Recirculating Linear Accelerators for Neutrino Factories

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`http://keil.home.cern.ch/keil/MuMu/Doc/ANL-Aug02/anltalk.pdf`

Programme – Educational Talk on RLA's

- Super-conducting RF system
  - Modules
  - RF cavity phasing
  - Beam loading
  - Bunch area and distortion
- Linac lattices
- Arc lattices: Choices, variation of arc parameters with energy
- Caveats
  - Discuss issues rather than solutions
  - Analytical results and *Mathematica* procedures applicable to any RLA
  - Do not pretend that examples are close to optimum
  - No off-crest acceleration
  - No spreaders, recombiners, peelers between linacs and arcs
  - Many unanswered questions

Performance parameters

† Muon flux $\dot{N}$/s	10^{11}
† Ejection energy/GeV	20
∗ Injection energy/GeV	2
∗ Number of passes	4
Normalised transverse emittance/mm	0.328
∗ Relative RMS energy spread at 20 GeV	
Normalised longitudinal emittance/mm	8.28
Bunch length at 20 GeV/mm	5.36

- Daggers mark ν physics parameters that I don't discuss
- Asterisks mark accelerator parameters that I assume to be fixed
- Emittances a products of standard deviations
- For longitudinal emittance of muons 1 eVs is 2.8374 m
- 'Hope' to achieve $\dot{N}$ with reasonable beam power on target without cooling
- 'Hope' that this RLA is cheaper than in Study I and Study II

RF System Issues

Parameters of super-conducting RF cavities and cryomodules

- Cavities derived from SNS super-conducting cavities with known cost and performance, also used for 8 GeV linac in Fermilab Proton Driver Study
- Six-cell cavities stretched from $\beta \approx 0.81$ to $\beta = 1$; other lengths from SNS
- Number of modules for 2 linacs is a multiple of 4
- Lengths include cryogenic boxes at both ends
- Maximum RF voltages similar to SNS and 8 GeV Linac Design Study at FNAL

Frequency f_{RF}	805 MHz	Shunt impedance R/Q	485 Ω
Unloaded quality factor Q	$> 5 \cdot 10^9$	Beam port radius a	37.9 mm

Cavities	1	2	3	4
Length/m	2.358	3.951	5.544	7.137
Voltage/MV	25	50	75	100
Number of modules	192	96	64	48

RF cavity phasing

- Six-cell cavities designed for speed $c\beta_d \approx c$
- Neighbouring six-cell cavities spaced $\lambda_{\mathrm{RF}}\beta_d$ apart
- Muons travel at speed $c\beta_p < c\beta_d$
- Effective acceleration in multi-cell cavity with π phase advance/cell, $\delta\beta = \beta_d - \beta_p$, and $n = 1, 2, 3$ labelling the cells downstream from the centre, to second order in $\delta\beta$ and n:

$$V = \frac{2\hat{V}}{\pi}\left[1 - \frac{\delta\beta}{\beta_d} - \frac{1}{2}\left(\frac{\pi\delta\beta}{2\beta_d}\right)^2\left(5 - 16n + 16n^2\right)\right]$$

- At entrance of first turn in first linac, where β_p is smallest, absolute values of 2nd and 3rd term at most $1.4 \cdot 10^{-3}$ and $3.2 \cdot 10^{-4}$, respectively.
- Cavity spacing makes vector sum of voltages smaller than scalar sum, find 0.946 for ratio on first turn in first linac, more for later turns
- Must adjust beam phase at linac entrances by choosing arc lengths

Beam loading in linac

Passage of muon bunches through RF cavities causes beam loading, i.e. extracts energy from energy W_s stored in RF cavities, and causes drop of accelerating voltage along bunch train.

- RF cavity parameters shunt impedance R, peak cavity voltage U and RF power P, unloaded quality factor Q , RF frequency f_{RF}, W_s and P are related by:

$$R = U^2/P \qquad\qquad Q = 2\pi f_{\mathrm{RF}} W_s/P$$

- Eliminating P and solving for W_s yields

$$W_s = U^2/[2\pi f_{\mathrm{RF}}(R/Q)] \approx 255 \text{ J}$$

- Compare W_s to energy W_e that passage of bunch train extracts from cavity. In NF with muon flux $\dot{N} \approx 10^{11}$/s, operating at repetition frequency $f_r \approx 4$ Hz we have:

$$W_e \geq \dot{N} e U/f_r \approx 0.1 \text{ J}$$

- Symbol $\geq$ takes care of muon decays between RLA and storage ring
- Note that 0.1 J/μs = 100 kW

Variation of acceleration along bunch train

Pass	0	1	2	3	4
W_s/J	255.78	255.68	255.58	255.48	255.38
U/MV	25	24.995	24.990	24.985	24.980
W_e/J	0.1	0.09998	0.09996	0.09994	0.09992

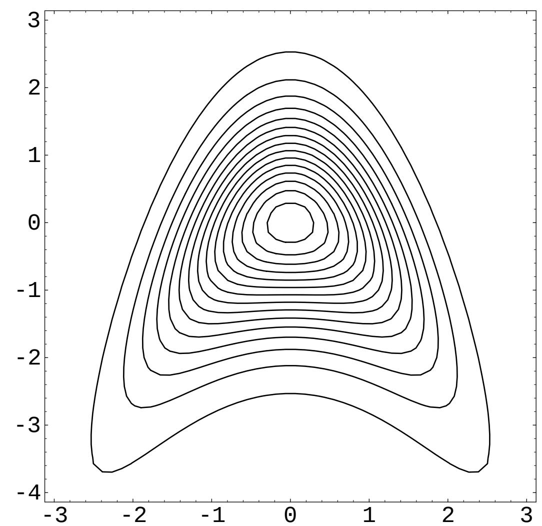
- Assume W_s in 0-th pass, calculate U and W_e
- Subtract W_e from W_s in $(n-1)$-th pass, get W_s in n-th pass
- Calculate U and W_e in n-th pass
- Head bunch gains sum of voltages from passes 0 to 4, i.e. $U = 99.97$ MV
- Tail bunch gains sum of voltages from passes 1 to 5, i.e. $U = 99.95$ MV
- Relative difference 0.02% small compared to energy spread in storage ring
- Calculation assumes cavities operated at resonant frequency
- There are better ways for beam loading calculations

Bunch distortion due to acceleration on crest I

- When bunches are accelerated on crest of RF wave form in an isochronous machine, muons that arrive earlier or later than centre of bunch are systematically accelerated less, described by normalised distribution $D(s,\eta)$:

$$D(s,\eta) = \frac{1}{2\pi\sigma_s\sigma_e} \exp\left[-\frac{1}{2}\left(\frac{s}{\sigma_s}\right)^2 - \frac{1}{2}\left(\frac{\eta + 2(\pi s/\lambda)^2}{\sigma_e}\right)^2\right]$$

- s is longitudinal coordinate along bunch, σ_s is standard deviation along s; η is relative energy error and σ_e its standard deviation in absence of distortion
- Distortion described by quadratic term with RF wavelength λ
- Mean $\bar{\eta}$ and standard deviation σ_{ed} of relative energy error in presence of distortion



$$\bar{\eta} = -\frac{2\pi^2\sigma_s^2}{\lambda^2} \qquad \sigma_{ed} = \sqrt{\frac{8\pi^4\sigma_s^4 + \lambda^4\sigma_e^2}{\lambda^4}}$$

Bunch distortion due to acceleration on crest II

- When energy spread in bunch σ_e vanishes, standard deviation σ_{ed} tends towards

$$\sigma_d = \frac{1}{\sqrt{2}} \left(\frac{2\pi\sigma_s}{\lambda} \right)^2 \quad (1)$$

- Distorted standard deviation σ_{ed} is obtained by adding σ_e and σ_d quadratically:

$$\sigma_{ed}^2 = \sigma_d^2 + \sigma_e^2 \quad (2)$$

- Banana-shaped distortion causes relation between σ_s and σ_{ed}, and upper limit on longitudinal emittance $\varepsilon_{\parallel}$
- In one set of conventional units, $\varepsilon_{\parallel}$ is defined as follows:

$$\varepsilon_{\parallel} = \sigma_e \gamma \sigma_s \quad (3)$$

- Argue that design of muon storage ring imposes an upper limit on σ_{ed}, and eliminate σ_e from (2) and (3). Introduce scaled variable $x = \sigma_s / \sigma_{s0}$ with maximum bunch length σ_{s0} when all energy spread σ_{ed} is due to distortion, by solving (1) for σ_{s0}:

$$\sigma_{s0} = \frac{\lambda}{2\pi} \sqrt{\sigma_{ed}\sqrt{2}}$$

Bunch distortion due to acceleration on crest III

- Longitudinal emittance $\varepsilon_{\|}$ is proportional to $\gamma\lambda$, and three halves power of σ_{ed} accepted by storage ring:

$$\varepsilon_{\|} = x\sqrt{1-x^4}\left(\frac{\gamma\lambda}{2\pi}\sqrt{\sigma_{ed}^3\sqrt{2}}\right)$$

- Maximum at $x = 3^{-1/4} \approx 0.759836$, when coefficient in front of large brackets has value $2^{1/2}3^{-3/4} \approx 0.620403$. For optimum bunch with maximum emittance $\varepsilon_{\|}$, bunch length σ_{os} becomes:

$$\sigma_{os} = \frac{\lambda}{2\pi}\sqrt{\sigma_{ed}\sqrt{2/3}}$$

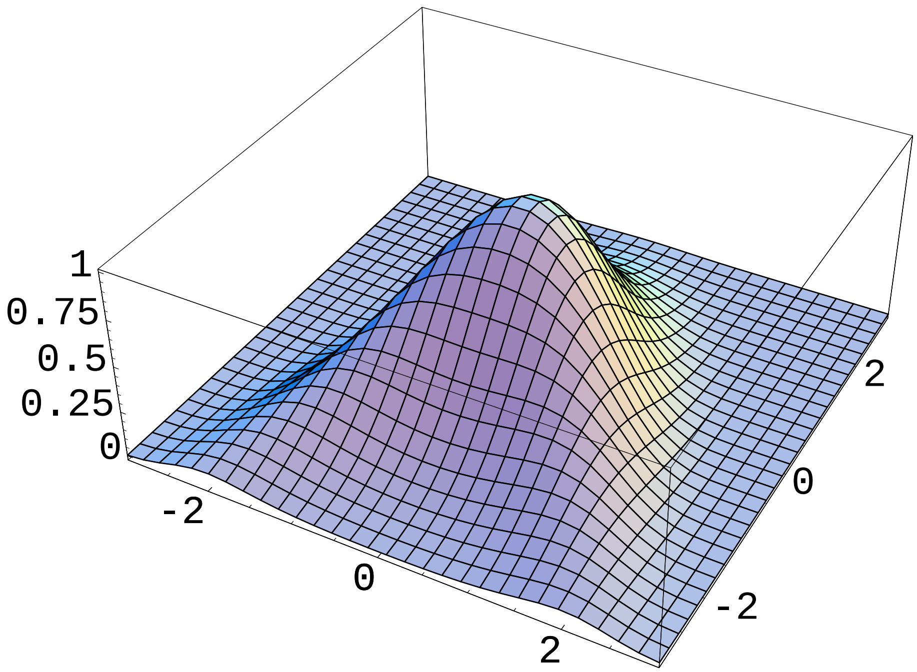
- Optimum value for σ_{od}:

$$\sigma_{od} = \sigma_{ed}/\sqrt{3}$$

- Optimum value of relative RMS energy spread in undistorted beam σ_{oe}:

$$\sigma_{oe} = \sigma_{ed}\sqrt{2/3}$$

Longitudinal emittance



E	20	GeV
λ	0.37241	m
σ_{ed}	0.01	
$\varepsilon_{\parallel}$	8.8278	mm
σ_{os}	5.36	mm
σ_{oe}	0.00816	

Discussion of the bunch shape

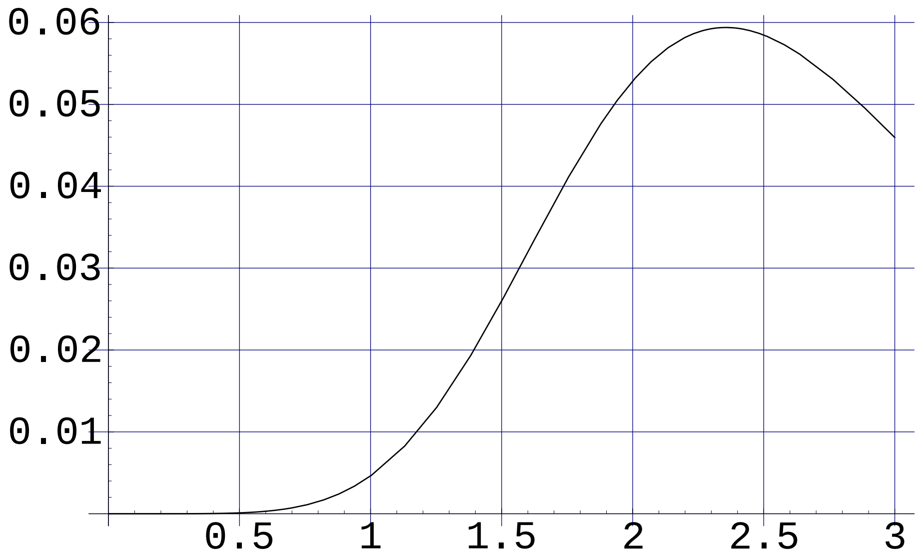
- Use scaled variables ξ and ζ. Measure bunch lengths in units of σ_{os}, and energy errors in units of σ_{oe}, such that level lines of the undistorted bunch density are circles. Distorted distribution function $F(\xi, \zeta)$ becomes:

$$F(\xi, \zeta) = \frac{1}{2\pi} \exp\left[-\frac{\xi^2}{2} - \frac{(\zeta + a\xi^2)^2}{2}\right]$$

- Parameter a describes distortion. It is relative energy difference between muons at centre of bunch and muons at $\pm$ an RMS bunch length σ_{os}, measured in units of RMS energy spread σ_{oe}. Numerically, find $a = 1/2$ for optimum bunches. Confirm this result analytically by comparing coefficients in distribution functions, and using transformations between scaled and unscaled variables.

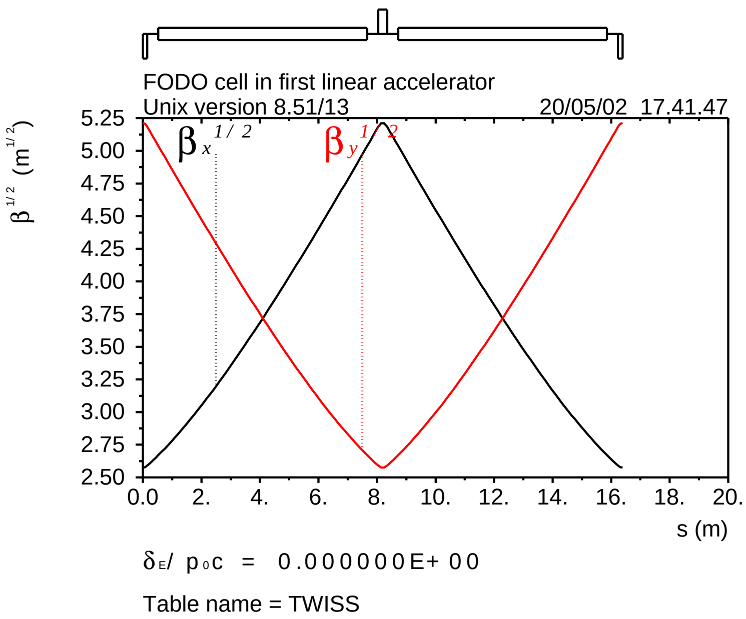
Contribution of bunch distortion to muon losses

What extra fraction of muons is lost due to banana-shaped distortion, compared to undistorted, circular bunches, when bunches are cut off at radius r? Since *Mathematica* did not find an analytical expression, I got answer numerically.



Lattice in Linacs

‘Golden’ cell FODO lattice for linac



- One cryomodule in half cell with 1 quadrupole and 1 to 4 RF cavities
- Length of half cell integer multiple of RF wavelength
- Phase advance

$$\sin(\mu/2) = (\sqrt{5} - 1)/2$$

in ‘Golden’ cells minimises $\hat{\beta}$ in focusing quadrupoles, and maximises transverse acceptance

- In thin-element approximation, ‘Golden’ cells with full period length L_p have:

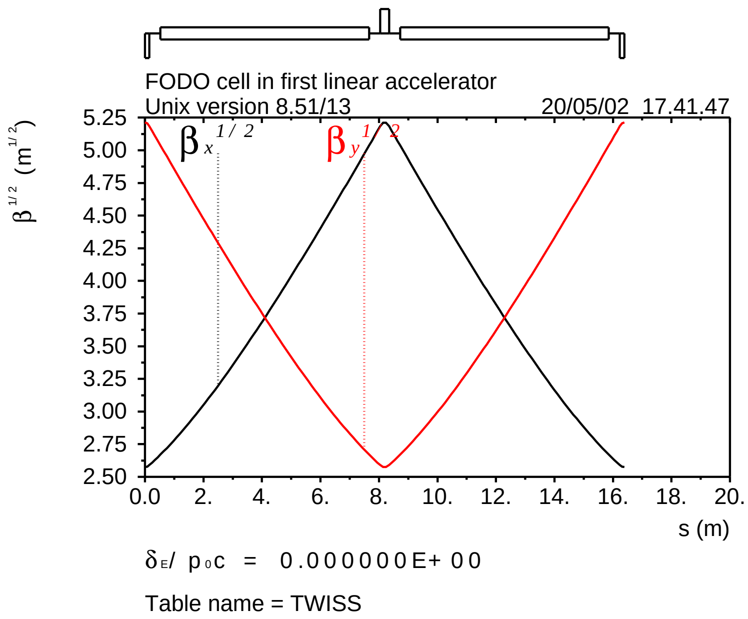
$$\hat{\beta} = \frac{L_p}{4}\sqrt{22 + 10\sqrt{5}}$$

Layout and optical functions of ‘Golden’ linac FODO cell

- Allowing $N_\sigma \approx 3$ RMS beam radii σ in the aperture radius a yields for the normalised emittance ϵ_n accepted by the FODO lattice, with usual relativistic factors β and γ:

$$\epsilon_n = \frac{\beta\gamma}{\hat{\beta}} \left(\frac{a}{N_\sigma} \right)^2$$

- Using $\hat{\beta}$ underestimates ϵ_n, since RF cavities do not fill half FODO cell

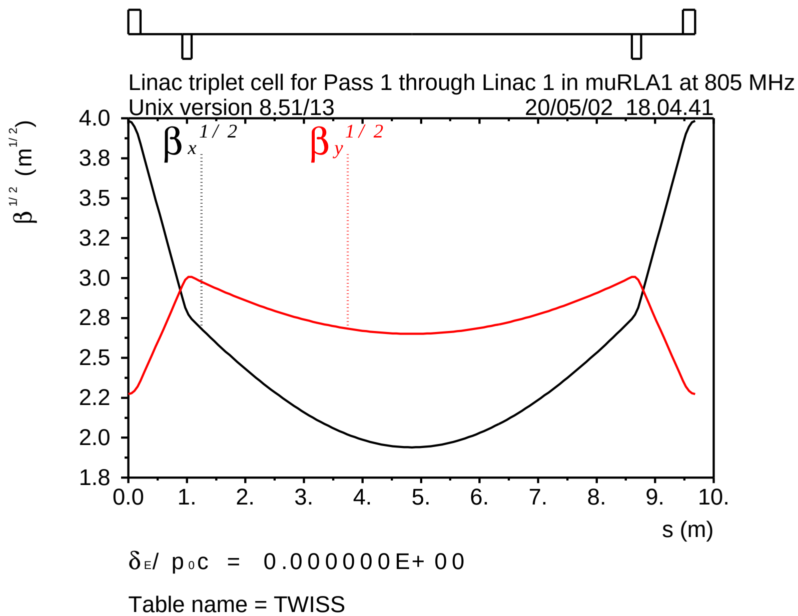


Parameters of ‘Golden’ FODO Cells in linac

- Only look at first period at entrance of linac, and ignore acceleration
- Adiabatic damping of betatron oscillations along linac ensures that emittance limitation occurs in first cell
- All quadrupoles along linac have the same gradient
- Quadrupole field at aperture radius 0.125 T, in Fermilab Recycler about 0.13 T

Cavities	1	2	3	4
Length of half cell/λ_{RF}	10	14	18	22
Length of half cell/m	3.724	5.214	6.703	8.193
Length of quadrupoles/m	0.671	0.480	0.373	0.305
Focal length of quadrupoles/m	3.013	4.218	5.423	6.628
Free space/m	0.695	0.783	0.786	0.751
$\hat{\beta}$/m	12.402	17.363	22.324	27.285
Emittance/mm	**0.244**	**0.174**	**0.136**	**0.111**
Length of 2 linacs/m	**715.0**	**500.5**	**429.0**	**393.3**

Linac triplet lattice design

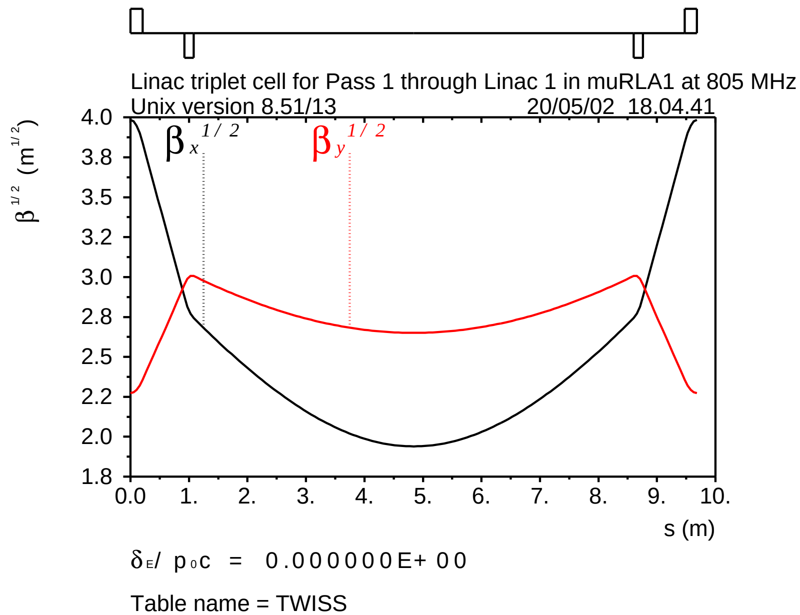


- RF cavities in long straight section between triplets
- Outer horizontally defocusing quadrupoles have twice focal length of central, horizontally focusing oquadrupole
- Triplet lattice stable for subsequent passes, where focal lengths of quadrupoles are larger in ratio of muon momenta

- At centre of long straight section, value of horizontal function $\beta_x = s/2$ is chosen such that $\beta_x = s$ at both ends of long straight section has smallest possible value
- Focal length f and vertical function β_y at centre of long straight section are adjusted such that $\alpha_x = \alpha_y = 0$ at centre of triplet

Layout and optical functions of linac triplet cell

- β_y at both ends of long straight section is result of matching
- Using β_y in normalised emittance underestimates ϵ_n, since RF cavities do not fill whole length of long straight section
- Quadrupole field at aperture radius is 0.5 T



Parameters of triplet cells in the linac

Cavities	1	2	3	4
Length of half triplet cell/λ_{RF}	7	9	11	13
Length of D quadrupoles/m	0.227	0.195	0.173	0.158
Length of half F quadrupoles/m	0.327	0.271	0.234	0.209
Focal length of D quadrupoles/m	2.227	2.712	2.917	3.201
Aperture radius of F quads/mm	54.5	52.7	51.3	50.2
Length of long straight section/m	2.987	4.509	6.020	7.525
Free space between quads/m	0.560	0.632	0.679	0.712
Hor. β_x at centre of long straight sect./m	1.607	2.352	3.097	3.841
Vert. β_y at centre of long straight sect./m	4.369	5.275	6.203	7.133
Hor. β_x at centre of F quad/m	10.269	12.217	14.175	16.117
Vert. β_y at centre of D quad /m	4.960	6.323	7.748	9.202
Emittance/mm	**0.609**	**0.478**	**0.390**	**0.328**
Length of 2 linacs/m	**1001.1**	**643.5**	**524.4**	**464.8**

Discussion of linear accelerator lattices

- FODO lattice has two cryomodules with 4 cavities in just under 16 m
- Triplet lattice has one cryomodule with 4 cavities in just under 10 m
- β-functions relevant for accepted emittance are about 27 m in FODO lattice, and about 10 m in triplet lattice
- Triplet lattice has between 2 and 3 times the emittance of FODO lattice with the same cryomodules
- Pay for this with three times the number of quadrupoles, their larger integrated gradients, and longer linacs
- Two cryomodules conceivable in long straight section of triplet lattice
- Pole-tip field of quadrupoles in FODO lattice deliberately chosen within reach of permanent magnets
- Pole-tip field of triplet quadrupoles well beyond that limit
- I underestimated ϵ_n, because I assumed that D quadrupoles had the same aperture radius as beam ports of SNS cavities

Transverse focusing along linacs

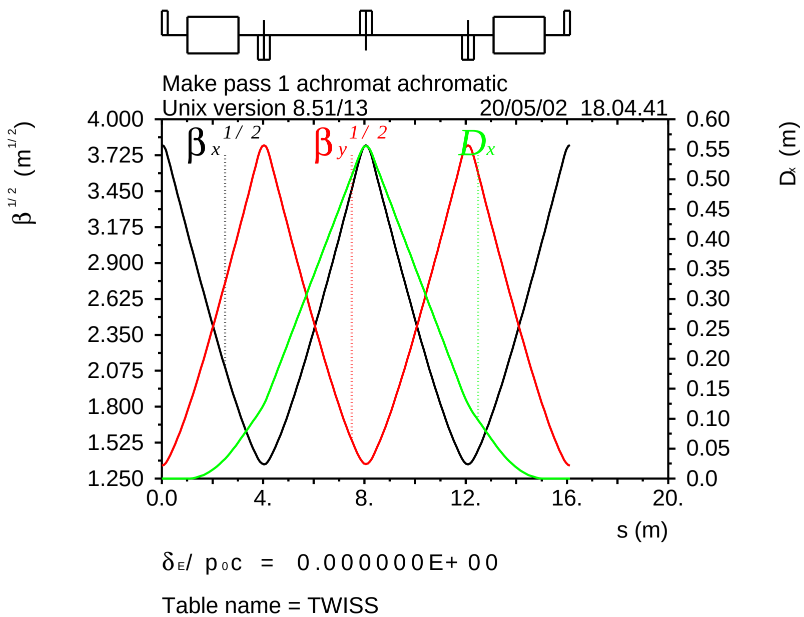
- So far, considered first cell in first linac
- Two extreme styles for focusing along linac:
  - Constant wavelength on first turn, increasing quadrupole strength in proportion to muon momentum, get shrinking wavelengths on later turns
  - Constant quadrupole gradient along linac, increasing wavelengths on all turns
- There are two extreme styles for focusing in second linac
  - First and second linac use the same quadrupoles. The phase advances at exit of first linac and entrance of second one are similar, and so is matching to linacs at both ends of arcs. Operating the RF system at a higher frequency is excluded.
  - Second linac has the same optimum phase advance at entrance as the first one. Quadrupole strengths are higher in ratio of energies, 4.25/2. Adiabatic damping of betatron oscillations between 2 and 4.25 GeV would then allow to reduce aperture, and perhaps to operate at $(3/2) \cdot 805 = 1207.5$ MHz, close to TESLA frequency.
- I use constant gradient and the same quadrupoles in both linacs

Lattice in Arcs

Arc lattice design styles

- Consider 2 arc lattice styles
  - Achromats
  - FODO cells
- Isochronous achromats well fit RLA's accelerating on crest of RF wave form. Longitudinal motion of beam is frozen. Muons do not change their longitudinal position along bunch. In the absence of strong space charge forces, bunch length and absolute energy spread are conserved through RLA, while relative energy spread is damped in proportion to muon energy.
- FODO cells well fit RLA's accelerating on slope of RF wave form. Synchrotron oscillations cause muons to move along bunch, and their energy offset to change.

Layout and optical functions of arc achromat



- For focusing, achromat consists of 2 FODO cells with equal horizontal and vertical phase advances $\mu_x = \mu_y$
- Dipoles installed in half FODO cells adjacent to every second horizontally focusing quadrupole
- β_x and β_y periodic with period length equal to half achromat length

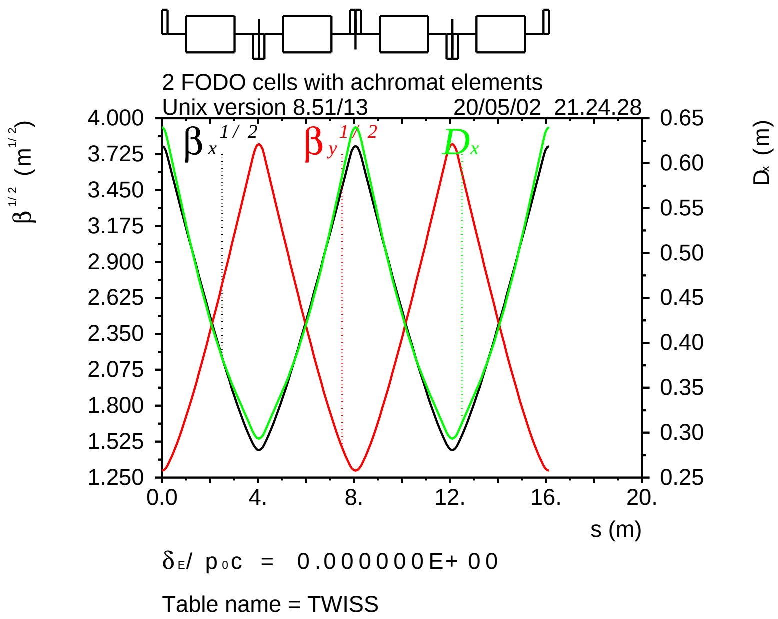
- Horizontal dispersion D_x vanishes at horizontally focusing quadrupoles next to both ends of achromat, reaches a maximum value in central, horizontally focusing quadrupole not surrounded by dipoles
- Conditions on D_x fix focal length f of quadrupoles, phase advances $\mu_x = \mu_y$ through achromat, maximum values of amplitude function $\hat{\beta}$ and dispersion $\hat{D}_x$

Thin-element formulae for achromats

$$
\begin{aligned}
f &= \frac{L(1+\sqrt{17})}{16} \\
\cos\mu &= \frac{\sqrt{17}-5}{4} \\
\hat{\beta} &= \frac{L(13+5\sqrt{17})}{4\sqrt{26\sqrt{17}-86}} \\
\hat{D_x} &= \frac{L\varphi(5+\sqrt{17})}{16}
\end{aligned}
$$

- Half length of achromat L
- Bending angle of full achromat φ
- No closed expression for $\mu/2\pi \approx 0.285176$ for half achromat

Layout and optical functions of 2 arc FODO cells



- Thin-element formulae for FODO cells

$$f = \frac{L}{4\sin(\mu/2)}$$

$$\hat{\beta} = \frac{L[1+\sin(\mu/2)]}{\sin\mu}$$

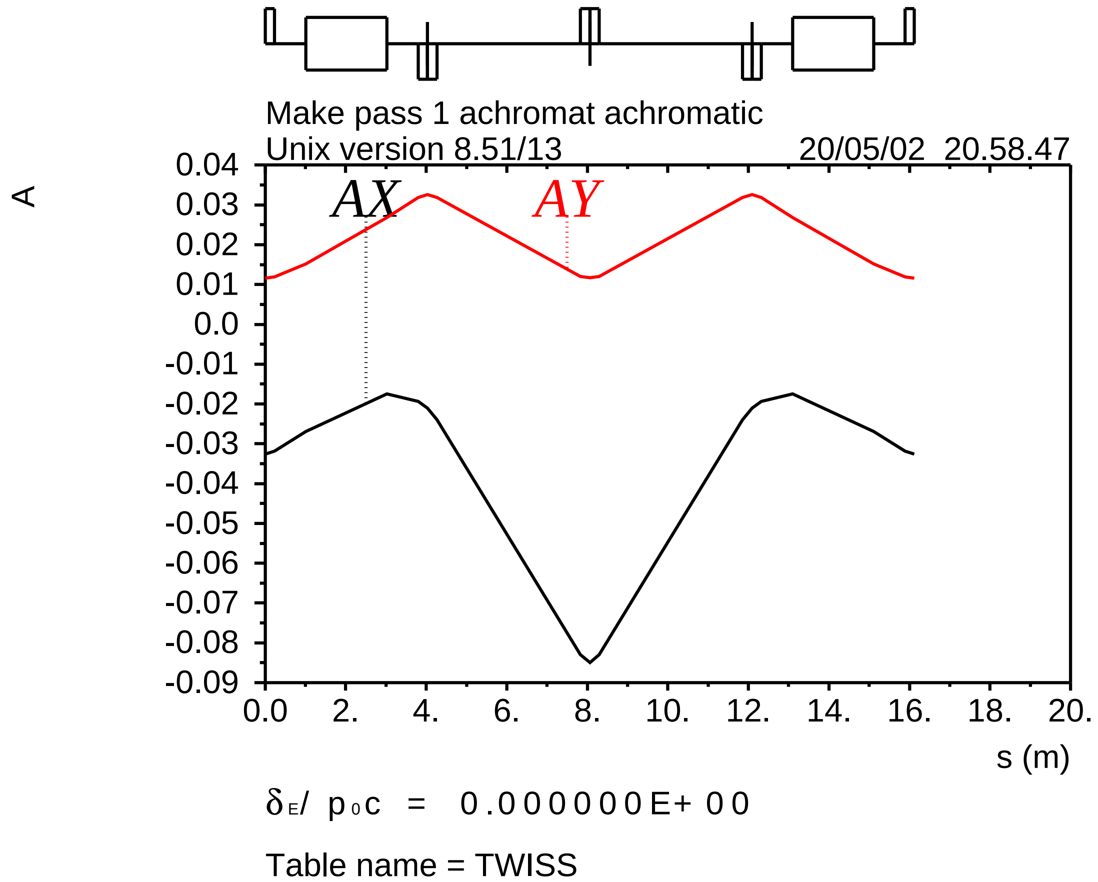
$$\hat{D_x} = \frac{L\varphi[1+\frac{1}{2}\sin(\mu/2)]}{4\sin^2(\mu/2)}$$

- FODO cell length L
- FODO cell bending angle φ
- At same μ, find the same f, $\hat{\beta}$ and also $\hat{D_x}$ as in achromat

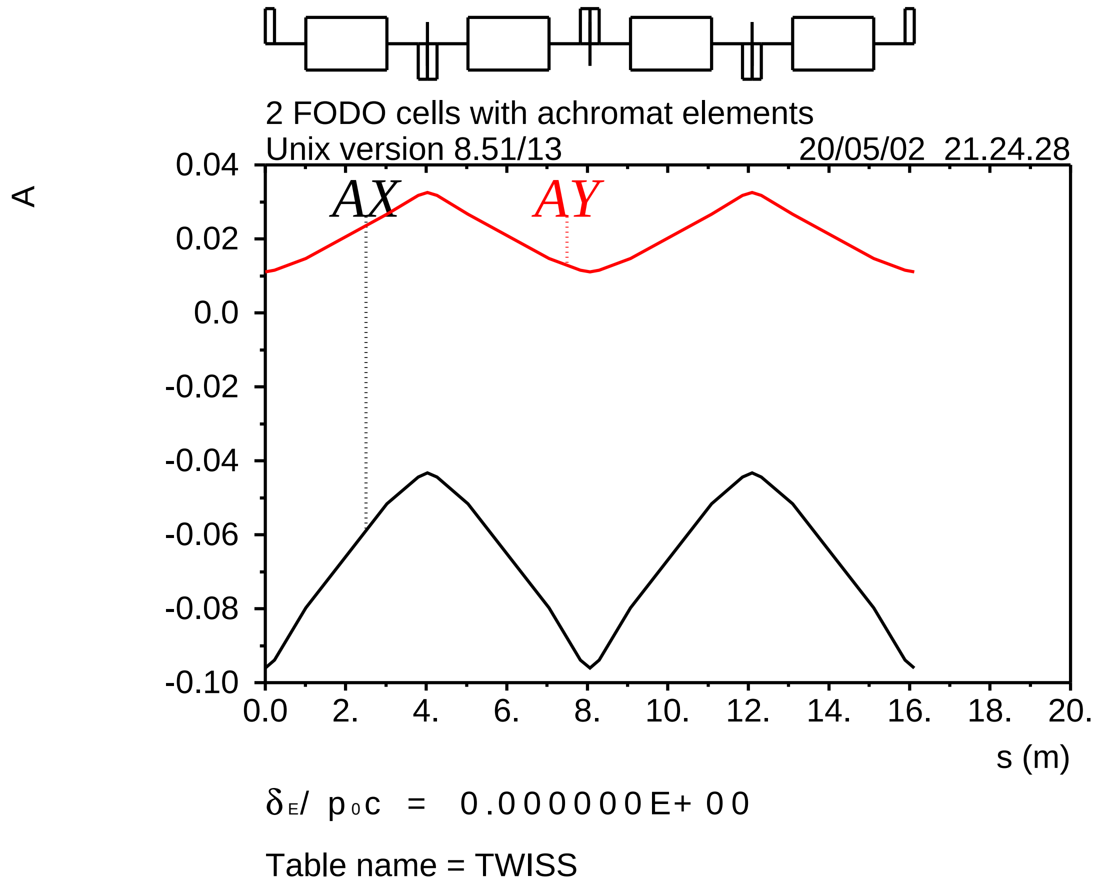
Comparison of arc lattices

- Arcs consisting of achromats are twice as long as arcs consisting of FODO cells, because on average only every second half cell contains a dipole
- The phase advance in achromat is uniquely determined by the conditions on D_x
- It is easy to make arcs not only achromatic, but also isochronous to first order, by arranging achromats in pairs, and letting D_x become negative in the outer F quadrupoles by relatively small variations of the quadrupole gradients
- FODO lattices cannot be made isochronous
- Dispersion suppressors are needed at either end of FODO arcs
- The phase advance in a FODO cell is a free parameter which can be chosen such that the aperture radius is minimised
- Next slides show horizontal and vertical apertures in arc lattices at 4.25 GeV, normalised emittance 0.328 mm and 4.7% relative energy spread

Aperture radii in metres for an achromat



Aperture radii in metres for 2 FODO cells



Variation of arc parameters with energy

- In an RLA, the muon energy E varies from pass to pass
- Specifically, in an RLA that accelerates muons from 2 to 20 GeV in four passes, the muon energies in the arcs at one end are 4.25, 8.75, 13.25 and 17.75 GeV, while they are 6.5, 11, and 15.5 GeV in the arcs at the other end.
- Assuming relative energy spread $\sigma_e = 1\%$ at 20 GeV, and isochronous arcs and bunch length independent of energy, energy spreads are about 4.7%, 2.3%, 1.5% and 1.1% in first set of arcs, and 3.1%, 1.8% and 1.3% in second one.
- Arcs for a given style have the same average radius R, can be stacked vertically
- To avoid increase of RMS beam radius and aperture for lower-energy arcs, dispersion $\hat{D}_x$ there must be smaller. One possible way of achieving this consists of varying both L and φ with energy E in proportion to $\sqrt{E}$, and hence varying $\hat{D}_x$ in proportion to E, thus compensating the $1/E$ variation of σ_e.
- A second way of scaling arc parameters with energy is making achromat cells short enough for $\hat{D}_x$ to be small enough at lowest energy, and to keep this cell length for arcs at higher energy.

Parameters of arc achromats with constant aperture A_x

Norm. emittance	0.328 mm	Rel. RMS energy spread at 20 GeV	1 %
Dipole field at 20 GeV	2 T	Quadrupole pole-tip field at 20 GeV	1 T

Beam energy/GeV	4.25	6.50	8.75	11.00	13.25	15.50	17.75
No. achromats	26	22	18	16	16	14	14
Arc cell length/m	8.06	9.53	11.64	13.10	13.10	14.97	14.97
Max. β-function/m	14.71	17.39	21.25	23.92	23.92	27.32	27.32
Max. dispersion/m	0.555	0.776	1.159	1.467	1.467	1.915	1.915
Rel. en. spread/%	4.70	3.07	2.28	1.81	1.50	1.29	1.12
Beam radius/mm	28.3	25.7	28.0	28.0	23.5	25.9	22.8
Focal length/m	2.58	3.05	3.73	4.19	4.19	4.79	4.79
Chamber rad./mm	**85.0**	**77.2**	**84.1**	**84.1**	**70.5**	**77.8**	**68.4**
Quad. length/mm	**467**	**549**	**658**	**736**	**743**	**839**	**844**
Free space/mm	**1548**	**1833**	**2253**	**2539**	**2532**	**2904**	**2898**

Parameters of arcs with constant dispersion D_x

Normalised emittance	0.328 mm	Rel. RMS en. spread at 20 GeV	1 %
Dipole field at 20 GeV	2 T	Quad. pole-tip field at 20 GeV	1 T
Dipole filling factor	0.5	Focal length	2.58 m
Arc cell cells	8.06 m	Dipole length	2.015 m
$\hat{\beta}$	14.71 m	$\hat{D}_x$	0.555 m

Beam energy/GeV	4.25	6.50	8.75	11.00	13.25	15.50	17.75
Rel. en. spread/%	4.70	3.07	2.28	1.81	1.50	1.29	1.12
Beam radius/mm	28.3	19.2	14.8	12.2	10.4	9.2	8.2
Chamber radius/mm	**85.0**	**57.7**	**44.4**	**36.5**	**31.3**	**27.5**	**24.7**
Quad. length/mm	**467**	**485**	**503**	**519**	**536**	**552**	**567**
Free space/mm	**1548**	**1530**	**1512**	**1496**	**1479**	**1464**	**1448**

Parameters of arc FODO cells with constant aperture A_x

Norm. emittance	0.328 mm	Rel. RMS energy spread at 20 GeV	1 %
Dipole field at 20 GeV	2 T	Quadrupole pole-tip field at 20 GeV	1 T

Beam energy/GeV	4.25	6.50	8.75	11.00	13.25	15.50	17.75
No. FODO cells	26	21	18	16	15	14	13
Arc cell length/m	8.06	9.98	11.64	13.10	13.97	14.97	16.12
Max. β-function/m	25.6	31.7	37.0	41.6	44.4	47.5	51.2
Max. dispersion/m	0.401	0.614	0.836	1.058	1.204	1.382	1.603
Rel. en. spread/%	4.70	3.07	2.28	1.81	1.50	1.29	1.12
Beam radius/mm	23.8	22.9	22.6	22.4	21.1	20.6	20.6
Focal length/m	2.13	2.64	3.08	3.46	3.69	3.96	4.26
Chamber rad./mm	71.3	68.8	67.9	67.2	63.4	61.8	61.9
Quad. length/mm	**474**	**566**	**644**	**712**	**759**	**808**	**861**
Free space/mm	**1541**	**1929**	**2267**	**2563**	**2734**	**2935**	**3170**

Parameters of arcs with constant dispersion D_x

Norm. emittance	0.328mm	Rel. RMS energy spread at 20 GeV	1%
Phase advance/2π	0.38483	Focal length	2.15 m
Arc cell length	8.06 m	Dipole length	2.015 m
$\hat{\beta}$	23.6 m	$\hat{D}_x$	0.409 m

Beam energy/GeV	4.25	6.50	8.75	11.00	13.25	15.50	17.75
Rel. en. spread/%	4.70	3.07	2.28	1.81	1.50	1.29	1.12
Beam radius/mm	23.7	16.8	13.4	11.4	10.0	9.0	8.2
Chamber rad./mm	**71.1**	**50.5**	**40.3**	**34.1**	**30.0**	**26.9**	**24.6**
Quad. length/mm	**468**	**509**	**546**	**581**	**614**	**646**	**676**
Free space/mm	**1547**	**1507**	**1469**	**1434**	**1401**	**1369**	**1339**

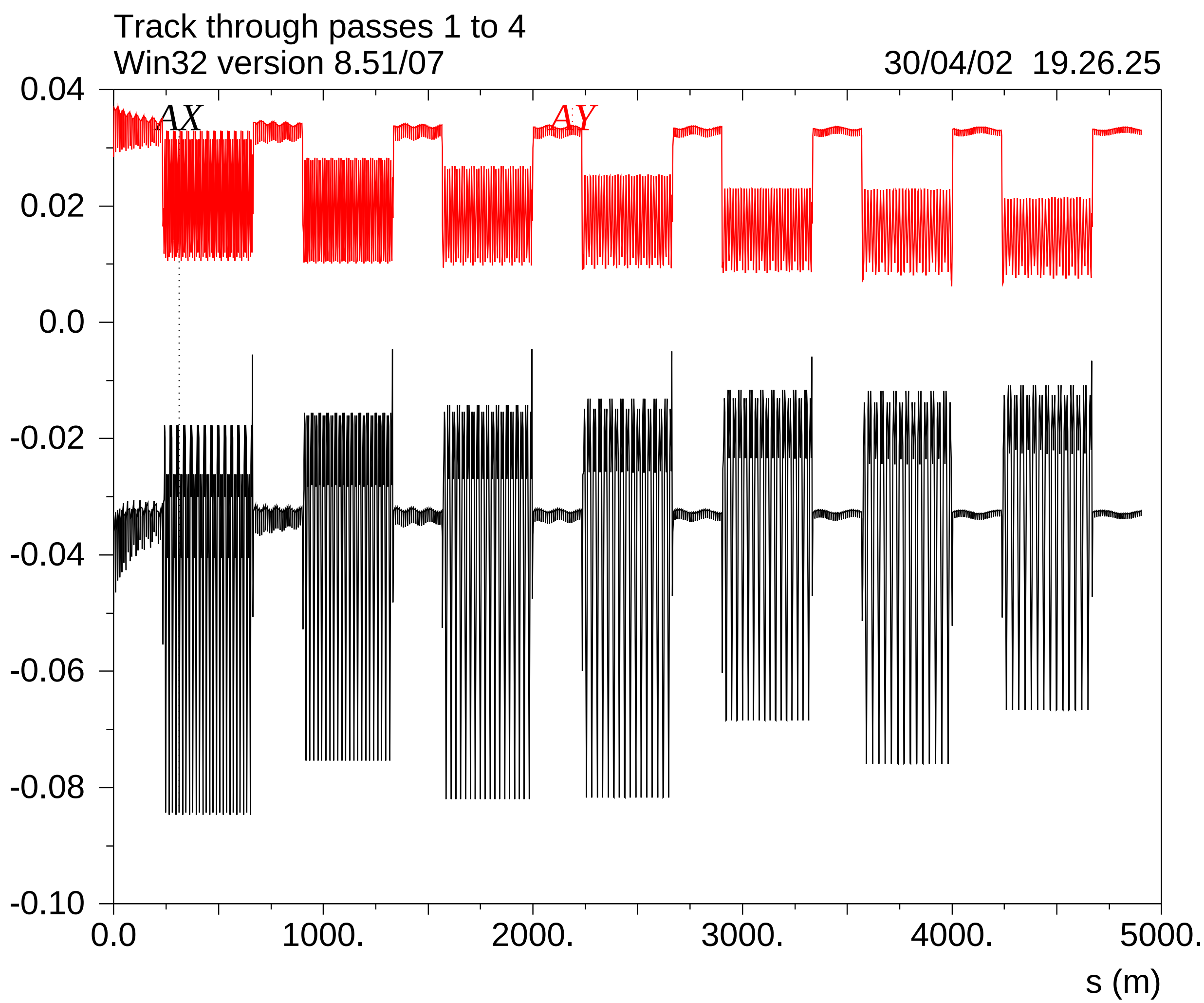
Comment on FODO cell parameters in arcs

- Increase phase advance $\mu/2\pi$ in FODO cells to minimise chamber radius
- For constant A_x:
  - Chamber radius becomes a minimum at $\mu/2\pi \approx 0.38483$ at 4.25 GeV and 0.40385 at 17.75 GeV. Use average $\mu/2\pi = 0.395$.
  - Comparing Slides 33 and 35 shows that increasing $\mu/2\pi$ has the desired effects of reducing maximum dispersion and vacuum chamber radius.
  - Maximum values of the β-functions increase.
  - Focal length of quadrupoles decrease, but reduction of chamber radius compensates it such that quadrupole lengths and free space do not change much.
- For constant number and length of FODO cells, at the value in the leftmost column for 4.25 GeV in Slide 33:
  - Between 4.25 and 17.75 GeV, the aperture radius shrinks by about a factor 3.
  - Quadrupole length and free space rather independent of E.
  - Minimal chamber radius at $\mu/2\pi \approx 0.38483$ at 4.25 GeV and 0.33715 at 17.75 GeV. Since aperture most critical at 4.25 GeV, use $\mu/2\pi \approx 0.38483$.

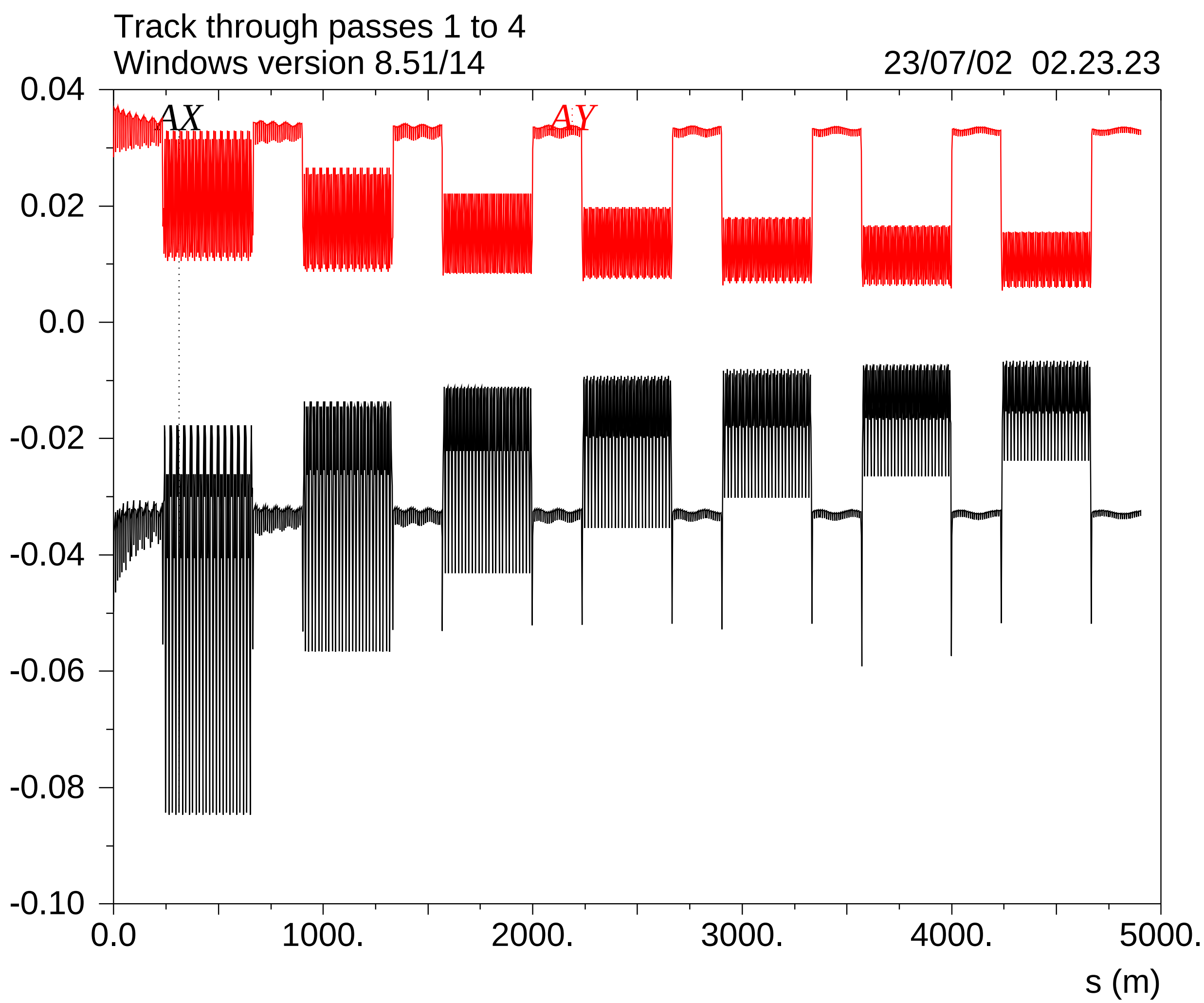
RLA from 2 to 20 GeV

- Assemble an RLA with the following ingredients:
  - Two linacs with triplets in fourth column of Slide 21 and identical quadrupoles
  - Each linac contains 12 cryomodules, each with 8 RF cavities and 2 quadrupole triplets, and possibly other things
  - Seven arcs consist of achromats as shown in Slides 33 or 34
  - Spreader and combiner or peeler sections not yet designed, replaced by betatron matching sections with 4 variable quadrupoles
- Thin-element calculations in *Mathematica* notebooks and packages start from dipole fields, quadrupole pole tip fields, etc., and feed lengths of magnets and straight sections, strengths of elements, etc. into MAD
- MAD reads data written with “expressions” and *Mathematica* files, does matching, etc. with finite-length elements, and draws graphs
- Nonlinear effects, tracking for dynamic aperture, study of errors and tolerances still to come

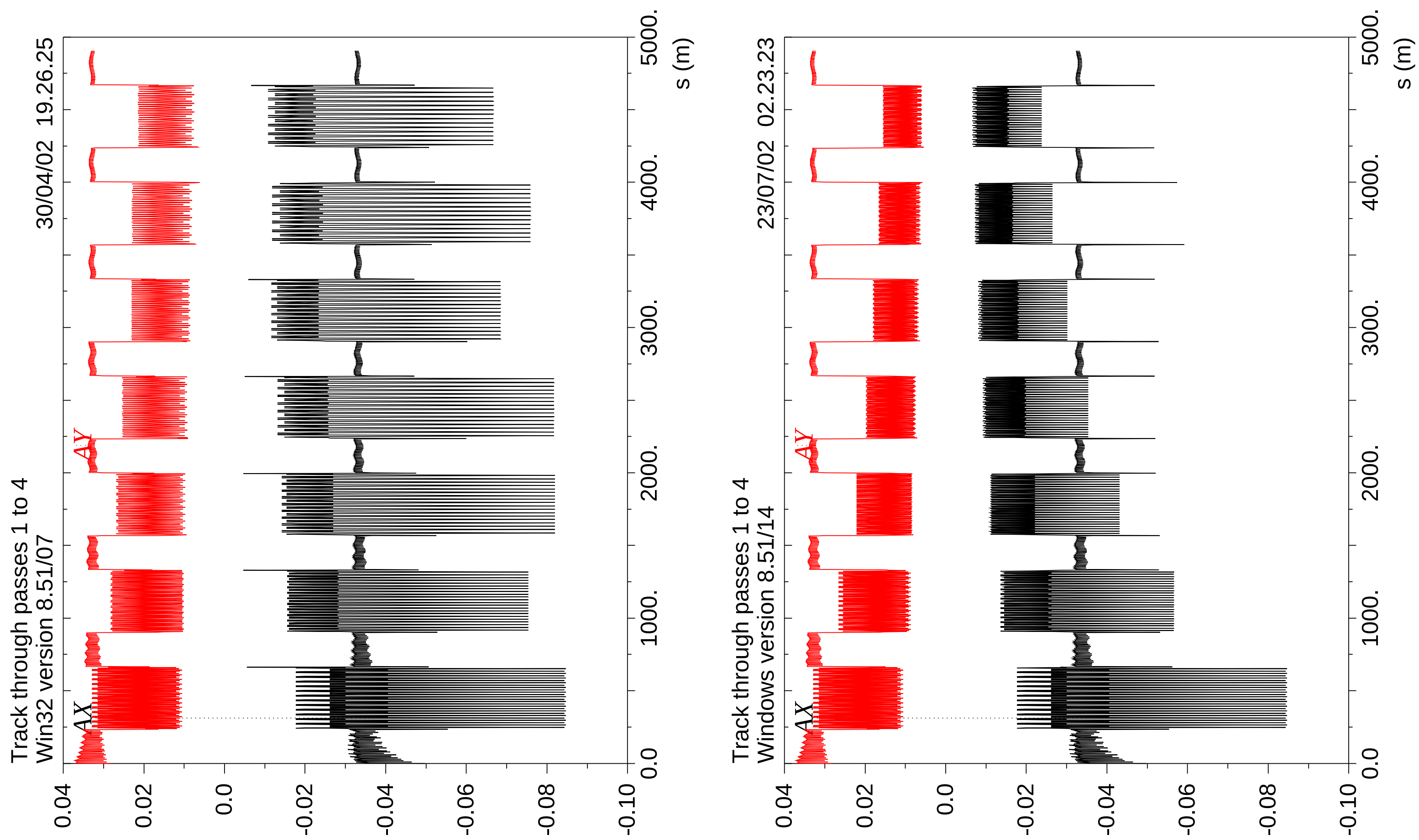
Aperture radii in m for RLA with constant arc aperture



Aperture radii in m for RLA with constant arc dispersion



Aperture radii in metres for the whole RLA



Plans for performance and cost estimates

- Take muon clouds from simulations, e.g. ICOOL simulations in Study II
- Calculate fraction within the accepted emittances of RLA
- Enhance performance by increasing:
  – injection energy
  – radius of beam ports in RF cavities
- Reduce cost by:
  – reducing number of recirculations to e.g. 3, increasing length of linacs by factor 4/3 and decreasing number of arcs from 7 to 5, if linac costs are less than arc costs
  – increasing injection energy and length of cells in linacs and arcs, and reducing number and strengths of components
- Consider combinations of performance enhancements and cost reductions