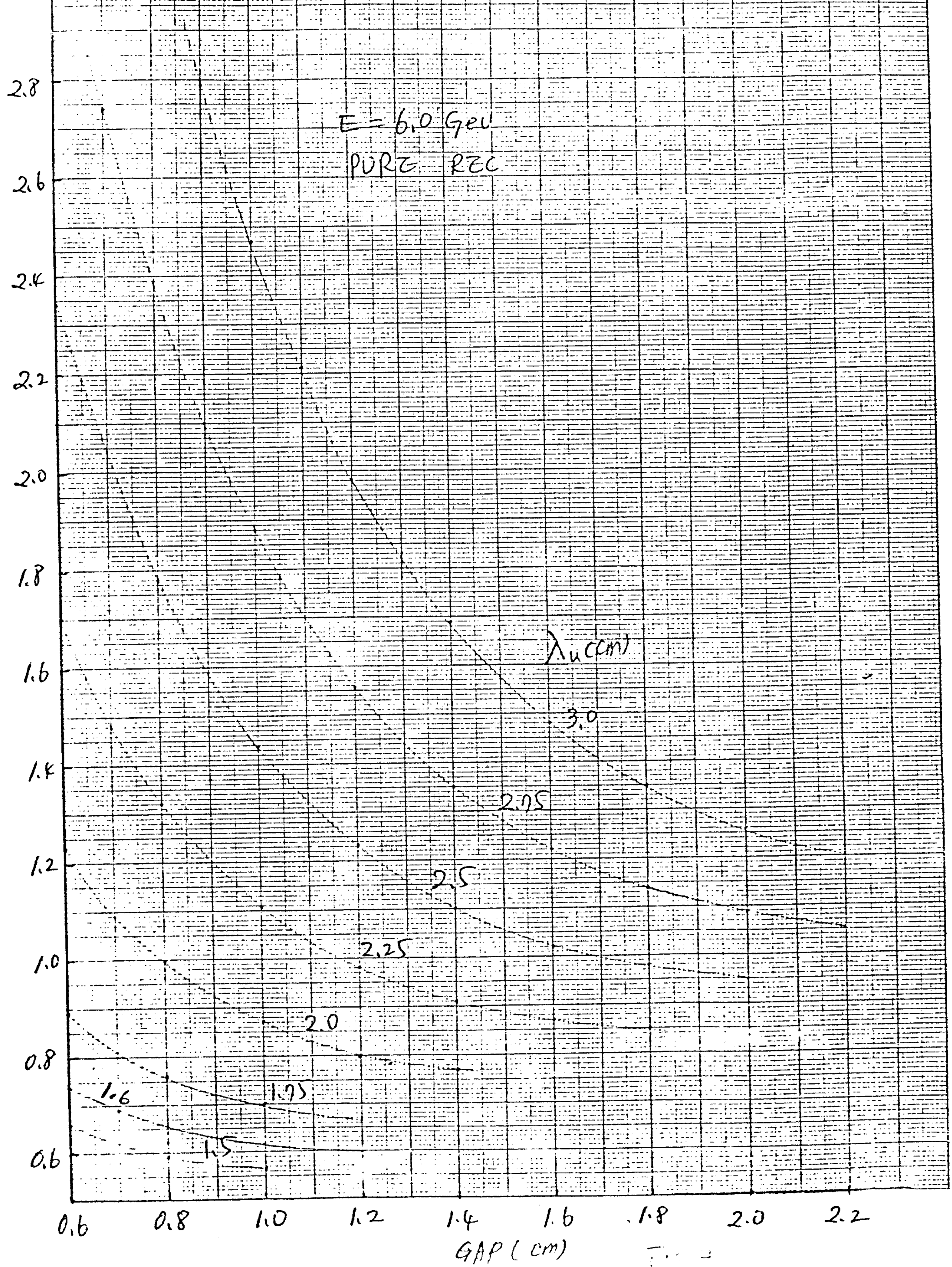
2. K. Halbach, J. de Phys., MT-8 (1984) C1-211.



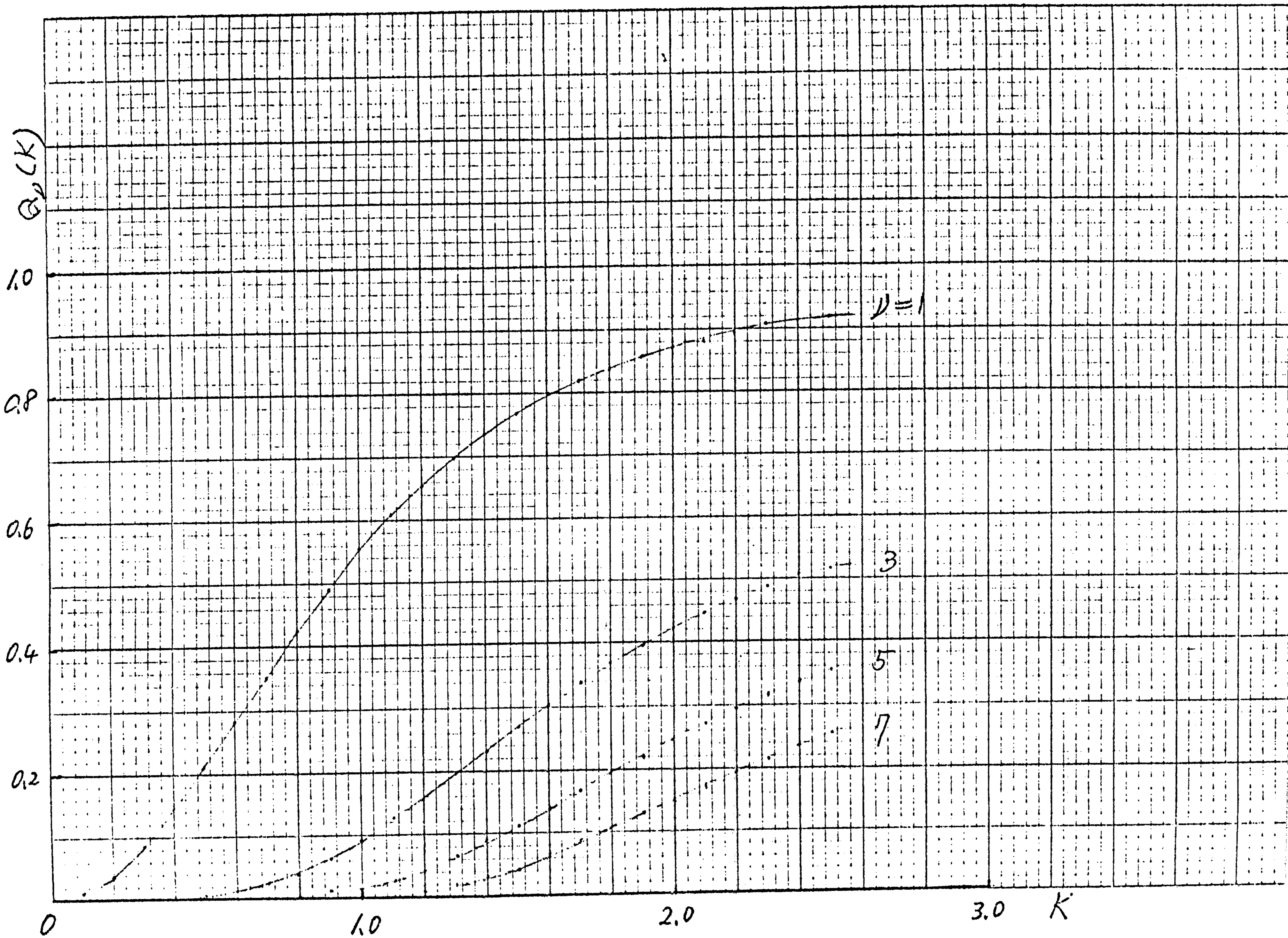
B_0 (Tesla)
1.0
0.5
0.1
HYBRID
$B_0 = 3.3\, e^{-g/\lambda_u (5.47 - 1.8 g/\lambda_u)}$
$B_0 = 1.62\, e^{-\pi g/\lambda_u}$
$h/\lambda_u = 1$
$h/\lambda_u = 1/4$
PURE REC
$B_0 = 1.30\, e^{-\pi g/\lambda_u}$
0
0.2
0.4
0.6
0.8
1.0
g/λ_u


E = 6.0 GeV
PURE REC
GAP = 0.8 cm
1.0
1.2
1.4
1.6
1.8
2.0
2.2
λ1 = 0.995A
0.870
0.803
0.747
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
1.25
1.5
1.75
2.0
2.25
2.5
2.75
3.0
λu (cm)

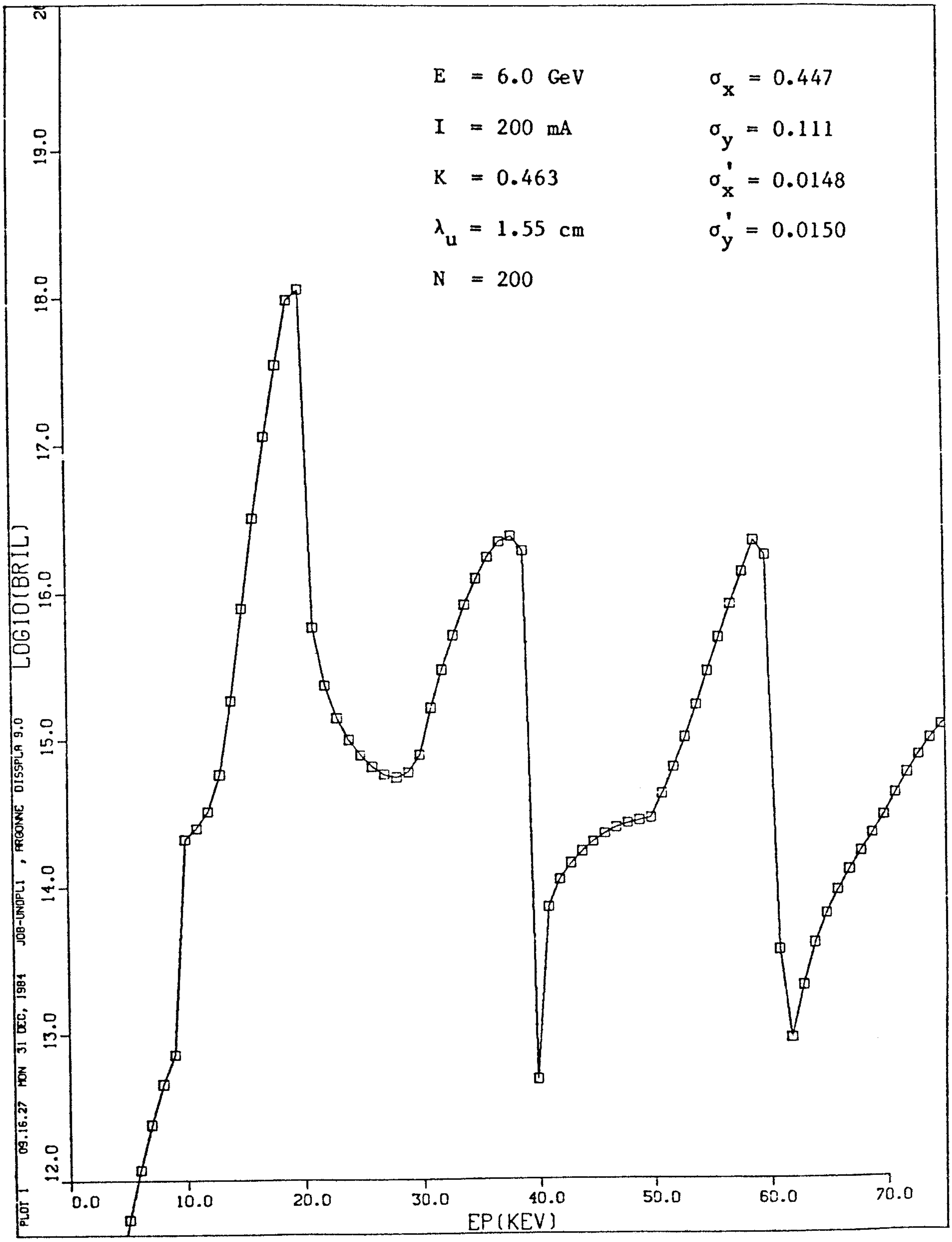

PURE REC
E = 6.0 GEV
λu (cm)
3.0
2.75
2.5
2.25
2.0
1.75
1.5
1.6
λ1 = 0.995A
0.920
0.803
0.707
K
GAP (cm)
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2



E = 6.0 Gev
PURE REC
λu (cm)
3.0
2.75
2.5
2.25
2.0
1.75
1.6
1.5
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
GAP (cm)

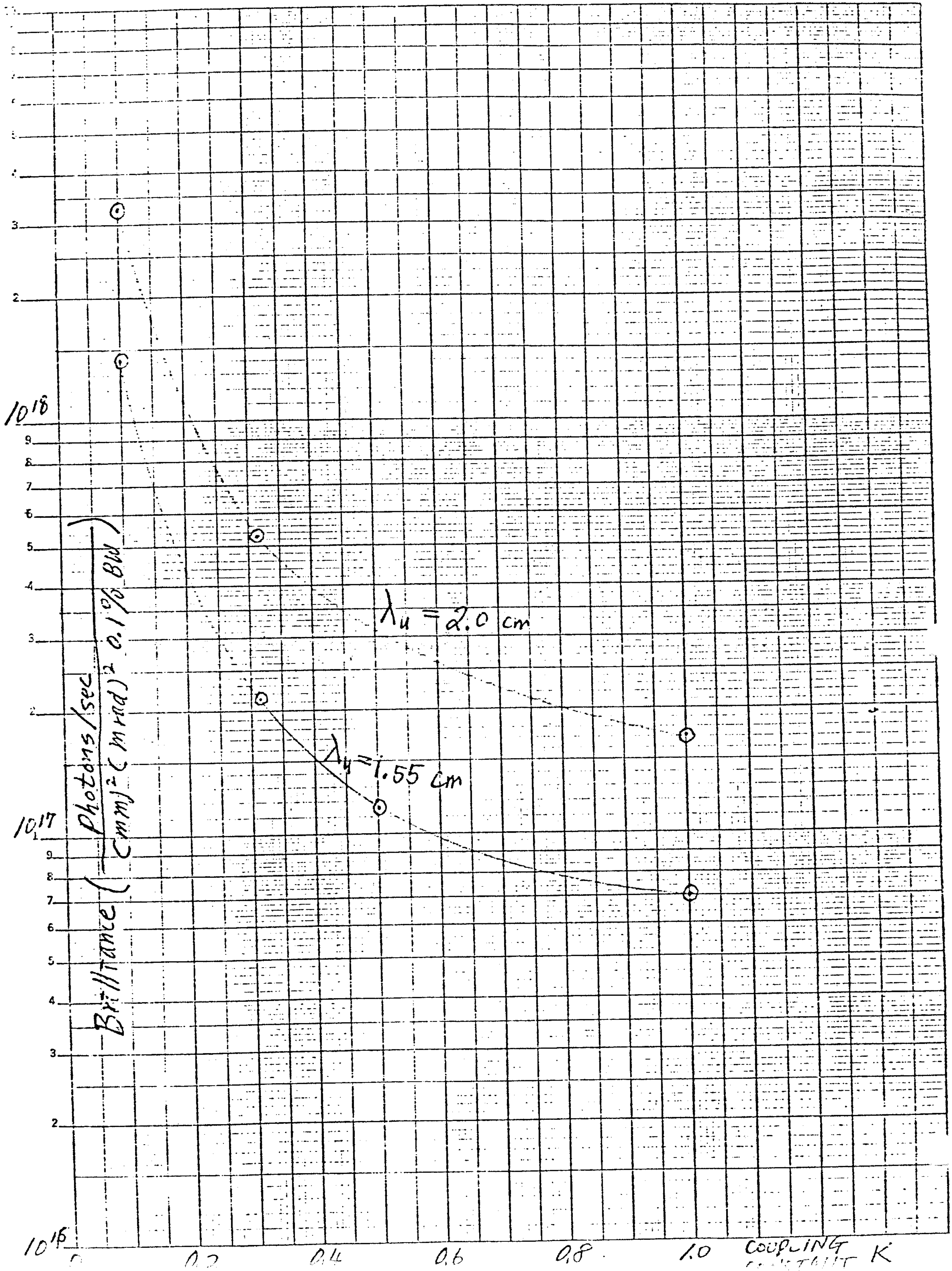


$G_\nu(K)$
1.0
0.8
0.6
0.4
0.2
0
1.0
2.0
3.0
K
$\nu=1$
3
5
7



E = 6.0 GeV
I = 200 mA
K = 0.463
λ_u = 1.55 cm
N = 200
σ_x = 0.447
σ_y = 0.111
σ_x' = 0.0148
σ_y' = 0.0150
LOG10(BRIL)
EP(KEV)
0.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
12.0
13.0
14.0
15.0
16.0
17.0
18.0
19.0
PLOT 1 09.16.27 MON 31 DEC, 1984 JOB-UNDPL1 , ARGONNE DISSPLA 9.0

Fig 6



10^{18}
10^{17}
10^{16}
Brilliance (Photons/sec / (mm)^2 (mrad)^2 0.1% BW)
$\lambda_u = 2.0$ cm
$\lambda_u = 1.55$ cm
0.2
0.4
0.6
0.8
1.0
COUPLING CONSTANT K

Table 1

UNP(CM)	GAP(CM)	B(TESLA)	K	WL1(A)	EV1(KEV)
1.5000	0.8000	0.3033	0.4249	0.5937	20.8877
1.5000	0.9000	0.2460	0.3446	0.5768	21.4968
1.5000	1.0000	0.1995	0.2795	0.5658	21.9172
1.5000	1.1000	0.1618	0.2267	0.5585	22.2028
1.5000	1.2000	0.1312	0.1838	0.5537	22.3947
1.5000	1.3000	0.1064	0.1491	0.5506	22.5228
1.5000	1.4000	0.0863	0.1209	0.5485	22.6079
1.5000	1.5000	0.0700	0.0981	0.5471	22.6642
1.5000	1.6000	0.0568	0.0795	0.5462	22.7014
1.5000	1.7000	0.0460	0.0645	0.5456	22.7259
1.5000	1.8000	0.0373	0.0523	0.5452	22.7421
1.6000	0.8000	0.3368	0.5033	0.6543	18.9501
1.6000	0.9000	0.2767	0.4135	0.6305	19.6681
1.6000	1.0000	0.2274	0.3398	0.6143	20.1845
1.6000	1.1000	0.1869	0.2792	0.6034	20.5488
1.6000	1.2000	0.1535	0.2295	0.5961	20.8023
1.6000	1.3000	0.1262	0.1885	0.5911	20.9770
1.6000	1.4000	0.1037	0.1549	0.5878	21.0967
1.6000	1.5000	0.0852	0.1273	0.5855	21.1782
1.6000	1.6000	0.0700	0.1046	0.5840	21.2337
1.6000	1.7000	0.0575	0.0860	0.5829	21.2713
1.6000	1.8000	0.0473	0.0706	0.5822	21.2967
1.7000	0.8000	0.3694	0.5865	0.7232	17.1454
1.7000	0.9000	0.3070	0.4875	0.6904	17.9597
1.7000	1.0000	0.2552	0.4053	0.6678	18.5691
1.7000	1.1000	0.2122	0.3369	0.6521	19.0150
1.7000	1.2000	0.1764	0.2800	0.6413	19.3358
1.7000	1.3000	0.1466	0.2328	0.6338	19.5639
1.7000	1.4000	0.1219	0.1935	0.6287	19.7247
1.7000	1.5000	0.1013	0.1609	0.6251	19.8373
1.7000	1.6000	0.0842	0.1337	0.6226	19.9159
1.7000	1.7000	0.0700	0.1112	0.6209	19.9706
1.7000	1.8000	0.0582	0.0924	0.6197	20.0086
1.8000	0.8000	0.4010	0.6741	0.8019	15.4639
1.8000	0.9000	0.3368	0.5662	0.7581	16.3562
1.8000	1.0000	0.2828	0.4755	0.7273	17.0502
1.8000	1.1000	0.2375	0.3993	0.7055	17.5762
1.8000	1.2000	0.1995	0.3354	0.6901	17.9671
1.8000	1.3000	0.1675	0.2817	0.6793	18.2535
1.8000	1.4000	0.1407	0.2366	0.6717	18.4611
1.8000	1.5000	0.1182	0.1987	0.6663	18.6104
1.8000	1.6000	0.0993	0.1669	0.6625	18.7171
1.8000	1.7000	0.0834	0.1401	0.6598	18.7931
1.8000	1.8000	0.0700	0.1177	0.6579	18.8471
1.9000	0.8000	0.4316	0.7658	0.8920	13.9020
1.9000	0.9000	0.3658	0.6491	0.8350	14.8502
1.9000	1.0000	0.3100	0.5502	0.7941	15.6153
1.9000	1.1000	0.2628	0.4663	0.7647	16.2155
1.9000	1.2000	0.2227	0.3953	0.7436	16.6761
1.9000	1.3000	0.1888	0.3350	0.7284	17.0234
1.9000	1.4000	0.1600	0.2840	0.7175	17.2820
1.9000	1.5000	0.1356	0.2407	0.7097	17.4727
1.9000	1.6000	0.1150	0.2040	0.7041	17.6123

FILE: UPARA OUTPUT A1 ANL VM/SP 305 CMS

1.9000	1.7000	0.0974	0.1729	0.7000	17.7140
1.9000	1.8000	0.0826	0.1466	0.6971	17.7878
2.0000	0.8000	0.4611	0.8613	0.9953	12.4589
2.0000	0.9000	0.3940	0.7361	0.9227	13.4392
2.0000	1.0000	0.3368	0.6291	0.8697	14.2586
2.0000	1.1000	0.2878	0.5376	0.8309	14.9232
2.0000	1.2000	0.2460	0.4595	0.8026	15.4491
2.0000	1.3000	0.2102	0.3927	0.7820	15.8573
2.0000	1.4000	0.1797	0.3356	0.7669	16.1693
2.0000	1.5000	0.1535	0.2868	0.7559	16.4051
2.0000	1.6000	0.1312	0.2451	0.7478	16.5817
2.0000	1.7000	0.1121	0.2095	0.7419	16.7132
2.0000	1.8000	0.0958	0.1790	0.7376	16.8105
2.1000	0.8000	0.4895	0.9601	1.1136	11.1346
2.1000	0.9000	0.4215	0.8267	1.0228	12.1237
2.1000	1.0000	0.3629	0.7118	0.9554	12.9785
2.1000	1.1000	0.3125	0.6129	0.9055	13.6943
2.1000	1.2000	0.2691	0.5278	0.8685	14.2781
2.1000	1.3000	0.2317	0.4544	0.8410	14.7442
2.1000	1.4000	0.1995	0.3913	0.8207	15.1099
2.1000	1.5000	0.1718	0.3369	0.8056	15.3929
2.1000	1.6000	0.1479	0.2901	0.7944	15.6097
2.1000	1.7000	0.1274	0.2498	0.7861	15.7744
2.1000	1.8000	0.1097	0.2151	0.7799	15.8988
2.2000	0.8000	0.5169	1.0621	1.2490	9.9280
2.2000	0.9000	0.4481	0.9207	1.1371	10.9049
2.2000	1.0000	0.3885	0.7982	1.0530	11.7759
2.2000	1.1000	0.3368	0.6920	0.9898	12.5278
2.2000	1.2000	0.2919	0.5999	0.9423	13.1593
2.2000	1.3000	0.2531	0.5201	0.9066	13.6775
2.2000	1.4000	0.2194	0.4509	0.8798	14.0946
2.2000	1.5000	0.1902	0.3909	0.8596	14.4253
2.2000	1.6000	0.1649	0.3389	0.8444	14.6842
2.2000	1.7000	0.1430	0.2938	0.8331	14.8849
2.2000	1.8000	0.1239	0.2547	0.8245	15.0395
2.3000	0.8000	0.5432	1.1669	1.4033	8.8363
2.3000	0.9000	0.4738	1.0179	1.2674	9.7836
2.3000	1.0000	0.4133	0.8879	1.1640	10.6527
2.3000	1.1000	0.3606	0.7746	1.0854	11.4249
2.3000	1.2000	0.3145	0.6757	1.0255	12.0919
2.3000	1.3000	0.2744	0.5894	0.9799	12.6540
2.3000	1.4000	0.2393	0.5142	0.9453	13.1181
2.3000	1.5000	0.2088	0.4485	0.9189	13.4947
2.3000	1.6000	0.1821	0.3913	0.8988	13.7961
2.3000	1.7000	0.1589	0.3413	0.8835	14.0347
2.3000	1.8000	0.1386	0.2977	0.8719	14.2218
2.4000	0.8000	0.5685	1.2743	1.5786	7.8552
2.4000	0.9000	0.4987	1.1180	1.4156	8.7593
2.4000	1.0000	0.4375	0.9808	1.2902	9.6107
2.4000	1.1000	0.3839	0.8605	1.1937	10.3877
2.4000	1.2000	0.3368	0.7549	1.1194	11.0771
2.4000	1.3000	0.2954	0.6623	1.0623	11.6733
2.4000	1.4000	0.2592	0.5810	1.0182	12.1778
2.4000	1.5000	0.2274	0.5097	0.9844	12.5968
2.4000	1.6000	0.1995	0.4472	0.9583	12.9395

2.4000	1.7000	0.1750	0.3923	0.9382	13.2162
2.4000	1.8000	0.1535	0.3442	0.9228	13.4373
2.5000	0.8000	0.5928	1.3842	1.7769	6.9785
2.5000	0.9000	0.5228	1.2207	1.5837	7.8298
2.5000	1.0000	0.4611	1.0766	1.4334	8.6507
2.5000	1.1000	0.4066	0.9495	1.3165	9.4186
2.5000	1.2000	0.3586	0.8373	1.2256	10.1172
2.5000	1.3000	0.3163	0.7385	1.1549	10.7365
2.5000	1.4000	0.2789	0.6513	1.0999	11.2733
2.5000	1.5000	0.2460	0.5743	1.0572	11.7293
2.5000	1.6000	0.2169	0.5065	1.0239	12.1104
2.5000	1.7000	0.1913	0.4467	0.9980	12.4243
2.5000	1.8000	0.1687	0.3940	0.9779	12.6799
2.6000	0.8000	0.6162	1.4963	2.0004	6.1988
2.6000	0.9000	0.5460	1.3260	1.7736	6.9916
2.6000	1.0000	0.4839	1.1751	1.5954	7.7722
2.6000	1.1000	0.4288	1.0414	1.4555	8.5192
2.6000	1.2000	0.3800	0.9228	1.3457	9.2147
2.6000	1.3000	0.3368	0.8178	1.2594	9.8459
2.6000	1.4000	0.2984	0.7247	1.1916	10.4058
2.6000	1.5000	0.2645	0.6422	1.1384	10.8921
2.6000	1.6000	0.2344	0.5691	1.0967	11.3071
2.6000	1.7000	0.2077	0.5044	1.0638	11.6559
2.6000	1.8000	0.1841	0.4470	1.0381	11.9452
2.7000	0.8000	0.6386	1.6105	2.2512	5.5082
2.7000	0.9000	0.5685	1.4336	1.9873	6.2397
2.7000	1.0000	0.5060	1.2761	1.7782	6.9735
2.7000	1.1000	0.4505	1.1360	1.6125	7.6900
2.7000	1.2000	0.4010	1.0112	1.4812	8.3717
2.7000	1.3000	0.3569	0.9001	1.3771	9.0041
2.7000	1.4000	0.3177	0.8013	1.2947	9.5774
2.7000	1.5000	0.2828	0.7132	1.2294	10.0863
2.7000	1.6000	0.2518	0.6349	1.1776	10.5296
2.7000	1.7000	0.2241	0.5652	1.1366	10.9095
2.7000	1.8000	0.1995	0.5031	1.1041	11.2306
2.8000	0.8000	0.6602	1.7266	2.5315	4.8984
2.8000	0.9000	0.5902	1.5434	2.2269	5.5682
2.8000	1.0000	0.5275	1.3796	1.9836	6.2513
2.8000	1.1000	0.4715	1.2331	1.7892	6.9305
2.8000	1.2000	0.4215	1.1023	1.6339	7.5894
2.8000	1.3000	0.3768	0.9853	1.5097	8.2133
2.8000	1.4000	0.3368	0.8807	1.4106	8.7907
2.8000	1.5000	0.3010	0.7872	1.3313	9.3139
2.8000	1.6000	0.2691	0.7037	1.2680	9.7789
2.8000	1.7000	0.2405	0.6290	1.2175	10.1851
2.8000	1.8000	0.2150	0.5622	1.1770	10.5349