TK-LS (3/4/85)

LS-15
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Chromaticity Correction and Betafunction Distortion

The required value of the chromaticity is otained by introducing sextupole magnets in the dispersive straight sections

$$\frac{p}{\nu}\frac{\Delta\nu}{\Delta p} = -\frac{1}{4\pi\nu}\int \beta(K - S\eta)ds,$$

where $K = \frac{1}{B\rho}\frac{dB_y}{dx}$ is the focusing strength of the lattice quadrupoles, $S = \frac{1}{B\rho}\frac{d^2B_y}{dx^2}$ the strength of the correction sextupoles and η is the dispersion function. About one half of the quadrupoles are located in dispersion-free straight sections. Furthermore, the natural chromaticity of the low-emittance lattice is large and one will have large harmonic components in the Fourier series expansion of $\beta(K - S\eta)$. Since the beta functions depend on the focusing strength, these Fourier components will effect the beta functions of the off-momentum particles. Analogous to the definition of chromaticity, one can write the momentum spread dependence of the beta function in the form $\frac{p}{\beta}\frac{d\beta}{dp}$. Using the Twiss parameter relations

$$\frac{d\beta}{ds} = -2\,\alpha$$

$$\frac{d\alpha}{ds} = K\beta - \frac{1+\alpha^2}{\beta}$$

one can show that this quantity $\frac{p}{\beta}\frac{d\beta}{dp}$ satisfies the equation

$$\frac{d^2}{d\phi^2}\left(\frac{p}{\beta}\frac{d\beta}{dp}\right) + 4\left(\frac{p}{\beta}\frac{d\beta}{dp}\right) = -2\beta^2\; p\,\frac{dK}{dp},$$

where $\phi = \int \frac{ds}{\beta}$ is the betatron phase advance. Setting $p\frac{dK}{dp} = (K - S\eta)$ and replacing $d\phi$ by $\nu d\psi$, one obtains

$$\frac{d^2}{d\psi^2}\left(\frac{p}{\beta}\frac{d\beta}{dp}\right) + (2\nu)^2\left(\frac{p}{\beta}\frac{d\beta}{dp}\right) = -2\nu^2\beta^2(K - S\eta) .$$

Expansion of $\nu\beta^2(K - S\eta)$ in the Fourier series

$$\nu\beta^2(K - S\eta) = \sum a_n e^{in N_s\psi},$$

where N_s is the number of superperiods,

$$a_k = \frac{1}{2\pi}\int_0^{2\pi} \nu\beta^2(K - S\eta)e^{-in N_s\psi} d\psi$$

$$= \frac{1}{2\pi}\int_0^{L_s} \beta(K - S\eta)\, e^{-inN_s\psi(s)} ds,$$

and L_s is the length of the superperiod, gives the periodic solution

$$\frac{p}{\beta}\frac{d\beta}{dp} = \sum \frac{2\nu\, a_n}{n^2-(2\nu)^2} e^{in N_s\psi(s)} . \tag{1}$$

The above equations apply for both transverse motion, with $K = K_x$, $\nu = \nu_x$, $\beta = \beta_x$, $\psi = \phi_x/\nu_x$ for the horizontal motion, and $K = K_y$, $\nu = \nu_y$, $\beta = \beta_y$, $\psi = \phi_y/\nu_y$ for the vertical motion. From Eq. (1) one sees that the beta function distortion is most sensitive to the Fourier components whose order is close to 2ν. Therefore, the choice of the locations and strengths of the sextupoles should not only be determined by maximizing the stability limits but also by minimizing the quantities

$$a_{m_x} = \frac{1}{2\pi} \int_o^{L_s} \beta_x (K_x - S\eta) e^{-i\, m_x N_s \psi_x(s)} ds,$$

with m_x close to $2\nu_x$,

and
$$a_{m_y} = \frac{1}{2\pi} \int_o^{L_s} \beta_y (K_y - S\eta) e^{-i m_y N_s \psi_x(s)} ds,$$

with m_y close to $2\nu_y$.