Undulator Parameters for the 7 GeV Option

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Advantages and disadvantages of the undulator parameters for the 7 GeV option are discussed. The first harmonic photon energy $E_1$ (keV) from an undulator in the forward direction is given by:

$$E_1 = \frac{0.948 E^2}{\lambda_u (1 + K^2/2)},$$

where $E$ (GeV) is the positron energy in the storage ring, $\lambda_u$ (cm) the undulator period, and $K = 0.934 B\lambda_u$ is the deflection parameter. When $E$ is increased from 6 to 7 GeV, $\lambda_u$ should be reduced by a factor of 1.36 to obtain the same photon energy for a given value of $K$.

In Fig. 1, $K$ values of the hybrid and pure SmCo$_5$ in the range of 0.2~3.5 are plotted as a function of the undulator gap in the range of 1.0~8.0 cm for several $\lambda_u$. Starting from $\lambda_u = 1.6$ cm, each $\lambda_u$ is selected by multiplying $\sim 1.36$.

In Figs. 2 and 3, the tunable ranges of $E_1$ for those selected $\lambda_u$ vs the gap are plotted for the case of 6 and 7 GeV, respectively. One observes that $\lambda_u$ of 6 GeV and 1.36 $\lambda_u$ of 7 GeV have approximately the same tunable range of $E_1$ except for the following four aspects.

1. In the 7 GeV case, there is an additional tunable range between the gap of 1.0~1.6 cm. This is a real advantage for $\lambda_u < 5$ cm and $E_1 > 1$ keV.

2. Since $K$ is proportional to $B\lambda_u$, and $B$ is a function of $\exp(-\text{gap}/\lambda_u)$, the tunable range of $E_1$ in 7 GeV is 1.36 times less sensitive with respect to the variation of the gap. In other words, for a given energy range $E_1$, the gap variation in 7 GeV is 1-36 times larger. This may mean that the tolerance of the gap in 7 GeV is slightly less stringent.

3. In the 7 GeV case, the dimensions of the undulators are 1.36 times larger. This may be an advantage in tolerance aspects of the dimensions and
magnetic field quality. The volume of the magnetic materials, however, increases by a factor of 2. This may affect the material costs.

4. Since the $\lambda_u$ in 7 GeV is 1.36 times larger, the number of undulator period $N$ for a given length of straight section is reduced by the same factor. This will reduce the brilliance by a factor of ~ 0.69 (6 m straight section in 6 GeV is equivalent to 8 m straight section in 7 GeV to obtain the same brilliance). In Fig. 4, the two cases are compared using the same positron beam parameters ($\sigma_x = 0.405$ mm, $\sigma_y = 0.098$ mm, $\sigma'_x = 0.018$ mrad, and $\sigma'_y = 0.007$ mrad). Figure 5 shows the variation of the brilliance at the first harmonic as a function of $N$ for fixed values of $\lambda_u = 4.0$ cm and $K = 0.55$. For $N > 100$, the brilliances are proportional to $N^{1.1}$ and $N^{1.25}$ for the single particle approximation and actual spectrum calculation, respectively.

The comparison for the same $K$ and the same photon energy is summarized as:

<table>
<thead>
<tr>
<th>Period, $\lambda_u$</th>
<th>6 GeV</th>
<th>7 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda_u$</td>
<td>$1.36 \lambda_u$</td>
</tr>
<tr>
<td>Gap, $g$</td>
<td>$g$</td>
<td>$0.6 + 1.36 \ g$</td>
</tr>
<tr>
<td>Brilliance for 5 m undulator</td>
<td>1</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Fig. 1. Deflection parameter $K$ vs undulator gap for hybrid SmCo$_5$. For $\text{Gap}/\lambda_u > 0.6$, magnetic fields for pure SmCo$_5$ are used.
Fig. 2. The first harmonic photon energy vs undulator gap for $E = 6$ GeV.
Fig. 3. The first harmonic photon energy vs undulator gap for $E = 7$ GeV.

Additional tunable range compared to 6 GeV is between the gap of 1.0 ~ 1.6 cm.
Fig. 4. Comparison of the brilliance of 6 GeV and 7 GeV.
Fig. 5. Brilliance variation as a function of the undulator period.

From single particle approximation
\[ \sim N^{1.1} \]

From spectrum calculation
\[ \sim N^{1.25} \]

- \[ E = 7 \text{ GeV} \]
- \[ \lambda_u = 4.0 \text{ cm} \]
- \[ K = 0.55 \]
- \[ \text{Gap} = 3.0 \text{ cm} \]
- \[ E_1 = 10 \text{ keV} \]