Using echo effect in accelerators: how to measure diffusion and multiply harmonics.

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Outline of the talk

- Echo effect in various media, spatial echo
- Transverse echo effect in accelerators
- Echo experiments in accelerator
- Using echo effect for seeding free electron laser
- Echo-seeding experiment at SLAC
- Noise amplification in seeding
- Conclusions

In this talk I would like to demonstrate how a simple and universal idea of the echo effect can be applied to accelerators, and how it can be put to use for practical applications. It is also an example of a fun topic in accelerator physics.

Echo in various media

Echo can be observed in a medium without dissipation which exhibits oscillations or waves. Usually these oscillations damp down due phase mixing, or decoherence, (Landau damping) caused by a frequency spread in the ensemble of the oscillators.

Excite the frequency ω_1, wait until the oscillation damps down, excite the frequency ω_2. The echo signal at the frequency $n\omega_2 \pm m\omega_1$ can be observed long after the first two excitations.

Echo effect in various media

- Nuclear magnetic spin echo in solids (1950)
- Photon echo in solids and gases (1964)
- Plasma wave echo (1967)
- Echo in a liquid with gas bubbles (1983)
- Echo in accelerators (1991), spin echo in accelerators (1995), echo effect for seeding FEL (2009)

Shadow echo

A simple illustration of the echo mechanism from the book by B. Kadomtsev, “Collective phenomena in plasma”.

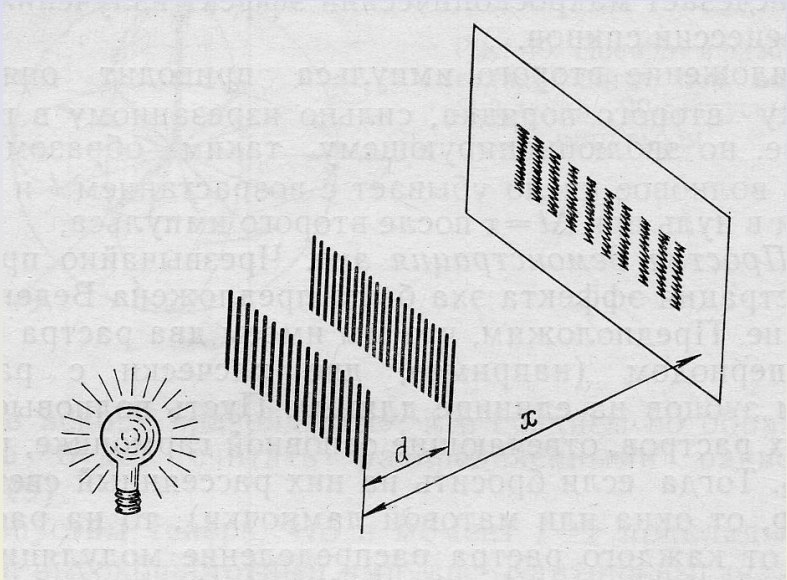
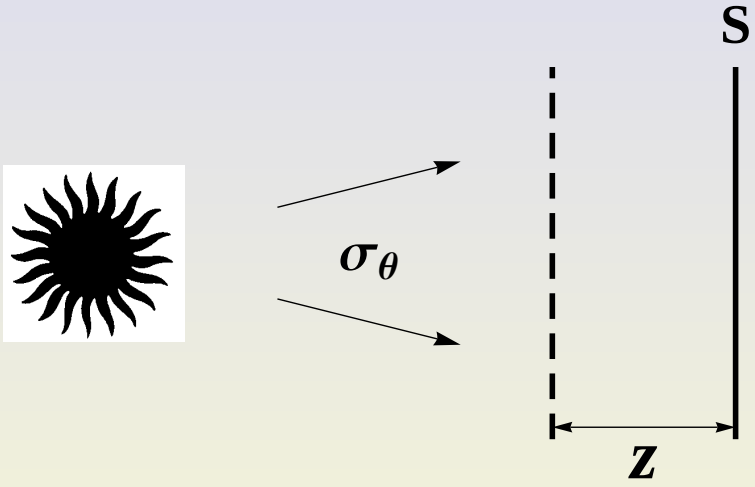


Рис. 50. Эхо на рассеянном свете

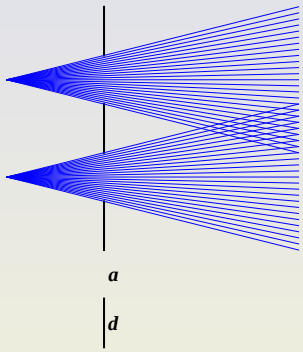
We expect that echo develops in time, but in this example the role of time is played by the distance x.

Mask is illuminated by light source

A thin mask with periodic slits of period $p = a + d$ is uniformly illuminated by light with angular spread σ_θ. Neglect diffraction and interference (small wavelength $\lambda \to 0$). The image is observed at the screen S.



Mask is illuminated by a light source

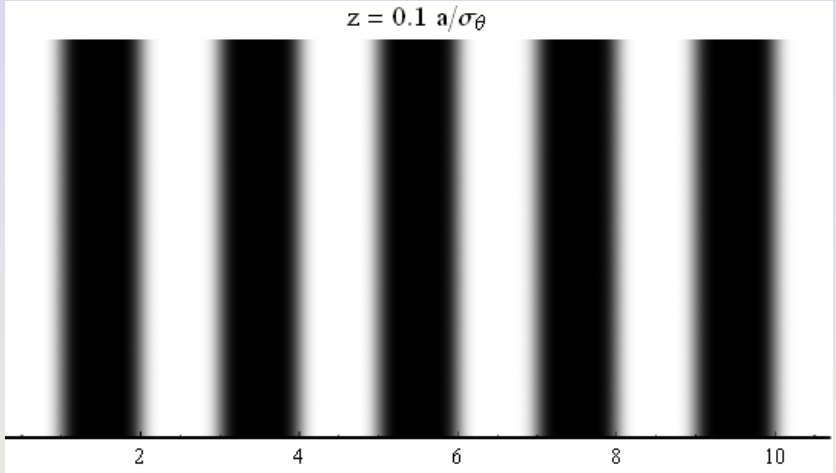


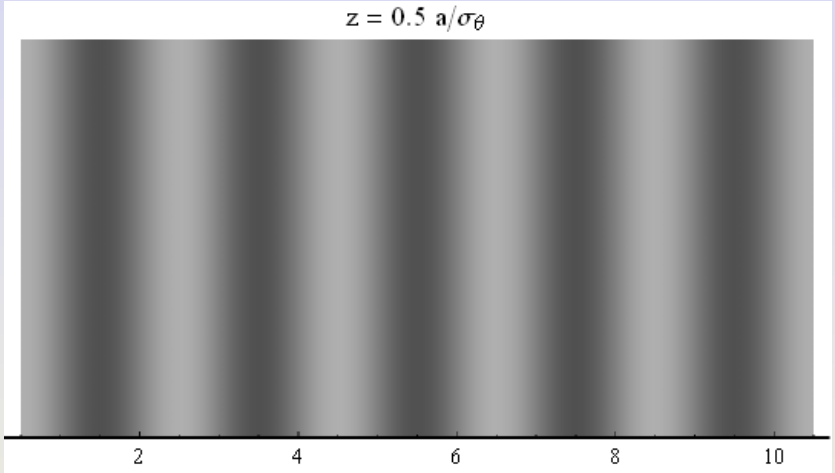
Rays begin to overlap at distance $z \sim d/\sigma_\theta$

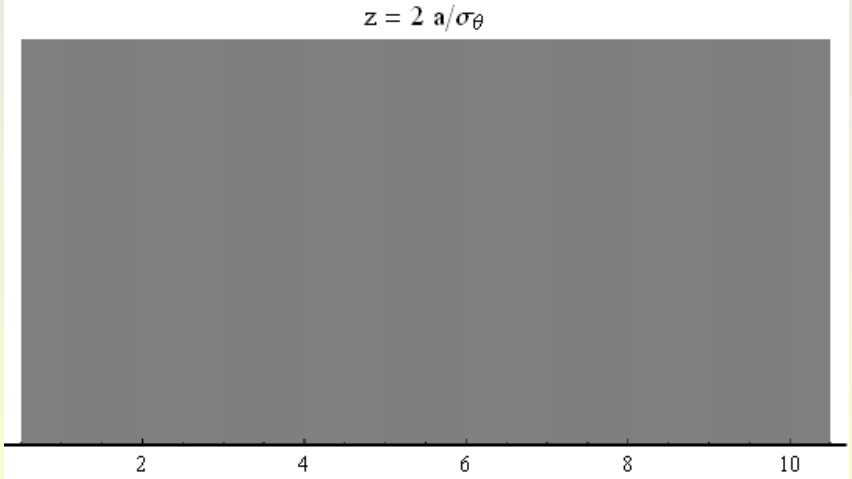
A simple numerical model can be easily simulated on a computer.

Screen patterns at various distances

One mask, varying position of the screen (slit width $a = p/2$)

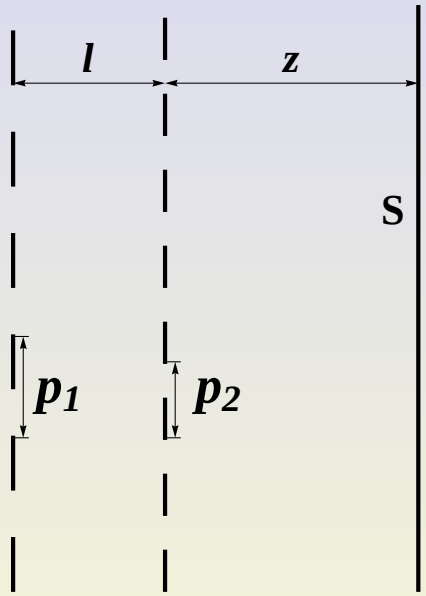






Shadow echo

Add another screen with period p_2 at distance l from the first one ($l, z \gg d/\sigma_\theta$).



If the screen is at the right position, one can observe a pattern on the screen—*shadow echo*:

$$z = \frac{n/p_1}{m/p_2 - n/p_1} l,$$

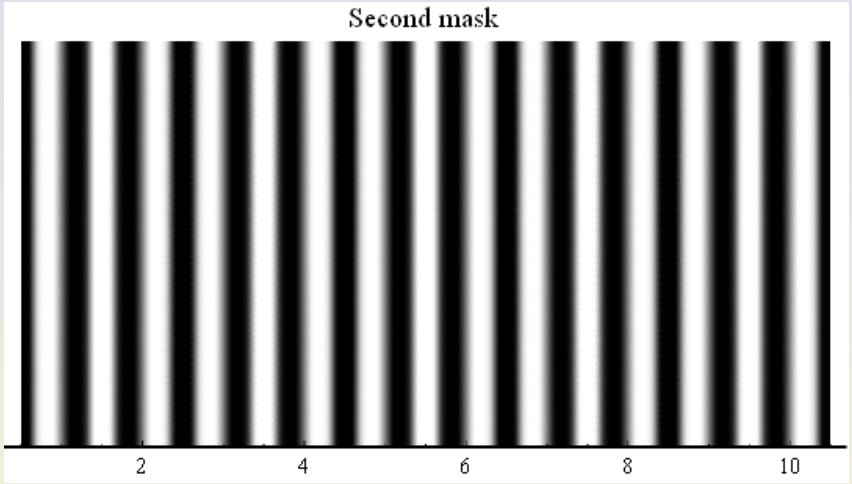
where m and n are integers.

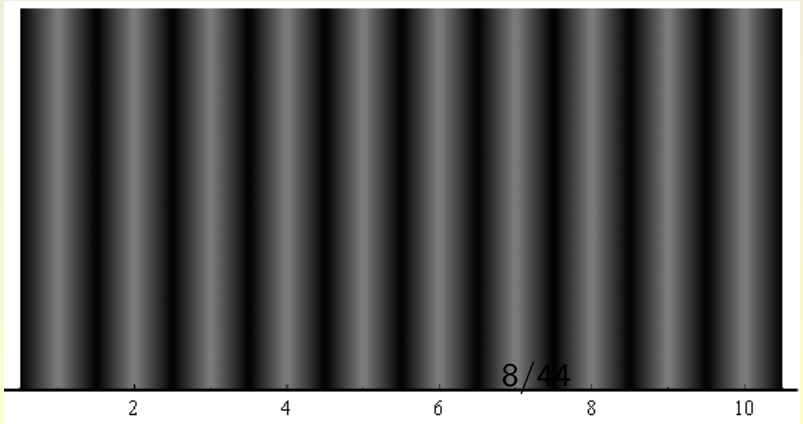
The patten has period p

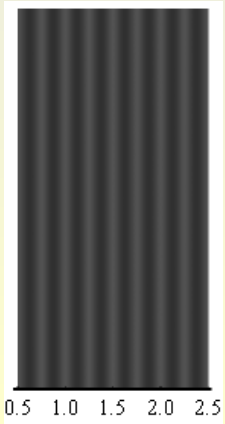
$$\frac{1}{p} = \left| \frac{n}{p_1} - \frac{m}{p_2} \right|.$$

Shadow echo

Example: $p_2 = \frac{1}{3}p_1$. For $m, n = 1$, echo should be observed at $z = \frac{1}{2}l$ with the period $p = \frac{1}{2}p_1$. For $m = 3$, $n = 1$, echo is observed at $z = \frac{1}{8}l$ with the period $p = \frac{1}{8}p_1$.

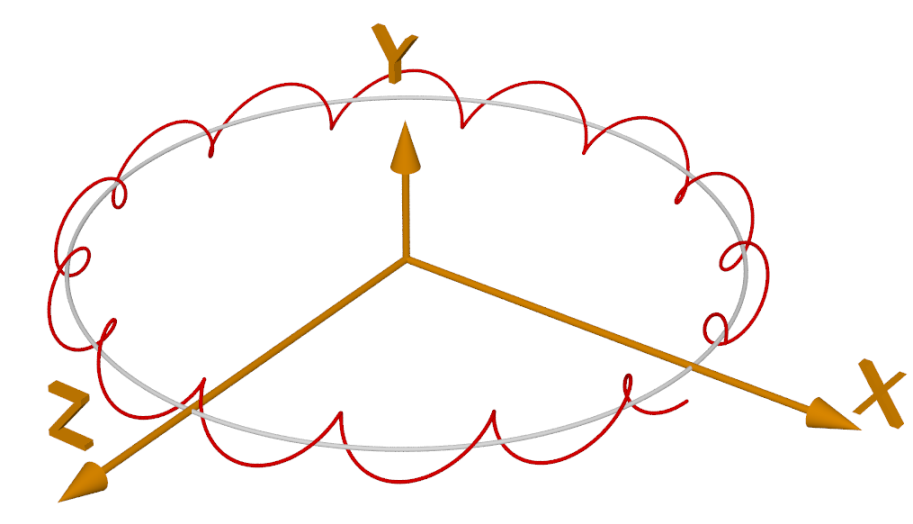






Oscillations in circular accelerators

Each particle in a circular accelerator executes oscillations around an equilibrium orbit.

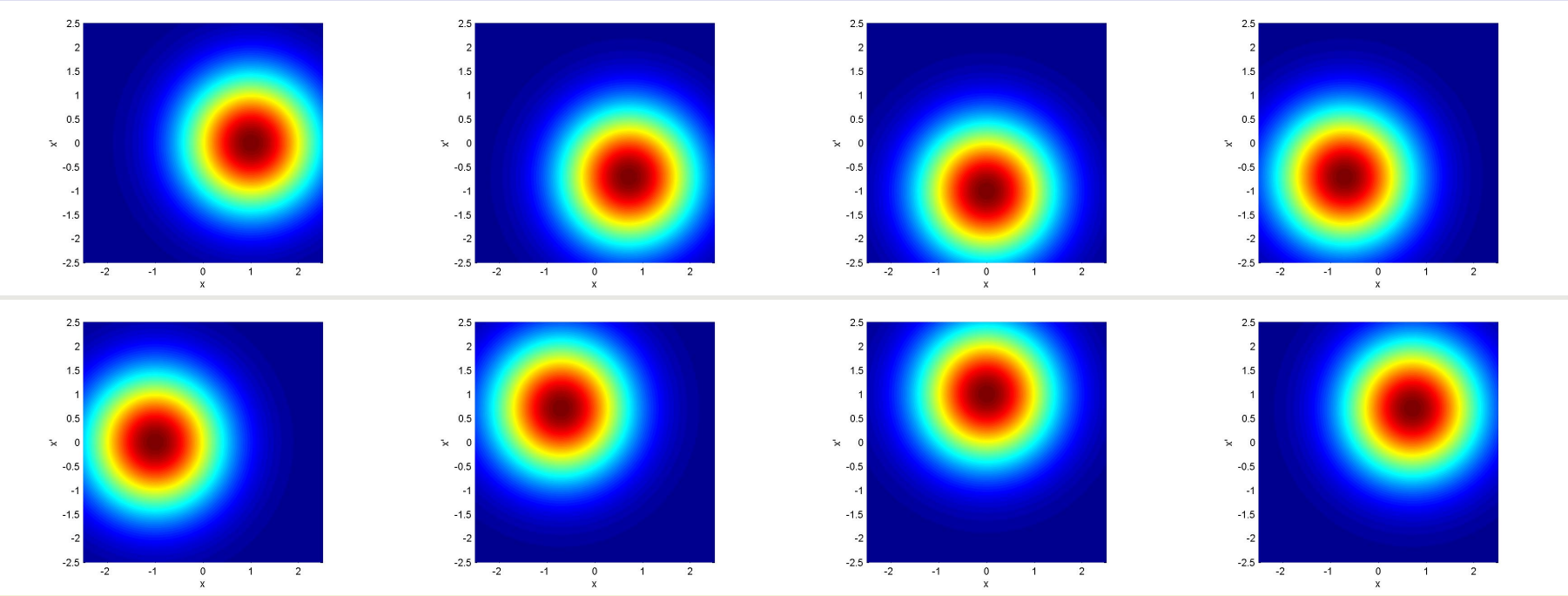


Transverse oscillations are called *betatron* oscillations, and longitudinal oscillations are called *synchrotron* one. The frequency of oscillations depends on particle's energy and amplitude.

A perturbed beam of particles starts to oscillate as a whole until the oscillations damp out due to phase mixing, or decoherence effect. Real damping and diffusion is very small in hadron accelerators, where the synchrotron radiation effects are negligible. Hence, one can expect an echo effect in accelerators.

Linear oscillations of the beam

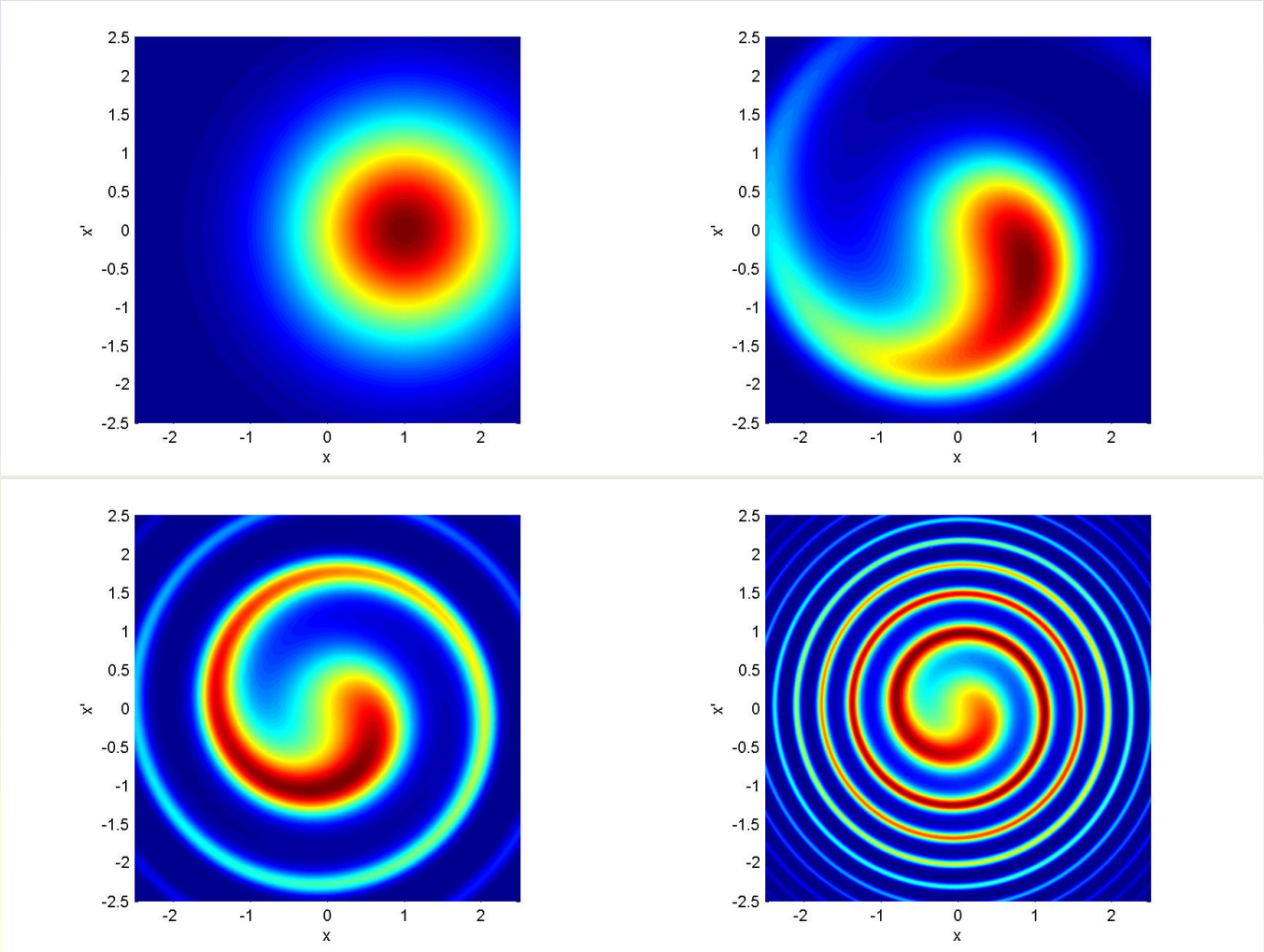
The beam is injected with an offset. In linear approximation the phase space rotates.



Beam position monitors in the ring would measure continuing oscillations with the betatron frequency.

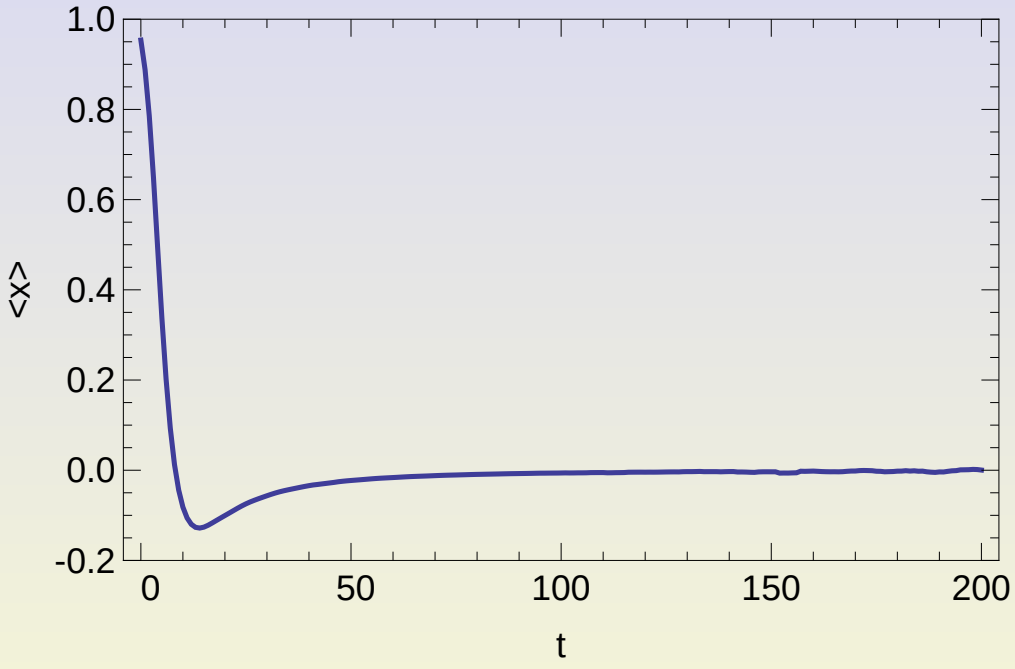
The rotation frequency depends on the amplitude

Phase space at $t = 0, 10, 30, 100$.



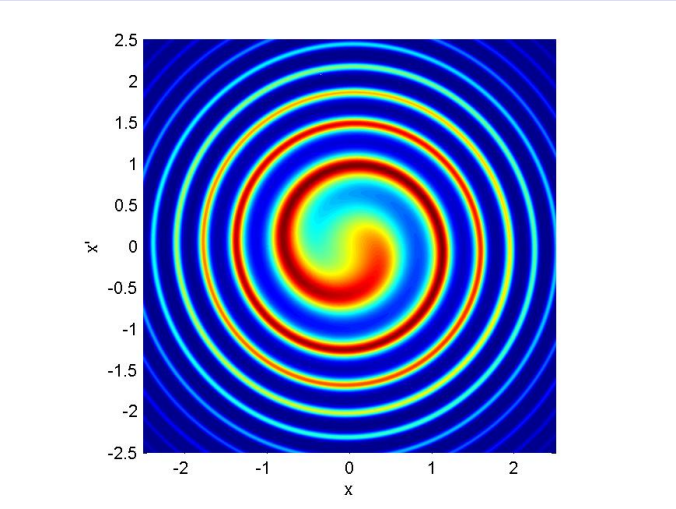
Averaged offset of the beam

Averaged offset $\langle x \rangle$ as a function of time



Quad kick

To observe echo, one has to make a pulsed quadrupole kick to the beam after it decoheres (Stupakov, 1991).



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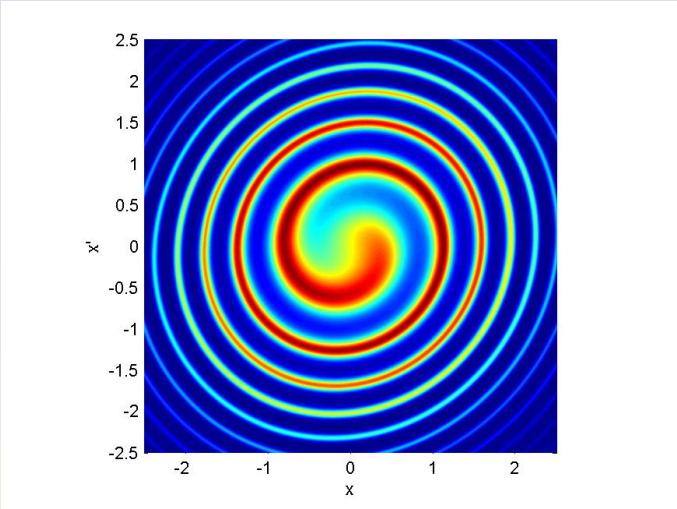
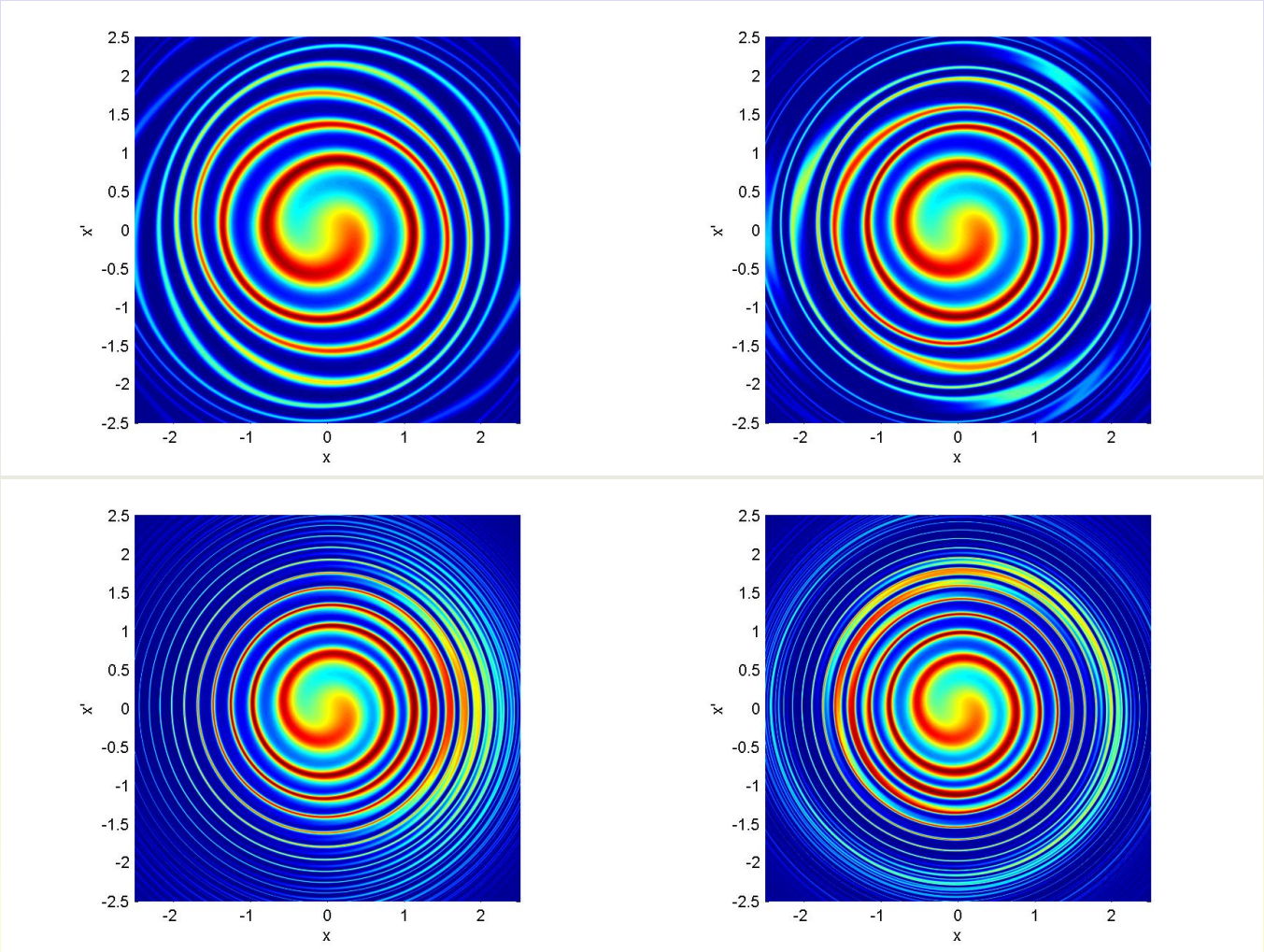


At time $t = 100$ a quadrupole kick is applied

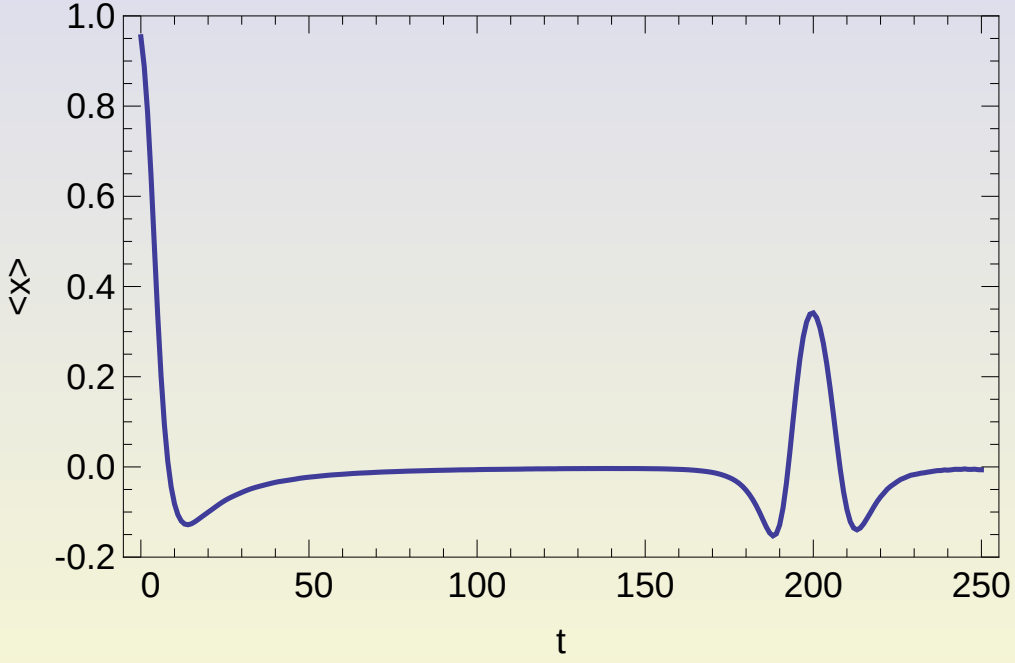
Evolution after the quad kick

Phase space at $t = 110, 130, 200, 230$



Averaged offset for the beam

Averaged offset $\langle x \rangle$ after the quad kick: the signal at $t \approx 200$ is the echo effect.

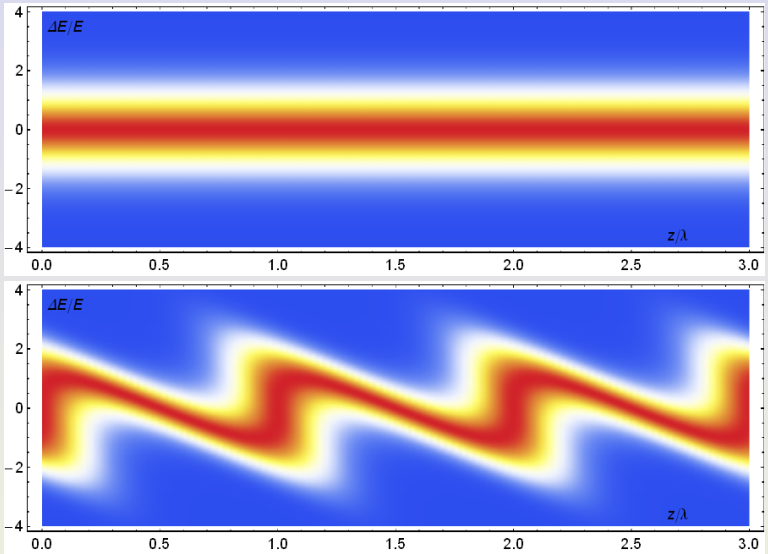


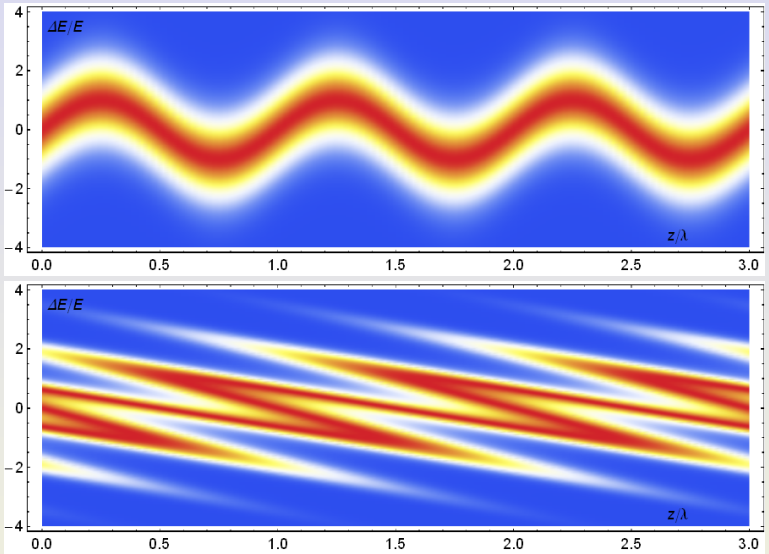
Modulated energy of the beam

First echo experiment in accelerators was performed in 1996 using the longitudinal degree of freedom of the beam. It involves the deviation of the longitudinal coordinate z (relative to a reference particle with nominal parameters) and the energy deviation $\Delta E = E - E_0$ from the nominal one. Two key elements are involved: *energy modulation* of the beam and a *slippage* effect in the ring.

Longitudinal phase space of the beam

Particles with larger energy travel along the orbit with larger radius and lag behind - the *slippage* effect.

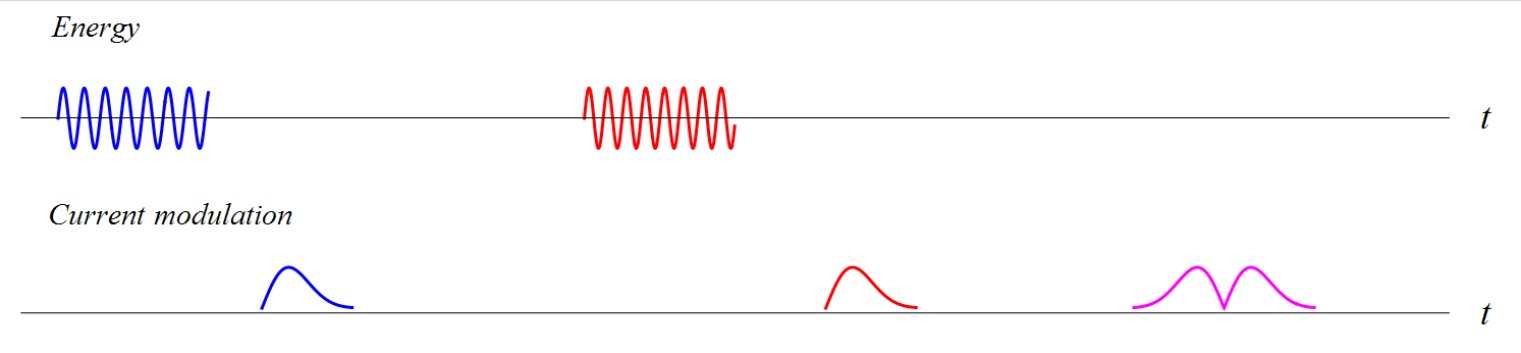




In fig. 3 the beam current is modulated along z, and can be measured as current modulation at a given location in the ring. In fig. 4 the current modulation disappears due to smearing out of the phase space.

Longitudinal echo in hadron accelerators

Experimental setup for echo observation.



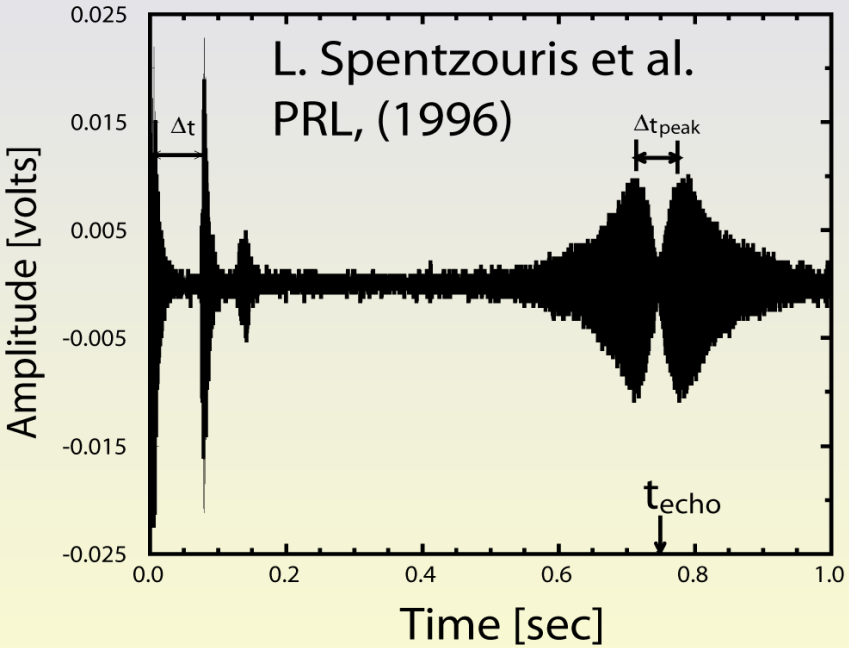
Apply RF kick at frequency $\omega_1 = h_1\omega_0$ (ω_0 is the revolution frequency)
Wait time $\tau \gg \tau_{\mathrm{damp}}$
Apply RF kick at frequency $\omega_2 = h_2\omega_0$
Wait time $\Delta t \gg \tau_{\mathrm{damp}}$
Measure beam modulation at a combination frequency. The lowest order echo is observed at the difference frequency $|h_2 - h_1|\omega_0$ at time

$$t_{\mathrm{echo}} = \frac{h_2}{|h_2 - h_1|}\tau$$

(similar to the spatial echo).

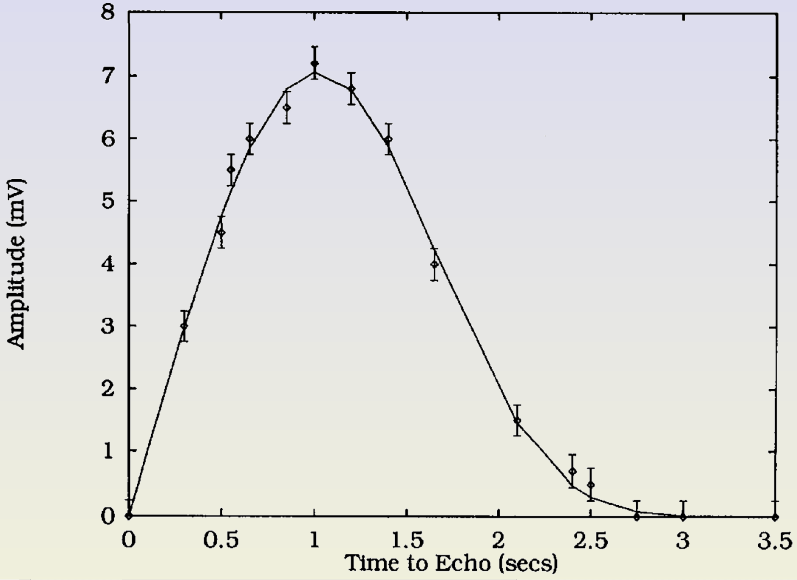
Echo experiment at Fermilab and CERN

Experiment at FNAL, at the Antiproton Accumulator with a coasting beam at 8.7 GeV. Ring parameters: $f_{rev} = 687$ kHz, $\Delta E = 3.2$ MeV, $\eta = 0.023$, $\Delta E_{mod}/\Delta E = 0.01$. The beam was modulated at $h_1 = 9$ and $h_2 = 10$. The echo signal was observed at $h = 1$.



Echo experiment at Fermilab and CERN

The echo signal is very sensitive to diffusion in the phase space when t_{echo} becomes large.



The measured collision rate is $3 \cdot 10^{-4}$ s.
Experiments on the CERN SPS, $E = 120$ GeV, ring where able to measure the diffusion time $\sim 1/3$ year. Echo experiments were also carried out in HERA at DESY and RHIC at BNL (with gold beams).

LCLS parameters

LCLS parameters

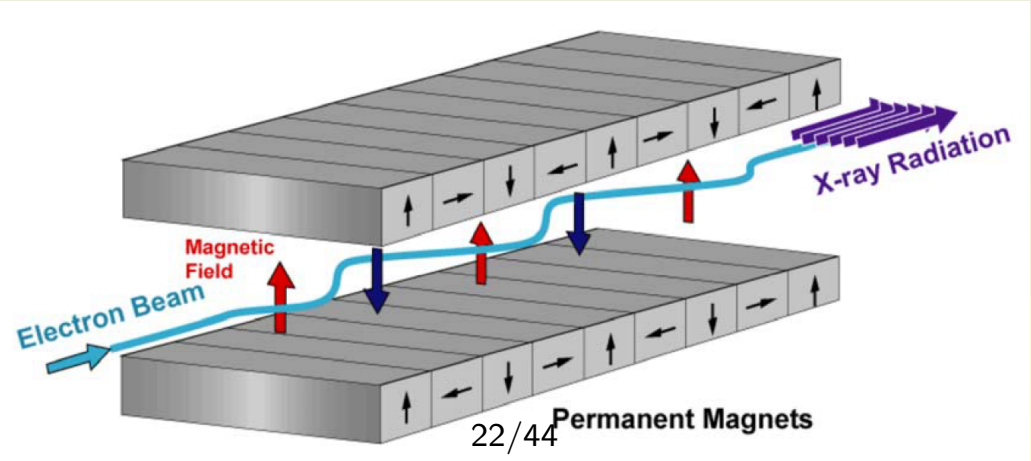
Electron beam energy	$< 13.6\,\mathrm{GeV}$
Electron bunch length	$3(?) - 100\,\mu\mathrm{m}$
Electron bunch transverse size	$\approx 30\,\mu\mathrm{m}$
Laser wavelength	0.15-1.5 nm
Photon number	$2 \cdot 10^{12}$
FEL power up to	40 GW

Electron velocity $v \approx c$, the Lorentz factor

$$\gamma = 1/\sqrt{1 - v^2/c^2} \approx 27000$$

The x-ray beam in the LCLS has excellent transverse coherence properties, however rather poor longitudinal coherence.

Essential element of FEL is undulator



FEL process viewed from the beam frame

Consider the FEL process in the beam reference frame. The bunch length in the beam frame

$$\tilde{l}_b = \gamma l_b = 27000 \times 20\,\mu\mathrm{m} \approx 54\,\mathrm{cm}$$

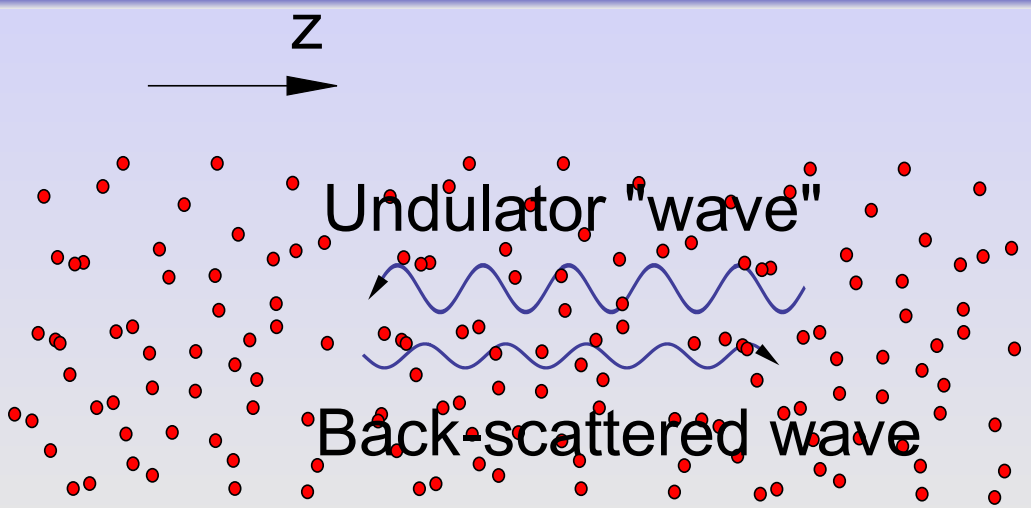
The undulator length in the beam frame

$$\tilde{l}_u = \frac{l_u}{\gamma} = \frac{100\,\mathrm{m}}{27000} \approx 4\,\mathrm{mm}$$

The magnetic field of the undulator is amplified by a factor of γ, $\tilde{B} = \gamma B_u = 27000 \times 1.3$ T. In addition, the beam sees the transverse electric field of the same magnitude $\tilde{E} \approx \gamma B$ (CGS system of units). In the limit $\gamma \gg 1$ the resulting field is very close to the field of a plane electromagnetic wave. It has wavelength

$$\tilde{\lambda} = \frac{\lambda_u}{\gamma} = \frac{3\,\mathrm{cm}}{27000} \approx 1\,\mu\mathrm{m}$$

FEL process viewed from the beam frame

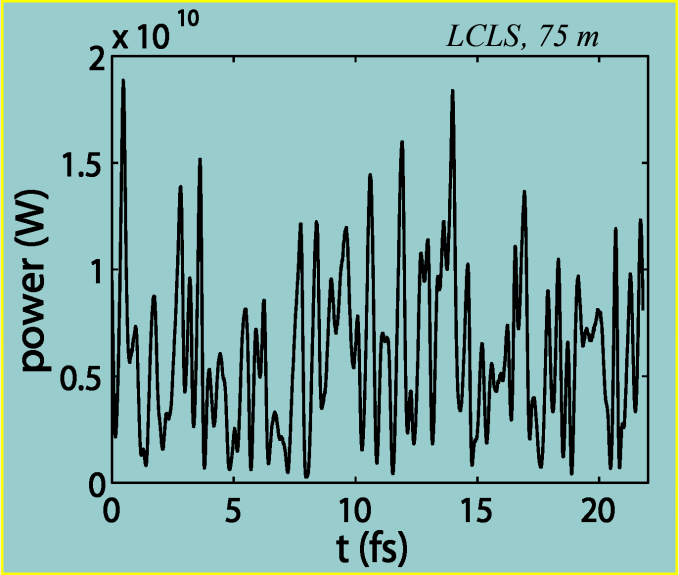


Thomson scattering: a backscattered wave combined with the incident field creates a standing wave. The ponderomotive force in this standing wave forces electrons to bunch in potential wells along z with the period $\pi/\tilde{k} \rightarrow$. This enhances the backscattering $\rightarrow$ increases bunching. The amplitude of the backscattered wave grows exponentially with time/length. In the lab frame

$$\lambda_{\mathrm{rad}} = \frac{\lambda_u}{2\gamma^2}(1 + K^2/2)$$

SASE FEL and problem of seeding

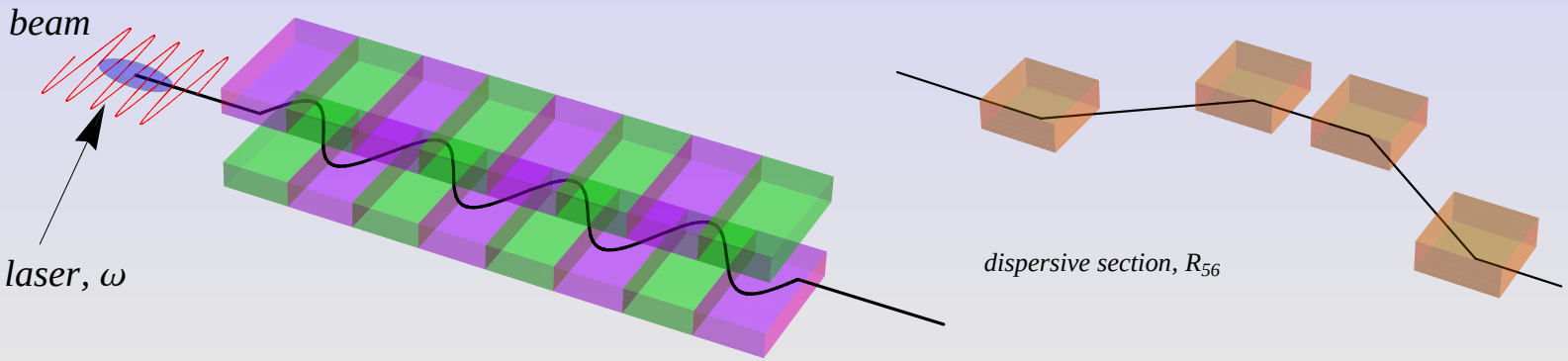
The SASE FEL (Self-Amplified Spontaneous Emission) radiation starts from initial shot noise in the beam, with the resulting radiation having an excellent spatial coherence, but a rather poor temporal one.



There are several approaches to introduce a coherent "seed" to an FEL. Echo effect in the beam can be used for seeding (Stupakov, PRL, **102**, 074801, 2009). One can use a frequency up-shift mechanism of the echo effect to generate high harmonics of the initial modulation.

How to modulate the beam energy and introduce slippage

One needs energy modulation and the slippage effect.

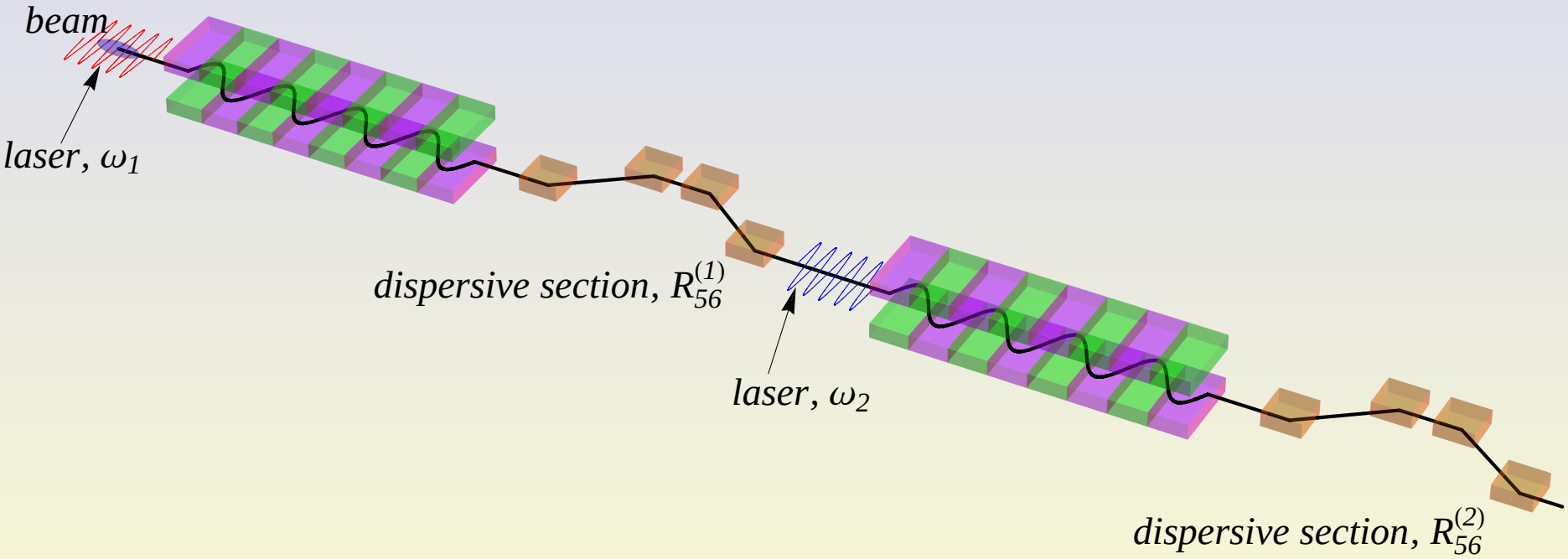


$$\omega_L = \frac{2k_u c\gamma^2}{1 + K^2/2}$$

The laser-beam interaction in the undulator generates energy modulation in the beam at the laser wavelength with some amplitude ΔE_{mod}. Four magnets (chicane) introduce variable slippage in the beam for particles of difference energies.

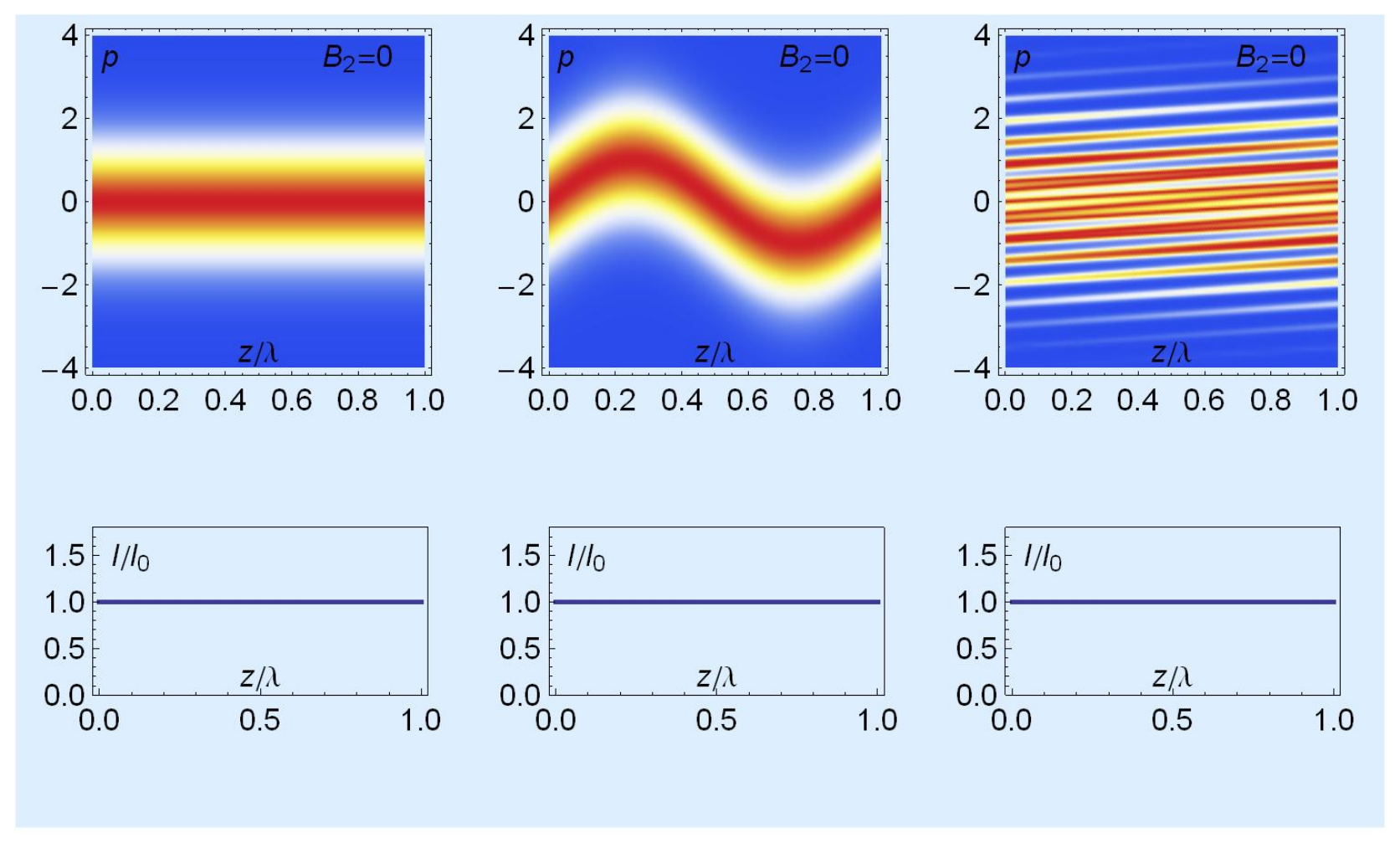
Using echo effect for seeding

Beam echo experiment requires two undulators, two chicanes and a laser beam. The beam propagates from left to right.

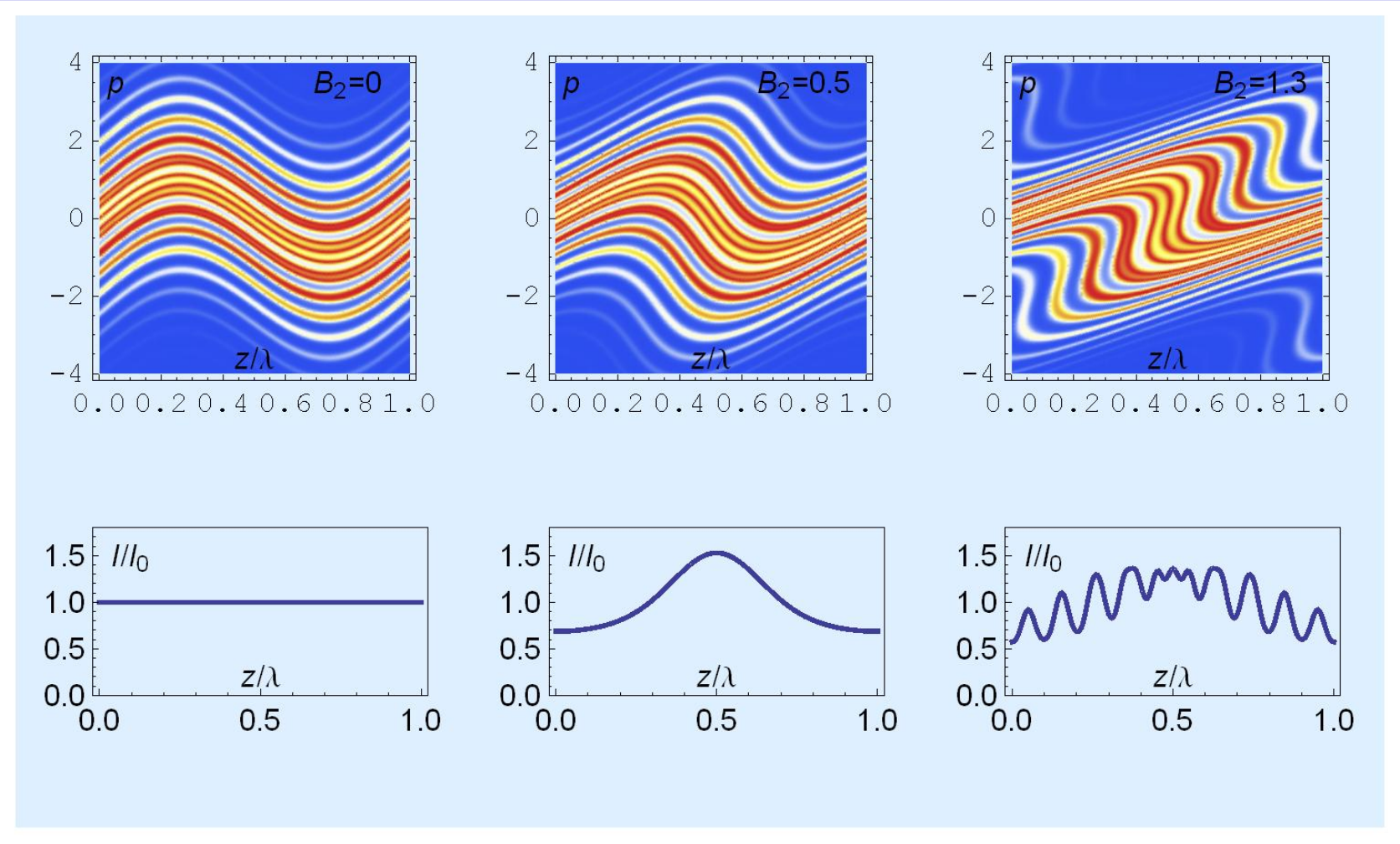


The modulated beam at the exit from the modulator is sent to an FEL undulator-radiator (not shown).

Phase plots at the first stage



Phase plots at the second stage



Seed at the 10th harmonic of the laser frequency with the modulation amplitude $\sim 16\%$.

EEHG promise

Echo Enabled Harmonic Generation (EEHG) seems promising for generation harmonics numbers up to 40-100. The harmonic amplitude scales as $\sim n^{-1/3}$. With the seed laser wavelength of about 200 nm, one can achieve seeding of soft x-ray FELs. Typical parameters

Electron beam energy	1-2 GeV
Energy spread	100-200 keV
Laser wavelength	200 nm
Harmonic number	20-100
FEL wavelength	a few nm

Several soft xray FEL projects in the world can benefit from the EEHG: FERMI@ELETTRA (Trieste, Italy), SwissFEL (PSI, Switzerland), NLS (UK), LBNL FEL (USA) ....

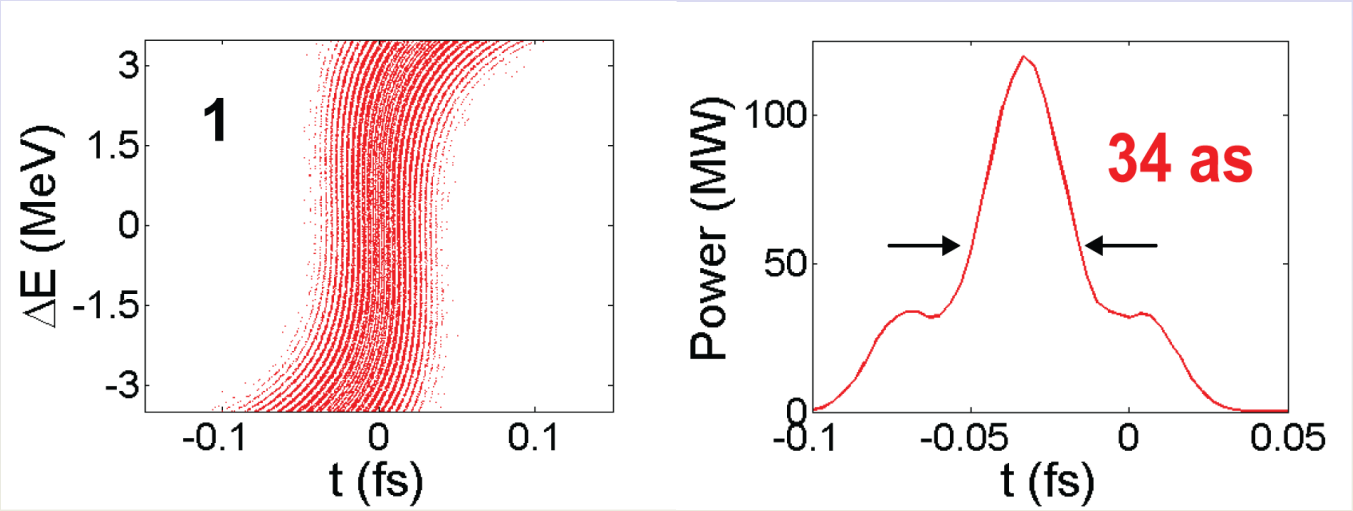
EEHG challenges

- High tolerances for the magnetic field of all elements, and ability to correct the field
- Good overlapping of laser beam with the electron beam
- Control of the phase space of the initial beam (beam emittance, energy spread, energy chirp, etc.)
- Particle interaction in the beam via Coulomb forces (space charge) and radiation (CSR) fields can lead to deterioration of the echo signal and noise amplification in the process of seeding

All these factor will eventually limit the maximally achievable harmonic number.

Attosecond pulses in EEHG

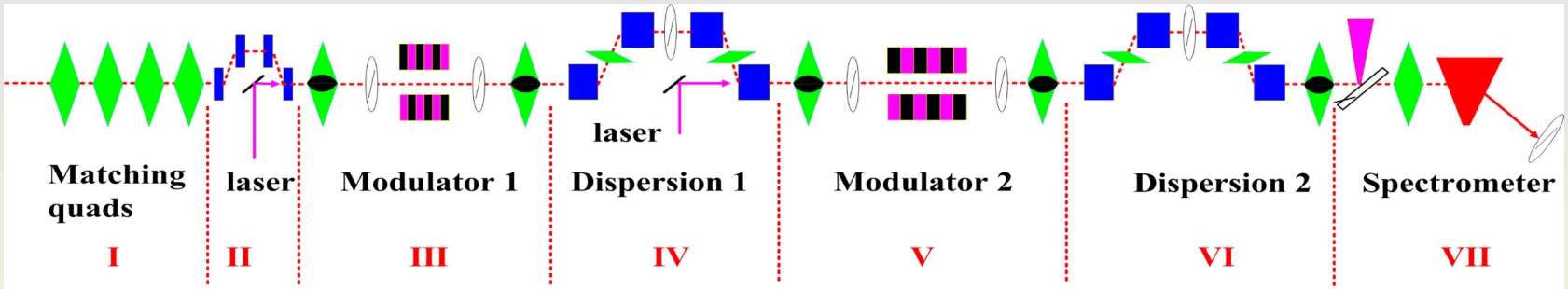
From D. Xiang et al. PRST-AB, **12**, 060701, 2009.



With some additional elements in the system one can generate an ultrashort, attosecond radiation from the seeded beam.

EEHG proof-of-principle experiment at SLAC

A proof-of-principe experiment to demonstrate EEHG is currently being conducted at SLAC at the NLCTA facility. NLCTA is equipped with an S-band injector (to ~ 100 pC), an X-band linac ($60 - 200$ MeV) and Ti:Sapphire laser systems. There is about 8-10 m of free space for a new experiment.



Layout of the ECHO-7 at the NLCTA.

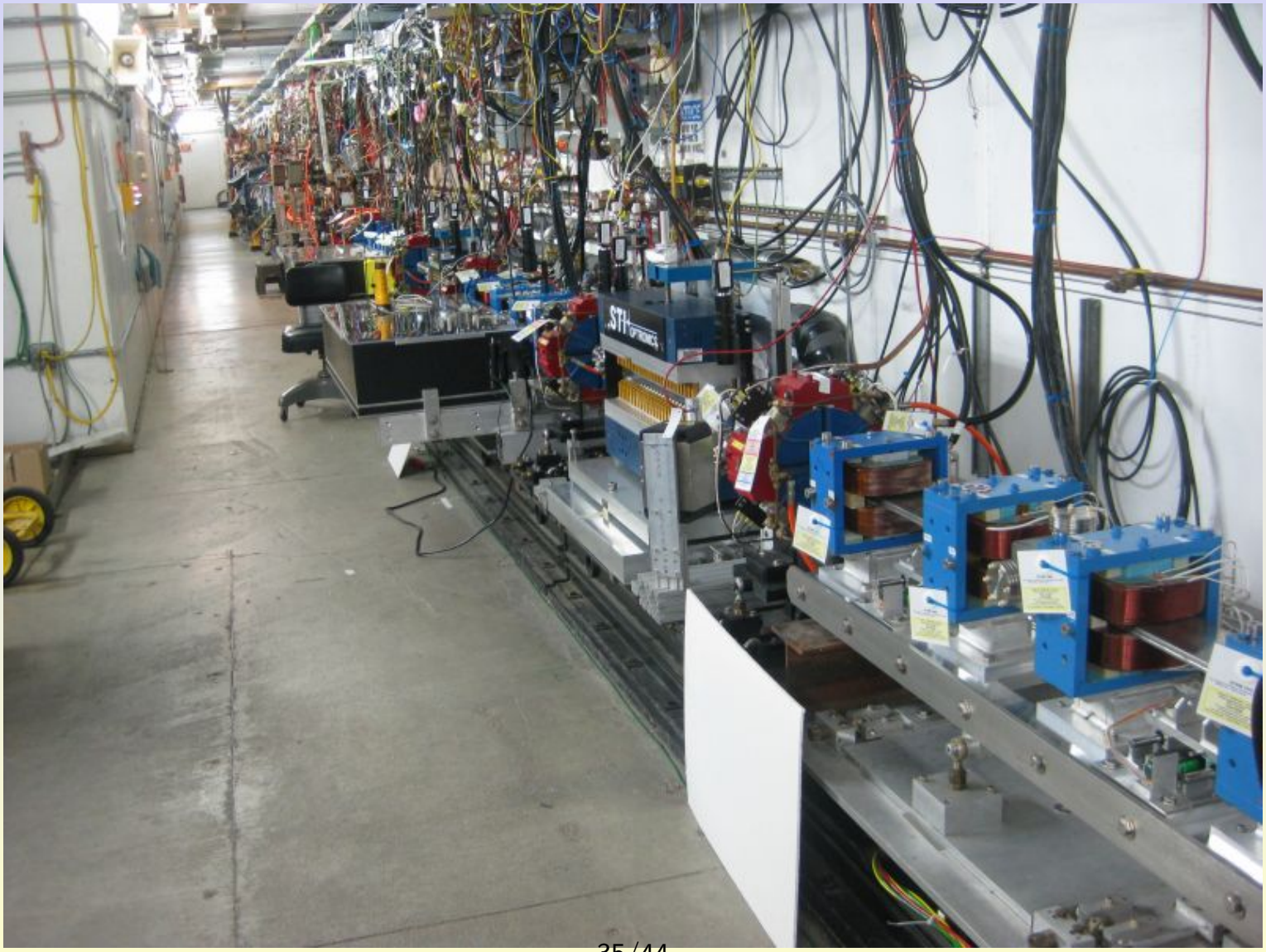
Echo experiment at NLCTA at SLAC

The parameters of the experiment:

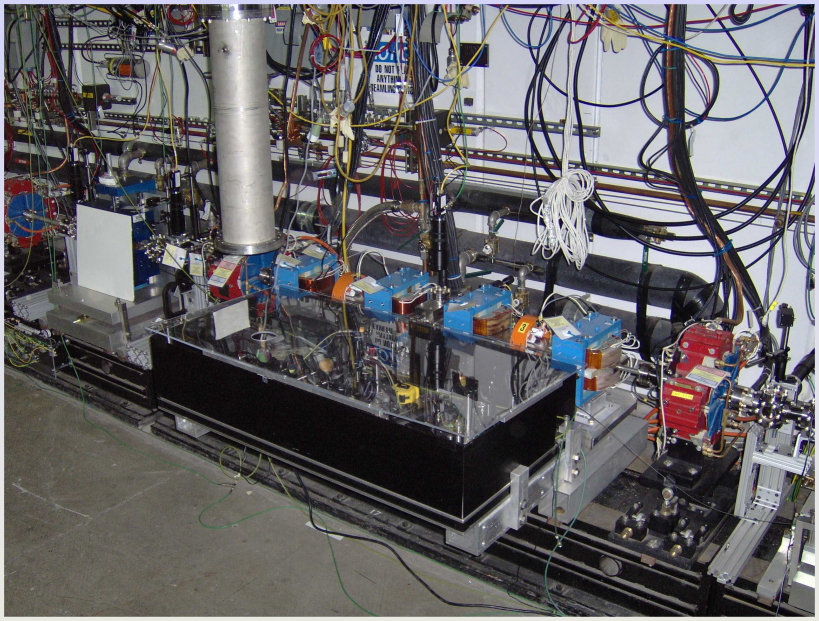
Electron beam energy	120 MeV
Normalized emittance	$< 8\,\mu$m
Slice energy spread	< 10 keV
First laser wavelength	795 nm
Second laser wavelength	1590 nm

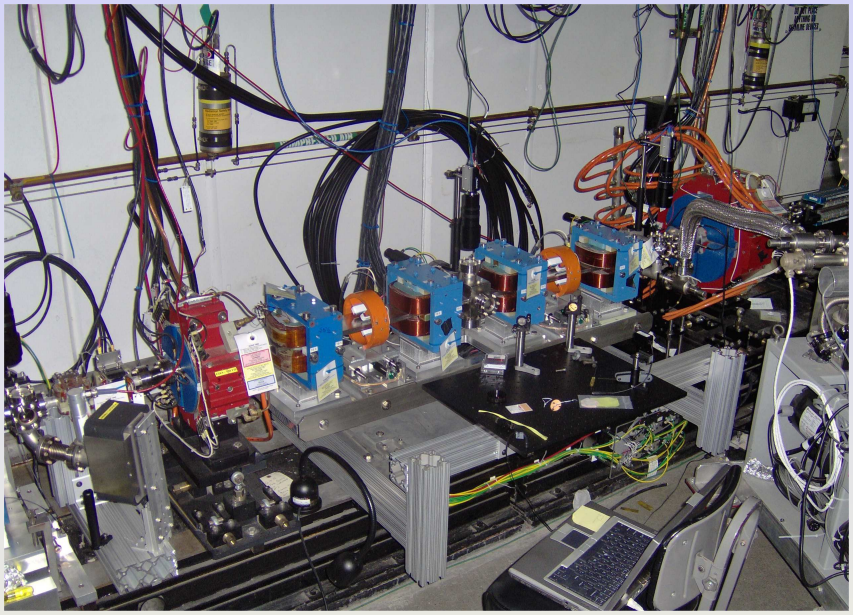
The echo induced modulation at 7th harmonic of the second laser (227 nm) will be generated in the beam.

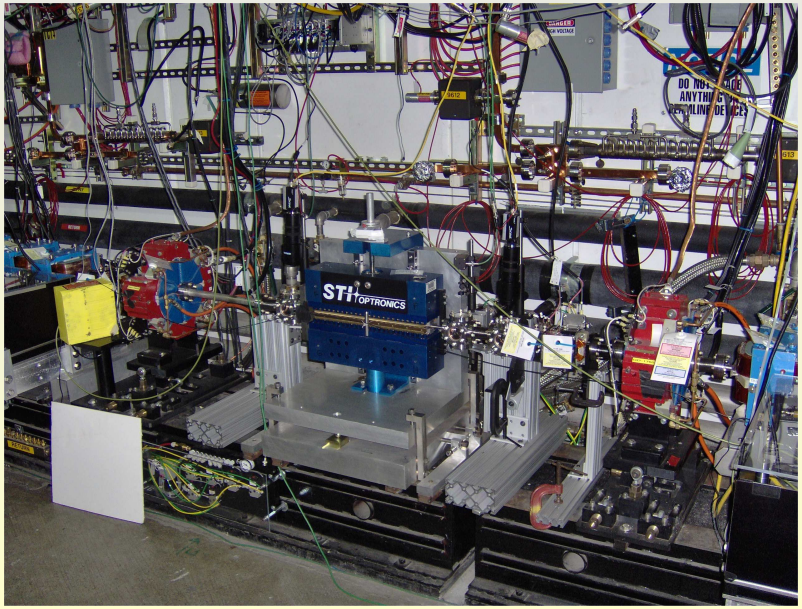
Experimental demonstration of the echo seeding at SLAC

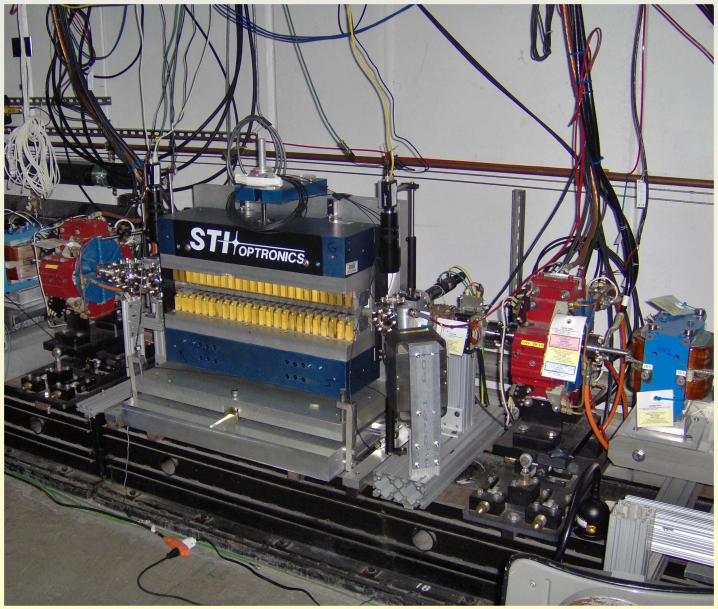


Experimental demonstration of the echo seeding at SLAC









Noise and radiation form-factor

We characterize the noise in the beam by its ability to radiate above the shot noise level. Radiation at frequency ω of an ensemble of N particles in 1D model is controlled by

$$\mathcal{F}(\omega) = \left| \sum_{j}^{N} e^{ikz_j} \right|^2 = \sum_{j,l=1}^{N} e^{ik(z_j - z_l)},$$

where $k = \omega/c$, and z_j is the z coordinate of j-th particle after the seeding device.

Separate the terms with $j = l$

$$N + \sum_{j \neq l} e^{ik(z_j - z_l)}$$

The first term is the shot noise.

$$F(k) = \frac{1}{N} \sum_{j \neq l} e^{ik(z_j - z_l)}$$

Interaction in the process of modulation

The beam is modulated in energy at wavelength λ_0 and then sent through a chicane with the strength R_{56}.

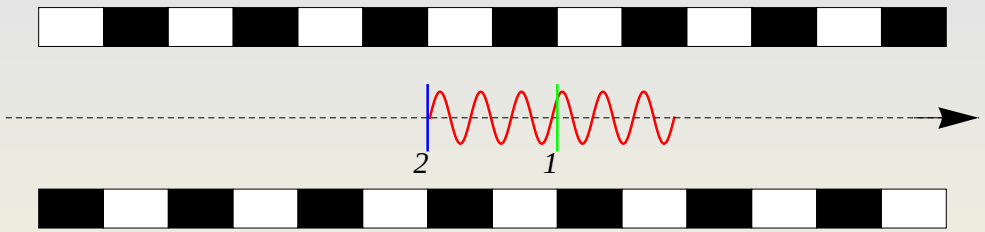
$$\hat{\eta}_i = \eta_i - \Delta\eta \sin(k_0 z_i) + \sum_{j \neq i} h(z_i - z_j)$$

$$\hat{z}_i = z_i + R_{56}\left[\eta_i - \Delta\eta \sin(k_0 z_i) + \sum_{j \neq i} h(z_i - z_j)\right]$$

$\Delta\eta$ is the amplitude of the energy modulation, $k_0 = 2\pi/\lambda_0$. h is the energy exchange function between the two particles in the undulator.

Interaction of particles in the undulator

Consider particles 1 and 2 of the beam traveling through an undulator, 1 is ahead of 2, $\zeta = z_1 - z_2 > 0$. The second particle emits electromagnetic wave (undulator radiation) which goes ahead and interacts with the first particle and changes its relative energy by $h(\zeta)$.



Interaction of particles in the undulator

Assuming a helical undulator, the energy change is (Stupakov&Krinsky, PAC03)

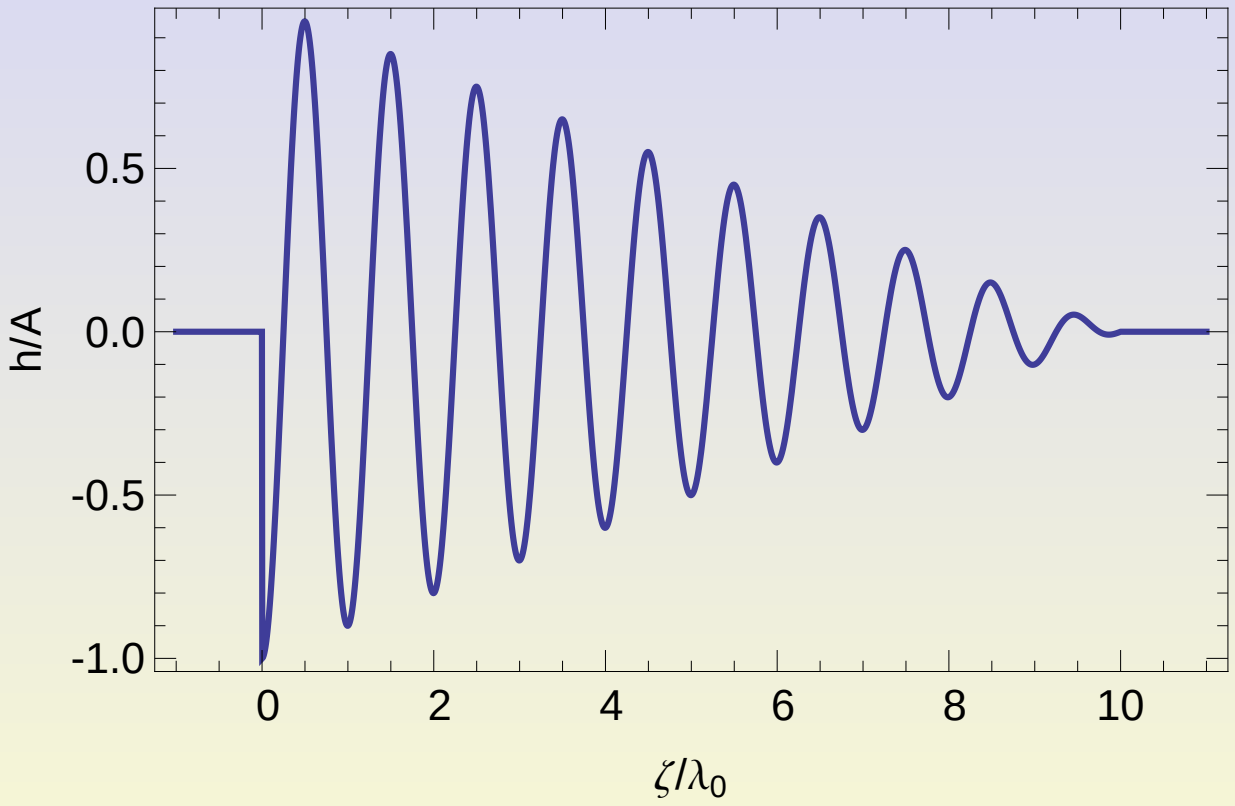
$$h(\zeta) = \begin{cases} -A\left(1 - \frac{\zeta}{\lambda_0 N_u}\right)\cos k_0\zeta, & N_u\lambda_0 > \zeta > 0 \\ 0, & N_u\lambda_0 < \zeta \text{ or } \zeta < 0 \end{cases}$$

where the parameter A is

$$A = 4\pi\frac{r_e L_u}{S\gamma}\frac{K^2}{1+K^2}$$

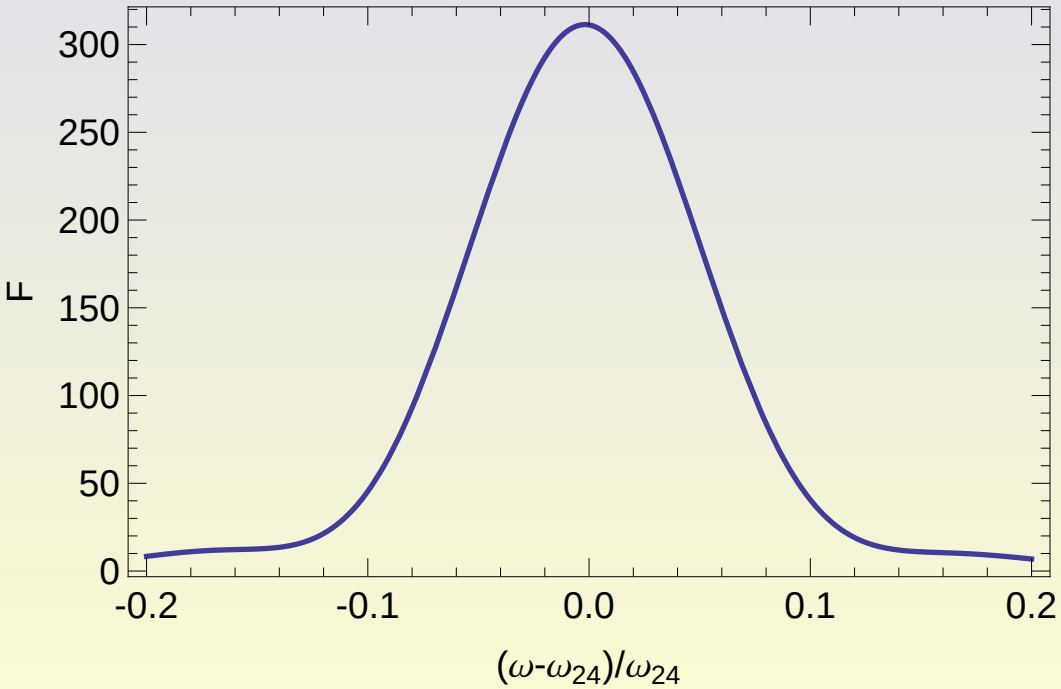
with L_u the number of undulator periods, K the undulator parameter, S the transverse area of the beam.

Interaction of particles in the undulator



Noise amplification in EEHG

Typical EEHG parameters: $E = 1.2$ GeV, $I_p = 800$ A, $\lambda_0 = 240$ nm, $N_u = 9$, $K = 1.84$, $\sigma_x = 100$ microns, $R_{56}^{(1)} = 8.2$ mm, $R_{56}^{(1)} = 0.35$ mm.



Conclusions

- The echo effect can be observed in various media, which, as we now understand, also include beams in accelerators. We've learnt that relativistic beams, in many cases, have a good "memory" of fine structures in the phase space, and the information about these structures can be recovered in a properly designed experiment.
- Several experiments on various hadron accelerators in the 1990th demonstrated that echo can be used for measuring weak dissipative or diffusive properties of the hadron beams.
- Recent theoretical developments indicate a promising feature of the echo effect as harmonic generator for seeded FELs for generation of soft x-rays. At SLAC we are now working on experimental demonstration of the echo-enabled harmonic generation.