

## Dependence of Brilliance on Beta Functions

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In LS-53, the beta functions through undulators and wigglers are optimized. Recently, K-J. Kim has shown that the effective source size and divergence are given by

$$s_{x,y} = (\sigma_{x,y}^2 + \sigma_R^2)^{1/2},$$

$$s'_{x,y} = (\sigma'_{x,y}^2 + \sigma'_R^2)^{1/2},$$

$$\text{with } \sigma_R = (\lambda L)^{1/2}/4\pi \text{ and } \sigma'_R = (\lambda/L)^{1/2}.$$

Here, the expression of the diffraction term  $\sigma_R$  is different from the formulas used in LS-53. With the new expression, the minimum phase space is in the neighborhood of  $\beta_x = \frac{L}{4\pi} \sim \frac{L}{4\pi} \beta_x / \beta_y$ . The source area, on the other hand, increases as the beta functions are increased.

In the straight section, if one uses the relation

$$\beta(s) = \beta [1 + (s - s_0^2)/\beta^2],$$

the effective beta function along the ID may be an average of  $\beta(s)$ ,

$$\langle \beta_{x,y} \rangle = \beta_{x,y} [1 + \frac{1}{12} (L/\beta_{x,y})^2],$$

which has the minimum at  $\beta_{x,y} \approx L/3.5$ . Here it is assumed that  $\beta(s)$  is symmetric with respect to the center of the ID. The above argument suggests that the maximum brilliance could be obtained at  $\beta \approx 2m$  for a 6-m long ID.

In Tables 1-5, the relative variations of the inverse phase space are shown for the undulator photon energies of 20-0.5 keV. In the range of  $\beta = 1 \sim 20$  m, the variations are rather insensitive to the beta functions. Similar Results have been obtained in LS-53. In the case of extreme wiggler

regime, the brightness does not depend on the divergence. Therefore, in Tables 6 and 7, the variations of the inverse source area, not the inverse phase space, are shown for the wigglers. The variations of the source area is relatively sensitive to the beta functions compared to that of the phase space.

SHK:ehr

# UNDULATOR PHASE SPACE

$E_0(E-08)=0.800$ , COUPLING=0.100, EX=0.7273E-08, EY=0.7273E-09  
 ID LENGTH(M)=5.200, PHOTON ENERGY(KEV)= 20.00, WAVELENGTH(A)= 0.62

MINIMUM PHASE SPACE=0.2154E-15 AT BX= 2.00 AND BY= 2.00

HORIZONTAL AXIS: BX, VERTICAL AXIS: BY  
 PLOT IS NORMALIZED WITH 4\*PIE\*PIE\*EX\*EY

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	97	97	97	96	96	96	96	96	96	96	96	96	96	96	96	96	95	95	95	
2	97	97	97	97	97	97	97	97	96	96	96	96	96	96	96	96	96	96	96	
3	97	97	97	96	96	96	96	96	96	96	96	96	96	96	96	95	95	95	95	
4	96	96	96	96	96	96	96	96	95	95	95	95	95	95	95	95	95	95	95	
5	95	95	95	95	95	95	95	95	95	95	95	95	95	95	94	94	94	94	94	
6	95	95	95	95	95	95	94	94	94	94	94	94	94	94	94	94	94	94	94	
7	94	94	94	94	94	94	94	94	94	94	93	93	93	93	93	93	93	93	93	
8	93	93	93	93	93	93	93	93	93	93	93	93	93	93	93	92	92	92	92	
9	93	93	93	93	93	93	92	92	92	92	92	92	92	92	92	92	92	92	92	
10	92	92	92	92	92	92	92	92	92	92	92	91	91	91	91	91	91	91	91	
11	92	92	92	92	91	91	91	91	91	91	91	91	91	91	91	91	91	90	90	
12	91	91	91	91	91	91	91	91	90	90	90	90	90	90	90	90	90	90	90	
13	90	90	90	90	90	90	90	90	90	90	90	90	90	90	89	89	89	89	89	
14	90	90	90	90	90	90	89	89	89	89	89	89	89	89	89	89	89	89	89	
15	89	89	89	89	89	89	89	89	89	89	89	89	88	89	88	88	88	88	88	
16	89	89	89	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	87	
17	88	88	88	88	88	88	88	88	88	88	87	87	87	87	87	87	87	87	87	
18	87	87	87	87	87	87	87	87	87	87	87	87	87	87	87	87	86	86	86	
19	87	87	87	87	87	87	87	87	87	86	86	86	86	86	86	86	86	86	86	
20	86	86	86	86	86	86	86	86	86	86	86	86	86	86	85	85	85	85	85	

Table 1. The inverse of the phase space, normalized with the inverse of the phase space at the short wavelength limit, is in the unit of %.

## UNDULATOR PHASE SPACE

EC(E-08)=0. 800 , COUPLING=0. 100 , EX=0. 7273E-08 , EY=0. 7273E-09  
 LB LENGTH(M)=5. 200 , PHOTON ENERGY(KEV)= 10. 00 , WAVELENGTH(A)= 1. 24

MINIMUM PHASE SPACE=0. 2219E-15 AT BX= 2. 00 AND BY= 2. 00

HORIZONTAL AXIS: BX, VERTICAL AXIS: BY  
 PLOT IS NORMALIZED WITH 4\*PIE\*PIE\*EX\*EY

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	93	93	93	93	93	93	93	93	93	92	92	92	92	92	92	92	91	91	91	91
2	94	94	94	94	94	94	94	93	93	93	93	93	93	93	92	92	92	92	92	92
3	93	93	93	93	93	93	93	93	93	92	92	92	92	92	92	91	91	91	91	91
4	92	92	92	92	92	92	92	92	91	91	91	91	91	91	91	90	90	90	90	90
5	91	91	91	91	91	91	91	90	90	90	90	90	90	90	90	89	89	89	89	89
6	90	90	90	90	90	90	90	89	89	89	89	89	89	89	89	88	88	88	88	88
7	89	89	89	89	89	89	88	88	88	88	88	88	88	88	87	87	87	87	87	87
8	88	88	88	88	88	87	87	87	87	87	87	87	87	87	86	86	86	86	86	86
9	87	87	87	87	87	86	86	86	86	86	86	86	86	86	85	85	85	85	85	85
10	86	86	86	86	86	85	85	85	85	85	85	85	85	84	84	84	84	84	84	84
11	85	85	85	85	85	84	84	84	84	84	84	84	84	84	83	83	83	83	83	83
12	84	84	84	84	83	83	83	83	83	83	83	83	83	83	82	82	82	82	82	82
13	83	83	83	83	83	82	82	82	82	82	82	82	82	82	81	81	81	81	81	81
14	82	82	82	82	82	82	81	81	81	81	81	81	81	81	80	80	80	80	80	80
15	81	81	81	81	81	81	80	80	80	80	80	80	80	80	79	79	79	79	79	79
16	80	80	80	80	80	80	80	79	79	79	79	79	79	79	79	79	79	78	78	
17	79	80	79	79	79	79	79	79	79	79	78	78	78	78	78	78	78	78	78	77
18	79	79	79	79	78	78	78	78	78	78	78	77	77	77	77	77	77	77	77	77
19	78	78	78	78	78	77	77	77	77	77	77	77	77	76	76	76	76	76	76	76
20	77	77	77	77	77	77	77	76	76	76	76	76	76	76	75	75	75	75	75	75

Table 2. The inverse of the phase space, normalized with the inverse of the phase space at the short wavelength limit, is in the unit of %.

## UNDULATOR PHASE SPACE

$E_0(E-08)=0.800$ , COUPLING=0.100, EX=0.7273E-08, EY=0.7273E-09  
 ID LENGTH(M)=5.200, PHOTON ENERGY(KEV)= 5.00, WAVELENGTH(A)= 2.48

MINIMUM PHASE SPACE=0.2346E-15 AT BX= 2.00 AND BY= 2.00

HORIZONTAL AXIS: BX, VERTICAL AXIS: BY  
 PLOT IS NORMALIZED WITH 4\*PIE\*PIE\*EX\*EY

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	89	88	88	88	87	87	87	87	86	86	86	86	85	85	85	85	84	84	84	84
2	89	89	89	89	88	88	86	88	87	87	87	87	86	86	86	86	85	85	85	85
3	88	88	88	87	87	87	87	86	86	86	86	86	85	85	85	85	84	84	84	84
4	86	86	86	86	85	85	85	85	84	84	84	84	84	83	83	83	83	82	82	82
5	84	84	84	84	84	83	83	83	83	82	82	82	82	81	81	81	81	81	81	80
6	83	83	82	82	82	82	81	81	81	80	80	80	80	80	80	80	79	79	79	79
7	81	81	81	80	80	80	80	80	79	79	79	79	78	78	78	78	77	77	77	77
8	79	79	79	79	79	78	78	78	78	77	77	77	77	77	76	76	76	76	75	75
9	78	78	78	77	77	77	77	76	76	76	76	76	75	75	75	75	74	74	74	74
10	76	76	76	76	76	75	75	75	75	75	74	74	74	74	73	73	73	73	73	73
11	75	75	75	75	74	74	74	74	73	73	73	73	72	72	72	72	72	71	71	71
12	73	74	73	73	73	73	72	72	72	72	72	71	71	71	71	71	70	70	70	70
13	72	72	72	72	72	72	71	71	71	71	70	70	70	70	70	69	69	69	69	69
14	71	71	71	71	71	70	70	70	70	70	69	69	69	69	69	68	68	68	68	68
15	70	70	70	70	69	69	69	69	68	68	69	68	68	68	67	67	67	67	67	66
16	69	69	69	68	68	68	68	68	67	67	67	67	67	66	66	66	66	66	66	65
17	68	68	68	68	67	67	67	67	66	66	66	66	66	65	65	65	65	65	65	64
18	67	67	67	66	66	66	66	66	65	65	65	65	65	64	64	64	64	64	64	63
19	66	66	66	65	65	65	65	64	64	64	64	64	64	63	63	63	63	63	63	63
20	65	65	65	65	64	64	64	64	63	63	63	63	63	62	62	62	62	62	62	62

Table 3. The inverse of the phase space, normalized with the inverse of the phase space at the short wavelength limit, is in the unit of %.

## UNDULATOR PHASE SPACE

$EC/E-09=0.800$ , COUPLING=0.100, EX=0.7273E-08, EY=0.7273E-09  
 IO LENGTH(M)=3.200, PHOTON ENERGY(KEV)= 1.00, WAVELENGTH(A)=12.40

MINIMUM PHASE SPACE=0.3259E-15 AT BX= 2.00 AND BY= 2.00

HORIZONTAL AXIS: BX, VERTICAL AXIS: BY  
 PLOT IS NORMALIZED WITH 4\*PIE\*PIE\*EX\*EY

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	61	62	61	61	60	59	59	58	57	56	56	55	55	54	53	53	52	52	51	51
2	64	64	64	63	62	61	61	60	59	59	58	57	57	56	55	55	54	54	53	53
3	61	62	61	61	60	59	59	58	57	56	56	55	55	54	53	53	52	52	51	51
4	58	59	58	58	57	55	55	55	54	54	53	53	52	51	51	50	50	49	49	48
5	56	56	56	55	54	54	53	52	52	51	50	50	49	49	48	48	47	47	46	46
6	53	53	53	52	52	51	50	50	49	49	48	48	47	46	46	45	45	45	44	44
7	51	51	51	50	49	49	48	48	47	46	46	45	45	44	44	43	43	43	42	42
8	48	49	48	48	47	47	46	46	45	45	44	43	43	43	42	42	41	41	40	40
9	46	47	46	46	45	45	44	44	43	43	42	42	41	41	40	40	40	39	39	38
10	45	45	45	44	44	43	43	42	42	41	41	40	40	39	39	39	39	38	37	37
11	43	44	43	43	42	42	41	41	40	40	39	39	38	38	38	37	37	36	36	36
12	42	42	42	41	41	40	40	39	39	39	38	38	37	37	36	36	36	35	35	35
13	41	41	41	40	40	39	39	36	38	37	37	36	36	36	35	35	35	34	34	34
14	39	40	39	39	39	38	38	37	37	36	36	35	35	35	34	34	34	33	33	33
15	38	39	38	38	37	37	37	36	36	35	35	34	34	34	33	33	33	32	32	32
16	37	38	37	37	37	36	36	35	35	34	34	34	33	33	32	32	32	31	31	31
17	36	37	36	36	36	35	35	34	34	34	33	33	32	32	32	31	31	31	30	30
18	36	36	36	35	35	34	34	34	33	33	32	32	32	31	31	31	30	30	29	29
19	35	35	35	34	34	34	33	33	32	32	32	31	31	31	30	30	30	29	29	28
20	34	34	34	34	33	33	32	32	32	31	31	31	30	30	30	29	29	28	28	28

Table 4. The inverse of the phase space, normalized with the inverse of the phase space at the short wavelength limit, is in the unit of %.

UNDULATOR PHASE SPACE

$E_0(E-08) = 0.600$ , COUPLING=0.100, EX=0.7273E-08, EY=0.7273E-09  
 ID LENGTH(M)=5.000, PHOTON ENERGY(KEV)= 0.50, WAVELENGTH(A)=24.80

MINIMUM PHASE SPACE=0.4276E-15 AT BX= 2.00 AND BY= 2.00

HORIZONTAL AXIS: BX, VERTICAL AXIS: BY  
 PLOT IS NORMALIZED WITH 4\*PIE\*PIE\*EX\*EY

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	46	47	46	45	44	43	42	42	41	40	39	39	38	37	37	36	36	35	34	34
2	48	49	48	47	46	45	44	44	43	42	41	40	40	39	38	38	37	37	36	36
3	46	47	46	45	44	43	42	42	41	40	39	39	38	37	37	36	36	35	34	34
4	43	41	43	43	42	41	40	39	38	38	37	36	36	35	34	34	33	33	32	32
5	41	41	41	40	39	38	38	37	36	35	35	34	34	33	32	32	31	31	30	30
6	38	39	38	36	37	36	35	35	34	33	33	32	32	31	31	30	30	29	29	28
7	36	37	36	36	35	34	34	33	32	32	31	30	30	29	29	29	28	28	27	27
8	35	35	35	34	33	33	32	31	31	30	30	29	29	28	28	27	27	26	26	26
9	33	34	33	32	32	31	31	30	29	29	28	28	27	27	26	26	26	25	25	24
10	32	32	32	31	31	30	29	29	28	28	27	27	26	26	25	25	24	24	24	23
11	31	31	31	30	29	29	28	28	27	27	26	26	25	25	24	24	24	23	23	23
12	29	30	29	29	28	28	27	27	26	26	25	25	24	24	23	23	23	22	22	22
13	28	29	28	28	27	27	26	26	25	25	24	24	23	23	23	22	22	22	21	21
14	28	28	28	27	26	26	25	25	24	24	24	23	23	22	22	22	21	21	21	20
15	27	27	27	26	26	25	25	24	24	23	23	22	22	22	21	21	21	20	20	20
16	26	26	26	26	25	24	24	23	23	22	22	21	21	21	20	20	20	19	19	19
17	25	26	25	25	24	24	23	23	22	22	22	21	21	20	20	20	19	19	19	19
18	25	25	25	24	24	23	23	22	22	21	21	21	20	20	20	19	19	19	18	18
19	24	24	24	24	23	23	22	22	21	21	21	20	20	19	19	19	19	18	18	18
20	24	24	24	23	23	22	22	21	21	20	20	20	19	19	19	18	18	18	17	

Table 5. The inverse of the phase space, normalized with the inverse of the phase space at the short wavelength limit, is in the unit of %.

## WIGGLER PHASE SPACE

EQ(E-08)=0. 800 , COUPLING=0. 100 , EX=0. 7273E-08 , EY=0. 7273E-09  
 ID LENGTH(M)=3. 000 , PHOTON ENERGY(KEV)=40. 000 , WAVELENGTH(A)= 0. 31

MINIMUM PHASE SPACE=0. 5052E-07 AT BX= 2. 00 AND BY= 2. 00

HORIZONTAL AXIS: BX, VERTICAL AXIS: BY  
 PLOT IS NORMALIZED WITH 2\*PIE\*SX\*SY

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	87	94	87	80	74	69	64	60	57	55	52	50	48	46	45	43	42	41	40	39
2	94	100	94	86	79	73	69	65	61	58	56	53	51	50	48	46	45	44	43	42
3	87	94	87	80	74	69	64	60	57	55	52	50	48	46	45	43	42	41	40	39
4	80	86	80	74	68	63	59	55	53	50	48	46	44	43	41	40	39	38	37	36
5	74	79	74	68	62	58	54	51	48	46	44	42	41	39	38	37	36	35	34	33
6	69	73	69	63	58	54	50	47	45	43	41	39	38	36	35	34	33	32	31	31
7	64	69	64	59	54	50	47	44	42	40	38	37	35	34	33	32	31	30	29	29
8	60	65	60	55	51	47	44	42	40	38	36	35	33	32	31	30	29	28	28	27
9	57	61	57	53	48	45	42	40	37	36	34	33	31	30	29	28	28	27	26	26
10	55	58	55	50	46	43	40	38	36	34	32	31	30	29	28	27	26	26	25	24
11	52	56	52	48	44	41	38	36	34	32	31	30	29	28	27	26	25	24	24	23
12	50	53	50	46	42	39	37	35	33	31	30	29	27	27	26	25	24	23	23	22
13	48	51	48	44	41	38	35	33	31	30	29	27	26	26	25	24	23	23	22	21
14	46	50	46	43	39	36	34	32	30	29	28	27	26	25	24	23	22	22	21	21
15	45	48	45	41	38	35	33	31	29	28	27	26	25	24	23	22	22	21	21	20
16	43	46	43	40	37	34	32	30	28	27	26	25	24	23	22	22	21	20	20	19
17	42	45	42	39	36	33	31	29	28	26	25	24	23	22	22	21	20	20	19	19
18	41	44	41	38	35	32	30	28	27	26	24	23	23	22	21	20	20	19	19	18
19	40	43	40	37	34	31	29	28	26	25	24	23	22	21	21	20	19	19	18	18
20	39	42	39	36	33	31	29	27	26	24	23	22	21	21	20	19	19	18	18	17

Table 6. The inverse of the effective source area, normalized with the inverse of the source area at the short wavelength limit, is in the unit of %.

## WIGGLER PHASE SPACE

E2(E-03)=0. 800 , COUPLING=0. 100 , EX=0. 7273E-09 , EY=0. 7273E-09  
 ID LENGTH(M)=3. 000 , PHOTON ENERGY(KEV)= 1. 000 , WAVELENGTH(A)= 12. 40

MINIMUM PHASE SPACE=0. 5083E-07 AT BX= 2. 00 AND BY= 2. 00

HORIZONTAL AXIS: BX, VERTICAL AXIS: BY  
 PLOT IS NORMALIZED WITH 2\*PI\*SX\*SY

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	87	93	87	80	74	68	64	60	57	54	52	50	48	46	45	43	42	41	40	39
2	93	99	93	85	79	73	68	64	61	58	55	53	51	49	48	46	45	44	43	41
3	87	93	87	80	74	68	64	60	57	54	52	50	48	46	45	43	42	41	40	39
4	80	86	80	73	68	63	59	55	52	50	48	46	44	42	41	40	39	38	37	36
5	74	79	74	68	62	58	54	51	48	46	44	42	41	39	38	37	36	35	34	33
6	68	73	68	63	58	54	50	47	45	43	41	39	38	36	35	34	33	32	31	31
7	64	68	64	59	54	50	47	44	42	40	38	37	35	34	33	32	31	30	29	29
8	60	64	60	55	51	47	44	42	40	38	36	34	33	32	31	30	29	28	28	27
9	57	61	57	52	48	45	42	40	37	36	34	33	31	30	29	28	28	27	26	25
10	54	58	54	50	46	43	40	38	36	34	32	31	30	29	28	27	26	26	25	24
11	52	56	52	48	44	41	38	36	34	32	31	30	29	28	27	26	25	24	24	23
12	50	53	50	46	42	39	37	35	33	31	30	29	27	26	26	25	24	23	23	22
13	48	51	48	44	41	38	35	33	31	30	29	27	26	25	25	24	23	23	22	21
14	46	50	46	43	39	36	34	32	30	29	28	26	25	25	24	23	22	22	21	21
15	45	48	45	41	38	35	33	31	29	28	27	26	25	24	23	22	22	21	20	20
16	43	46	43	40	37	34	32	30	28	27	26	25	24	23	22	22	21	20	20	19
17	42	45	42	39	36	33	31	29	28	26	25	24	23	22	22	21	20	20	19	19
18	41	44	41	38	35	32	30	28	27	26	24	23	23	22	21	20	20	19	19	18
19	40	43	40	37	34	31	29	28	26	25	24	23	22	21	20	20	19	19	18	18
20	39	42	39	36	33	31	29	27	25	24	23	22	21	21	20	19	19	18	18	17

Table 7. The inverse of the effective source area, normalized with the inverse of the source area at the short wavelength limit, is in the unit of %.