LS-102
November 1987
K. Symon

Preliminary Thoughts on the Aladdin Experiments

Keith Symon

I suggest three sets of experiments for comparing tracking results with experimental data on the dynamic aperture in Aladdin, as follows. (See the tune diagram, Fig. 1.)

1. Aperture determined by a single dominant resonance.

When the aperture is determined by a single dominant nonlinear resonance, it can be calculated analytically, as well as by numerical tracking. The experimental aperture should therefore agree with the predicted one, if we know what we are doing.

I suggest the third integral resonance $n_z = 7\ 1/3$. We should choose n_x as far from other resonances as possible, say $n_x = 7.08$. The resonance $3n_z = 22$ is driven by a sextupole term with a cos(22 r) dependence on azimuth, where r = 2!s/C and C is the circumference. In order to drive the resonance with a known driving term, we should excite a single skew sextupole with a controlled excitation. We should be near enough the resonance that it is dominant, but far enough away so that the aperture depends on the sextupole excitation and not on other imperfection sextupole terms. There are then three steps:

1. Determine analytically the predicted aperture, as a function of the tune and the sextupole amplitude.

2. Determine the predicted aperture by tracking, and compare with the results of step 1.

3. Determine the experimental aperture on Aladdin and compare.

2. Dominance by a group of intersecting resonances.

If the working point is near the intersection of two or more resonances on the tune diagram (Fig. 1), then the motion cannot be found analytically, but we can find one constant of the motion [Ref. 1]. We can make some predictions and educated guesses about the motion. In particular, on certain rays from the intersection, we expect, and find [Ref. 1], that the dynamic aperture will be smaller than predicted by tracking the motion for a few hundred or even a few thousand turns.

I suggest the neighborhood of $n_x = 7\ 1/6$, $n_z = 7\ 1/3$, although some analysis should be done to verify that we expect the same interesting phenomena near this point that were found near a different intersection in Ref. 1. Another interesting intersection is $n_x = n_z = 7\ 1/4$, but that intersection involves mainly fourth-integral resonances.

The theorem of corresponding motions, proved in Ref. 1, tells us that if we are close enough to the resonance intersection so that only the intersecting resonances matter, then the motions at different points on a single ray through the intersection are identical, if amplitudes are scaled in proportion to distance from the intersection, and if times are scaled in inverse proportion. We can use this result to determine when we are close enough to the intersection. Again, we want to stay far enough from the intersection so that the results are not determined by stray imperfection terms.

The intersecting resonances are:

$$\begin{aligned} 3\,n_z &= 22 \\ 2\,n_x - n_z &= 7 \\ 2\,n_x + 2\,n_z &= 29 \end{aligned}$$

The first two are driven by skew sextupole terms with a cos(22 r) dependence and a cos(7 r) dependence. The last is driven by an octupole term with a cos(29 r) dependence. We should provide excitation of a sextupole and an octupole winding for this purpose. Since the two third-integral resonances involve odd multiples of n_z, they cannot be excited by elements symmetrical about the horizontal midplane; hence the need for a skew sextupole. We may omit the octupole and study the two third-integral resonances alone, provided we are not left at the mercy of stray octupole terms, which should be quite small.

The program again has three steps:

1. Study the problem analytically, determine the constant of the motion, and see what we can predict or suggest about the motion. A selection of working points on typical rays over a semicircle about the intersection should be studied, as in Ref. 1. Try to find some rays where the motion is expected to remain finite for hundreds of thousands of turns before becoming unstable, so that tracking results may be inadequate.

2. Study the motion numerically, determine that the theorem of corresponding motions is satisfied at the selected points, and compare the analytically predicted or suggested apertures with the calculated ones.

3. Do experiments at the selected points and compare the measured dynamic apertures with the predicted ones.

3. Many non-intersecting resonances.

When the motion is affected by three or more resonances that do not intersect in a point, there is little that can be said analytically. One may anticipate stochastic phenomena.

It is suggested that we try using the standard working point: $n_x = 7.139$, $n_z = 7.229$. This should be as good a place as any in the area, since it is presumably chosen to be away from troublesome resonances. We should excite a skew quadrupole, and one or more sextupoles and octupoles to provide driving terms for each of the resonances in the general area. If we cannot get enough excitation of one sextupole and octupole to bring the dynamic aperture inside the chamber, we can try several, spaced and phased to emphasize the desired driving terms.

There are now two steps:

1. Do tracking studies to find several different sets of excitations of the driving terms which are practical, and which bring the dynamic aperture inside the vacuum chamber.

2. Do the corresponding experiments on Aladdin and compare.

Note that one advantage of the suggested experiments is that we can inject, accelerate, and get set for the experiment, and then excite the driving terms slowly, watching the aperture as we go.

Reference

1. H. K. Meier and K. R. Symon, "Analytical and Computational Studies on the Interaction of a Sum and a Difference Resonance". Proc. Int. Conf. on High-Energy Accelerators and Instrumentation - CERN 1959, pp. 253-262.

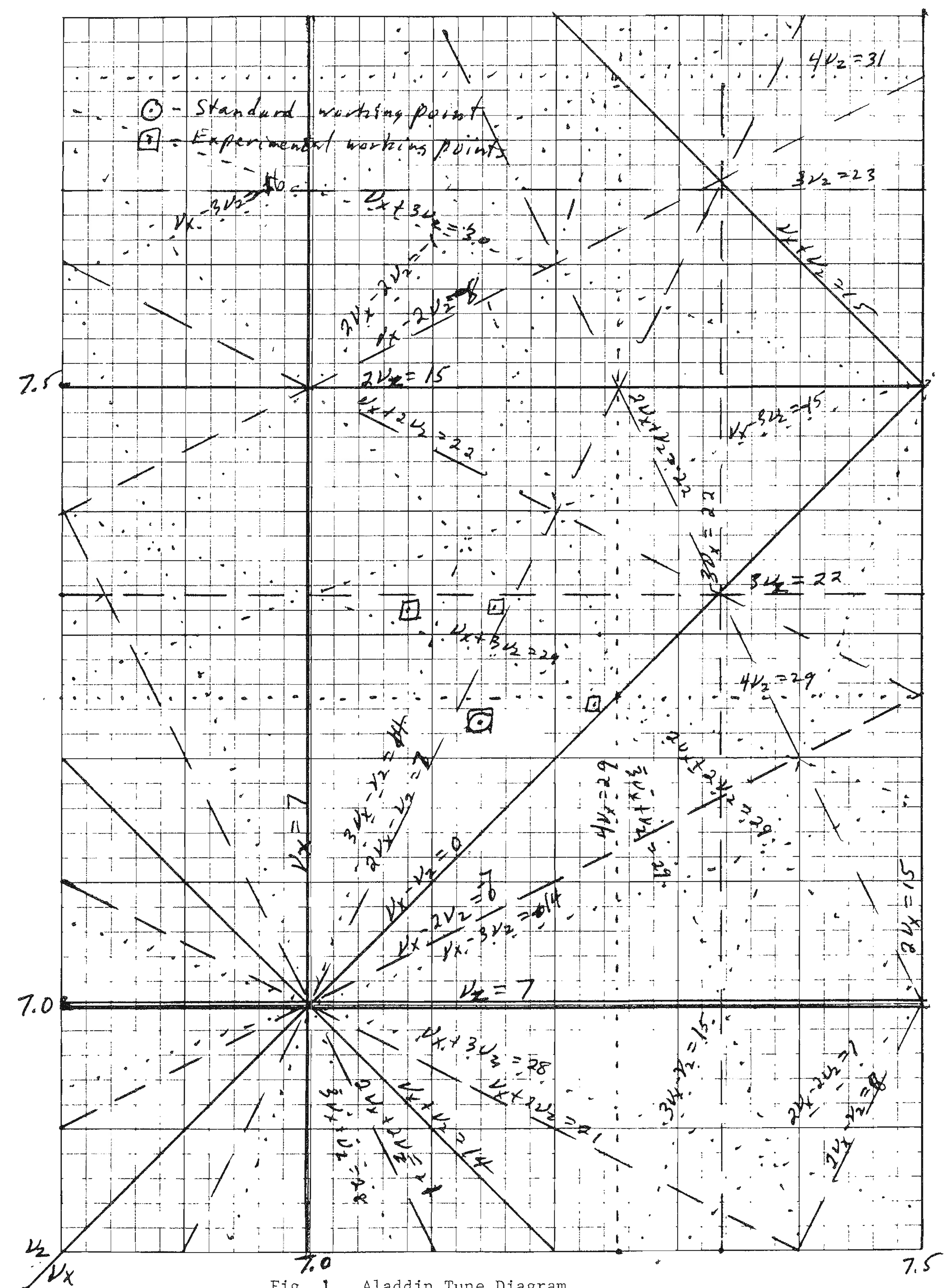

Fig. 1. Aladdin Tune Diagram