Limitations of a strong conditioner

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- Initial motivation to improve LCLS x-ray FEL design
- Presentation is somewhat historical according to our efforts
- Find limitations and draw some conclusions

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MOTIVATION

SASE FEL needs very bright electron beam…

$$\varepsilon_N < \gamma \frac{\lambda_r}{4\pi}$$

transverse emittance: $e_N \lesssim 1$ *mm* at 1 Å, 15 GeV

$$\sigma_\delta < \rho \approx \frac{1}{4}\left(\frac{1}{2\pi^2}\frac{I_{pk}}{I_A}\frac{\lambda_u^2}{\beta\varepsilon_N}\left(\frac{K}{\gamma}\right)^2\right)^{1/3}$$

energy spread:
$s_d \lesssim 0.05\%$ at $I_{pk} = 4$ kA,
$K \approx 4$, $l_u \approx 3$ cm, …

Energy spread is easy, but <u>emittance</u> is a real challenge
(present RF-guns produce $e_N > 2$ *m*m)

Requirement is eased if <u>correlation</u> is established
between energy and ‘emittance’ ($d \sim x^2$) → *“FEL conditioning”*

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Radio-Frequency Beam Conditioner for Fast-Wave Free-Electron Generators of Coherent Radiation

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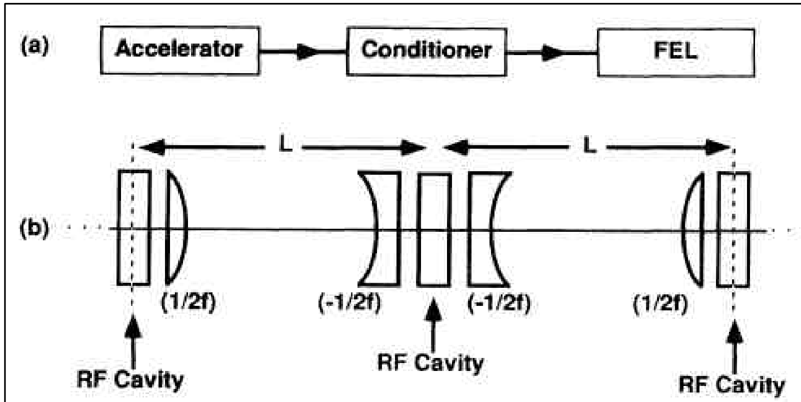
…a very good idea.
How can we use it?

A method for conditioning electron beams is proposed to enhance gain in resonant electron-beam devices by introducing a correlation between betatron amplitude and energy. This correlation reduces the axial-velocity spread within the beam, and thereby eliminates an often severe constraint on beam emittance. Free-electron-laser performance with a conditioned beam is examined and analysis is performed of a conditioner consisting of a periodic array of FODO channels and idealized microwave cavities excited in the TM_{210} mode. Numerical examples are discussed.

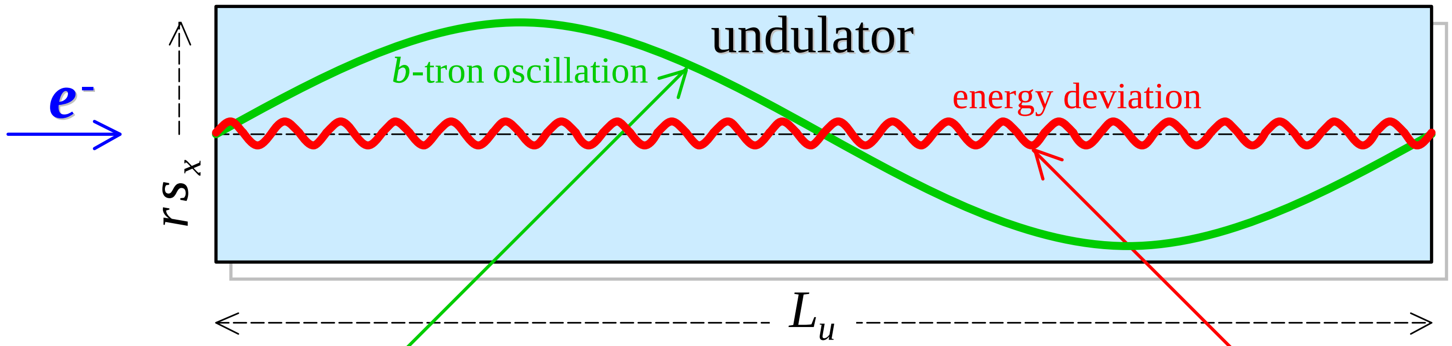


the beam and the FEL, as depicted in Fig. 2(a). We consider the simplest example of such a conditioner corresponding to a periodic lattice, with period as depicted in Fig. 2(b), consisting of a FODO array and suitably phased microwave cavities operating in the TM_{210} mode.

For example, for a 30-Å FEL, with $I \sim 80$ A, $mc^2\gamma_0 \sim 1240$ MeV, $\varepsilon_n \sim 2\times10^{-6}\pi$ m, $\lambda_w \sim 2$ cm, $B \sim 0.66$ T, and plasma density $n_p \sim 1.5\times10^{13}$ cm^{-3}, we find extremely high gain, $L_G/2 \sim 2.1$ m (without conditioning $L_G/2 \sim 26$ m). However, $mc^2\Delta\gamma_c \sim 17$ MeV and the corresponding conditioner would be several hundred meters long [17].

nificantly, from the Panofsky-Wenzel theorem one expects a radio-frequency quadrupole (RFQ) effect with a phase-dependent focal length of order $f_l \sim \gamma/2al$. As a result the beam head and tail will have slightly different lattice parameters and will be mismatched upon injection. We will consider only the limit $f_l \gg f$, where this effect is small. In general one expects that this effect can be eliminated with proper matching at the conditioner entrance and exit, for example, with an RFQ [12].

FEL Electron Beam Conditioning…



path length lag due to b-tron oscillation:

$$\Delta s_r \approx -\int_0^{L_u} \frac{x'^2(s)}{2} ds$$

$$x'(s) = r\sqrt{\frac{\varepsilon_u}{\beta_u}}\sin(s/\beta_u)$$

$$\Delta s_r \approx -r^2 \frac{\varepsilon_u}{\beta_u}\int_0^{L_u} \frac{\sin^2(s/\beta_u)}{2} ds$$

$$= -\frac{1}{4}\frac{\varepsilon_u}{\beta_u} L_u r^2$$

path length change due to energy offset:

relative slippage

$$\Delta s_\delta \approx \eta \cdot L_u \delta_u = \frac{1}{\gamma_u^2}\left(1 + K_u^2/2\right) \cdot L_u \delta_u$$

$$= 2\frac{\lambda_r}{\lambda_u} L_u \delta_u$$

…FEL Electron Beam Conditioning

Multiply Δs_r by 2 to include both x <u>and</u> y, and total path is sum of ***b***-tron and energy effects…

$$\Delta s = \Delta s_\delta + 2\Delta s_r = 2\frac{\lambda_r}{\lambda_u} L_u \delta_u - \frac{1}{2}\frac{\varepsilon_u}{\beta_u} L_u r^2 = 0$$

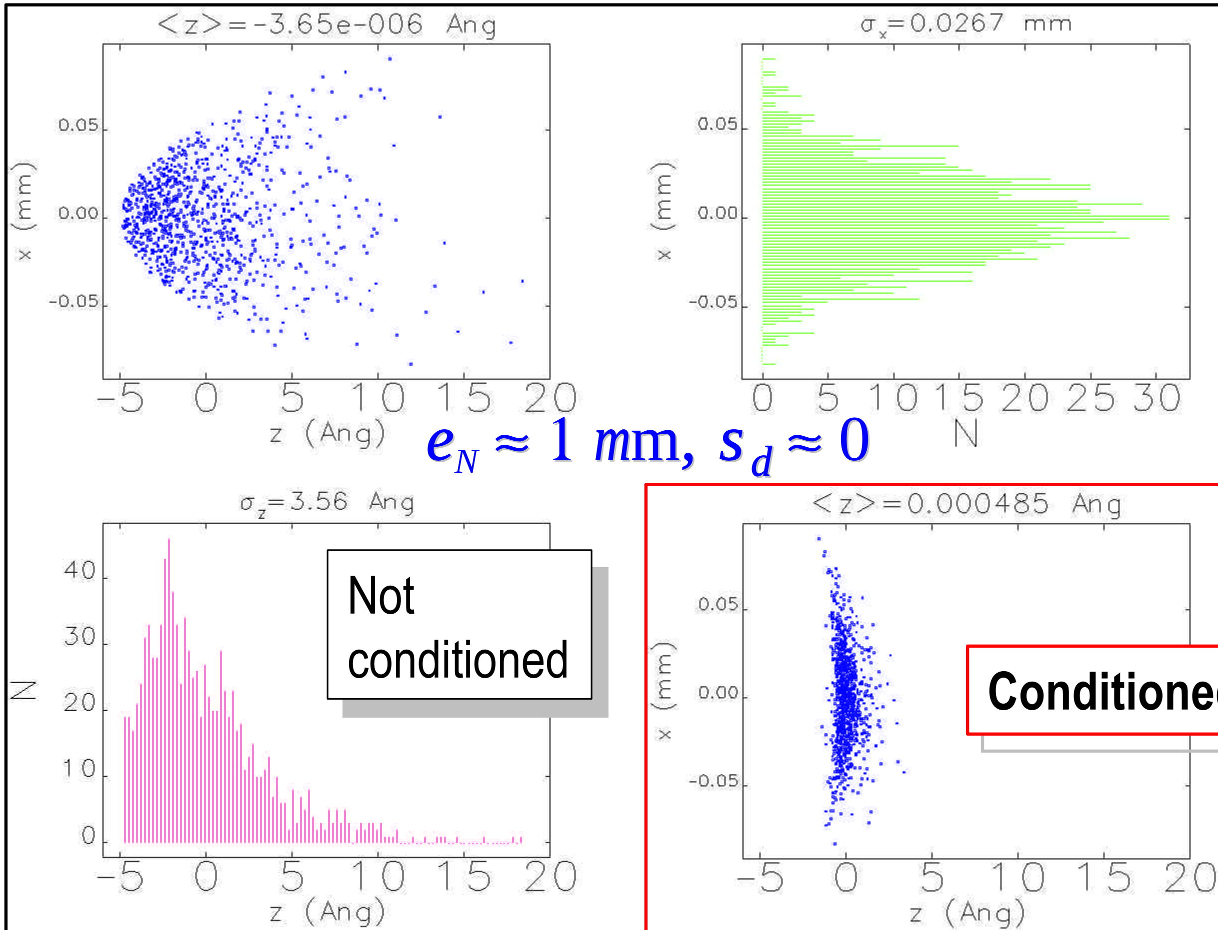
$$\boxed{\delta_u = \frac{1}{4\gamma_u}\frac{\lambda_u}{\lambda_r}\frac{\varepsilon_N}{\beta_u} r^2}$$ **conditioned**

Relative energy deviation, d_u, of each e^- should be increased in proportion to the square of its normalized ***b***-tron amplitude, r

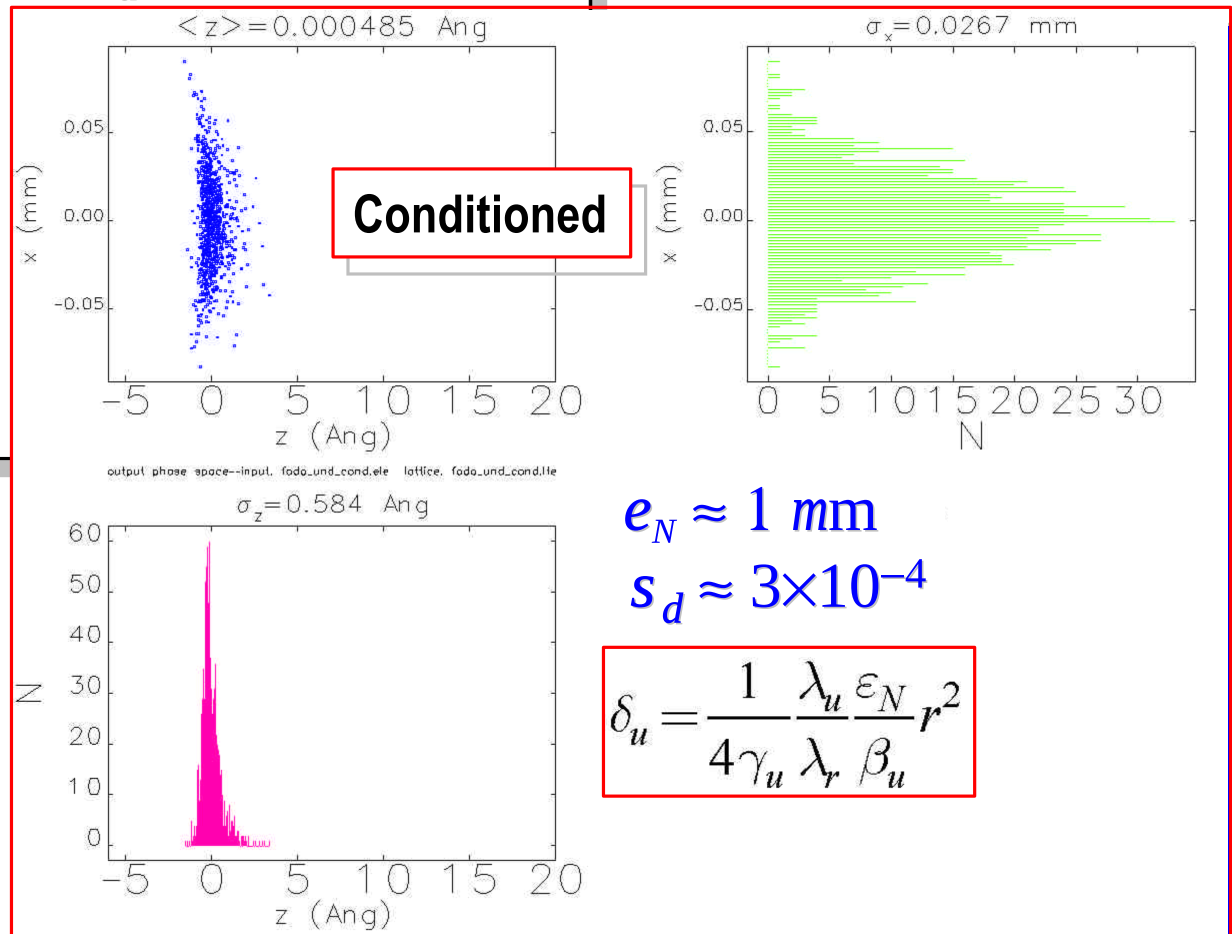
$$\boxed{r^2 = \frac{x^2 + (\beta_u x')^2 + y^2 + (\beta_u y')^2}{\varepsilon_u \beta_u}}$$ (natural focusing: $a_{x,y} = 0$)

Slippage in Tracking Run

(with *LCLS*-undulator-like FODO lattice)



$e_N \approx 1$ mm, $s_d \approx 0$



$e_N \approx 1$ mm

$s_d \approx 3\times10^{-4}$

$$\delta_u = \frac{1}{4\gamma_u}\frac{\lambda_u}{\lambda_r}\frac{\varepsilon_N}{\beta_u}r^2$$

Most publications add conditioner after accelerator, before FEL. What about conditioning prior to acceleration and compression?

$$\boxed{\delta_u = \frac{1}{4\gamma_u}\frac{\lambda_u}{\lambda_r}\frac{\varepsilon_N}{\beta_u}r^2}$$

(Practical Issues)

- Locate conditioner near start of accelerator at low energy (weaker conditioning fields needed)…

- After conditioner e^- bunch is compressed from s_{z0} to s_{zf}, and accelerated from g_0 to g_u …

- Acceleration ***reduces*** conditioned relative energy spread, but compression ***increases*** it…

- Energy deviation needed at low-energy conditioner is…

Conditioning for LCLS

$$\delta = \frac{\sigma_{z_f}}{\sigma_{z_0}} \frac{\gamma_u}{\gamma_0} \delta_u = \frac{1}{2\gamma_0} \frac{\epsilon_N}{\sigma_{z_0}} a r^2 \, ,$$

where

$$a \equiv \frac{1}{2} \frac{\lambda_u}{\lambda_r} \frac{\sigma_{z_f}}{\beta_u}$$

$a \approx 30$ for LCLS

Recall that

$$r^2 = \frac{x^2 + (\beta_u x')^2 + y^2 + (\beta_u y')^2}{\epsilon_u \beta_u}$$

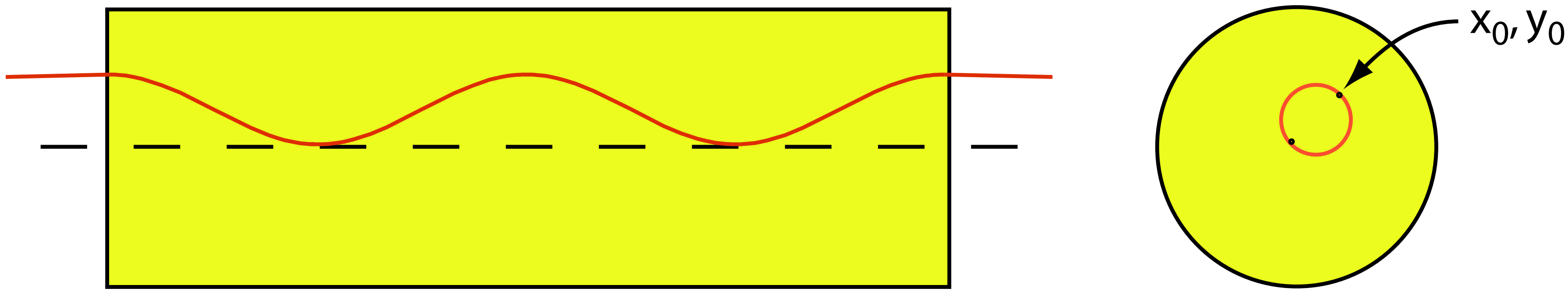
Assume $r = \sqrt{2}$—particles with the amplitude of betatron oscillations $x_a = y_a = \sigma_x (= \sigma_y)$.

$$\Delta E_{\text{cond}} = \gamma m c^2 \delta = m c^2 a \frac{\epsilon_N}{\sigma_{z_0}}$$

For LCLS, before BC1, $\sigma_{z_0} = 1$ mm and

$$\Delta E_{\text{cond}} = 18 \text{ keV}$$

Using solenoid for conditioning



If a particle enters solenoid parallel to the axes, $r = \frac{1}{2}\sqrt{x_0^2 + y_0^2}$.

$$k = \frac{1}{2}\frac{\omega_H}{c} = \frac{eB}{2E_b}$$

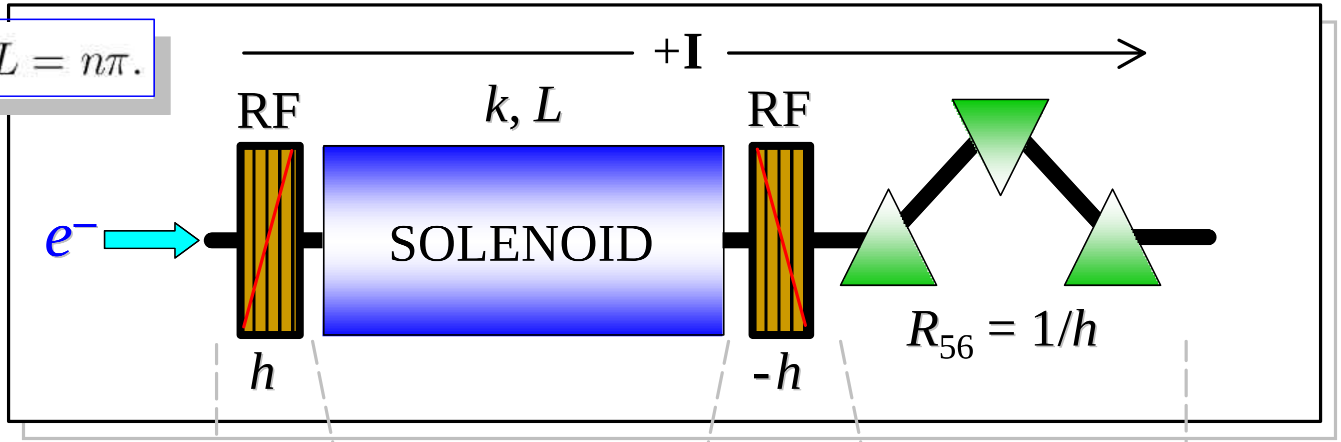
$$\Delta z = -\frac{1}{2}k^2(x_0^2 + y_0^2)L$$

1. Chirp beam in energy before the solenoid, $\delta = hz_0$
2. Send the beam through the solenoid, $z = z_0 - \Delta z$
3. Chirp with $-h$ after the solenoid

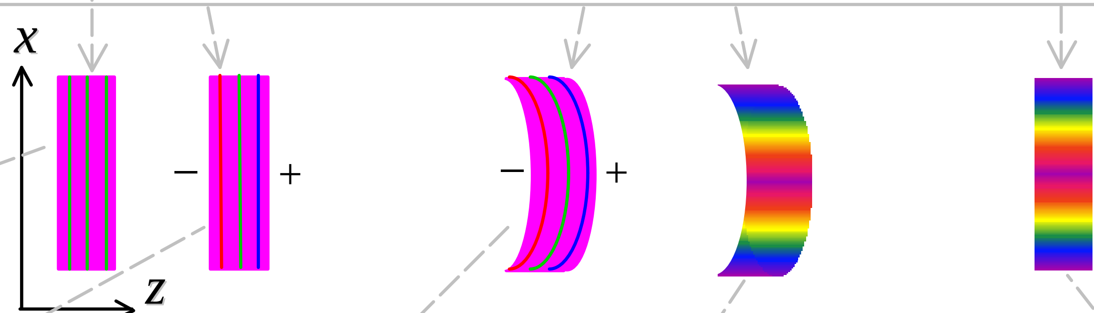
$$\delta = hz_0 - hz = h\Delta z = \frac{1}{2}hk^2(x_0^2 + y_0^2)L$$

A ‘One-Phase’ Conditioner (for simplicity)

$|k|\beta \gg 1,\ |k|L = n\pi.$



e^- bunch along line



$$\begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \\ z_0 \\ \delta_0 \end{pmatrix}$$

$$x_1 = x_0$$
$$x_1' = x_0'$$
$$z_1 = z_0$$
$$\delta_1 \approx h z_0$$

$$x_2 = x_0$$
$$x_2' = x_0' + k^2 L h x_0 z_0$$
$$z_2 = z_0 - k^2 L\left(x_0^2 + y_0^2\right)$$
$$\delta_2 \approx h z_0$$

$$x_3 = x_0$$
$$x_3' = x_0' + k^2 L h x_0 z_0$$
$$z_3 = z_0 - \frac{1}{2} k^2 L\left(x_0^2 + y_0^2\right)$$
$$\delta_3 \approx \delta_0 + \frac{1}{2} k^2 L h\left(x_0^2 + y_0^2\right)$$

$$x_4 = x_0$$
$$x_4' = x_0' + k^2 L h x_0 z_0$$
$$z_4 \approx z_0 - \frac{1}{h}\delta_0$$
$$\delta_4 \approx \delta_0 + \frac{1}{2} k^2 L h\left(x_0^2 + y_0^2\right)$$

$T_{216} = T_{436} = -T_{511} = -T_{533} = k^2 L/2$

…A ‘One-Phase’ Conditioner (for simplicity)

$$\boxed{\begin{aligned} x &= x_0 \\ x' &= x_0' + k^2 L h x_0 z_0 \\ z &\approx z_0 - \frac{1}{h}\delta_0 \\ \delta &\approx \delta_0 + \frac{1}{2}k^2 L h\left(x_0^2 + y_0^2\right) \end{aligned}}$$

transverse aberration

one-phase conditioning

Energy conditioning is provided for *h* > 0…

$$\boxed{r^2 \equiv \frac{x_0^2 + y_0^2}{\beta \epsilon_0}\,.}$$

Equate to earlier result…

$$\delta = \frac{\sigma_{z_f}}{\sigma_{z_0}} \frac{\gamma_u}{\gamma_0} \delta_u = \frac{1}{2\gamma_0} \frac{\varepsilon_N}{\sigma_{z_0}} a \cdot r^2, \qquad a \equiv \frac{1}{2} \frac{\lambda_u}{\lambda_r} \frac{\sigma_{z_f}}{\beta_u}$$

$$\boxed{k^2 L h \beta \sigma_{z0} = \frac{1}{2} \frac{\lambda_u}{\lambda_r} \frac{\sigma_{zf}}{\beta_u} \equiv a\,,}$$

Conditioner parameters (left) are set by FEL parameters (right)

Solenoid conditioner for LCLS

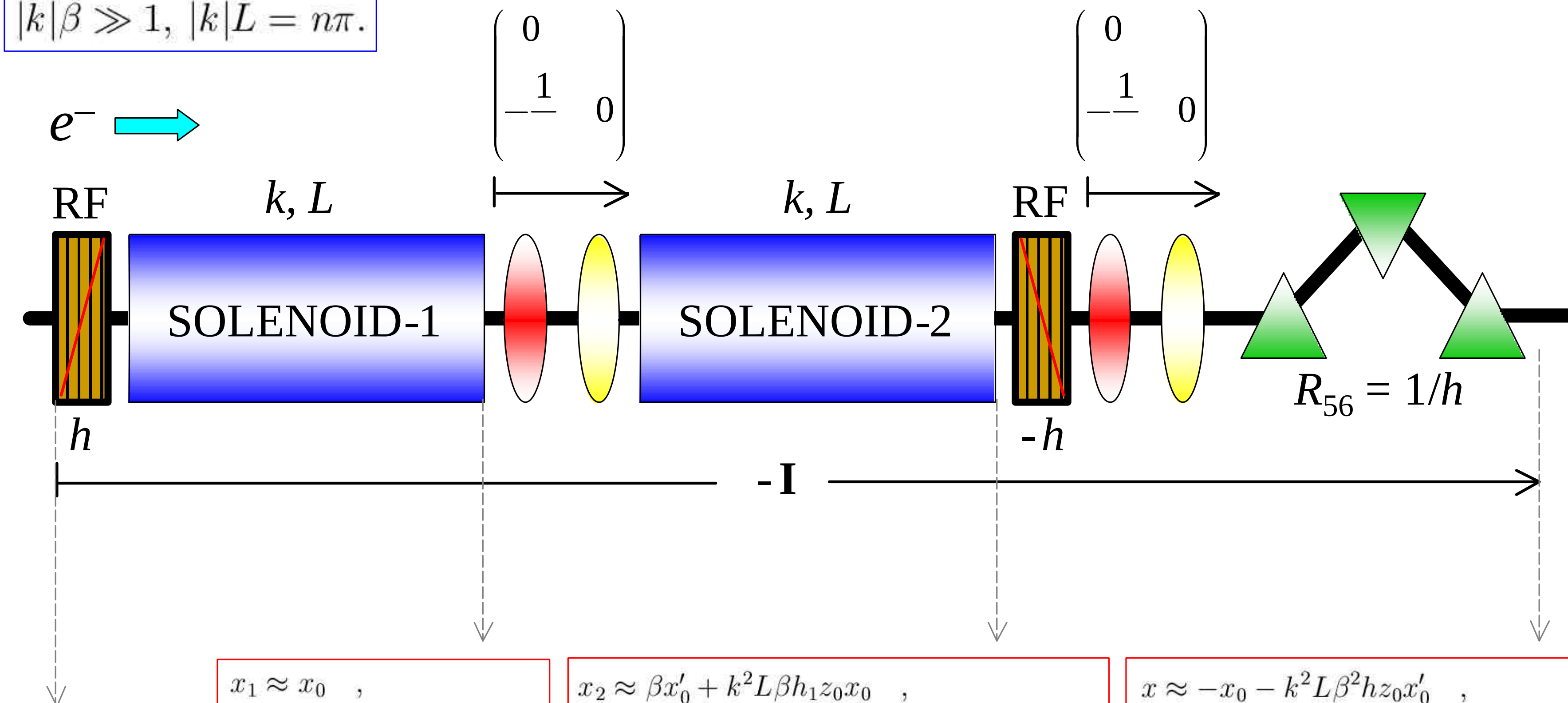
For LCLS, $a = 33$.

$B = 3.5$ T,
$E_b = 100$ MeV,
$\beta = 100$ m,
rms energy chirp $= 4 \times 10^{-3}$
(bunch length ~ 1 mm)
$k = 5.25$ m^{-1},
$L = 3$ m.

Beam makes 5 revolutions inside the solenoid.

‘Two-Phase’ FEL Conditioner

$$|k|\beta \gg 1,\ |k|L = n\pi.$$



$$(x_0, x_0', y_0, y_0', z_0, \delta_0)$$

$$x_1 \approx x_0 \quad ,$$
$$x_1' \approx x_0' + k^2 L h_1 z_0 x_0 \quad ,$$
$$z_1 \approx z_0 - \frac{1}{2}k^2 L(x_0^2 + y_0^2)$$
$$\delta_1 \approx h_1 z_0 \quad ,$$

$$x_2 \approx \beta x_0' + k^2 L \beta h_1 z_0 x_0 \quad ,$$
$$x_2' \approx -\frac{1}{\beta}x_0 + k^2 L \beta h_1 z_0 x_0' \quad ,$$
$$z_2 \approx z_0 - \frac{1}{2}k^2 L[x_0^2 + (\beta x_0')^2 + y_0^2 + (\beta y_0')^2]$$
$$\delta_2 \approx h_1 z_0 \quad .$$

$$x \approx -x_0 - k^2 L \beta^2 h z_0 x_0' \quad ,$$
$$x' \approx -x_0' + k^2 L h z_0 x_0 \quad ,$$
$$z \approx z_0 + R_{56}\delta_0 \quad ,$$
$$\delta \approx \frac{1}{2}h k^2 L[x_0^2 + (\beta x_0')^2 + y_0^2 + (\beta y_0')^2]$$

(see also N. Vinokurov; NIM A **375**, 1996, pp. 264-268)

Conditioning and Emittance Growth

Transverse emittance growth due to solenoid chromaticity…

$$\epsilon_x^2 = \langle (x-\overline{x})^2 \rangle \langle (x'-\overline{x'})^2 \rangle - \langle (x-\overline{x})(x'-\overline{x'}) \rangle^2 .$$

$$x \approx x_0 ,$$

$$x' \approx x'_0 + k^2 L h z_0 x_0 ,$$

$$\overline{x} = \langle x \rangle = 0$$

$$\overline{x'} = \langle x' \rangle = 0$$

$$\langle x x' \rangle = 0$$

$$\begin{aligned} \epsilon_x^2 &= \langle x^2 \rangle \langle x'^2 \rangle \\ &\approx \langle x_0^2 \rangle \langle (x'_0 + k^2 L h z_0 x_0)^2 \rangle \\ &= \epsilon_{x0}^2 [1 + (k^2 L h \beta \sigma_{z0})^2] , \end{aligned}$$

where $\langle x_0^2 \rangle = \beta \epsilon_{x0}$, $\langle x_0'^2 \rangle = \epsilon_{x0}/\beta$, and $\langle z_0^2 \rangle = \sigma_{z0}^2$,

Relative transverse emittance growth…

$$\frac{\epsilon_x}{\epsilon_{x0}} \approx k^2 L h \beta \sigma_{z0} \gg 1 ,$$

…is set by FEL parameters, <u>not</u> conditioner…

$$a \equiv \frac{1}{2} \frac{\lambda_u}{\lambda_r} \frac{\sigma_{zf}}{\beta_u} \approx \frac{\epsilon_x}{\epsilon_{x0}}$$

‘RF-Quad’ Effect

Conditioner adds kick dependent on z_0…

Head ($z_0 > 0$) is de-focused and tail is focused (RF-quad effect)…

$$\boxed{x' \approx x'_0 + \underbrace{k^2 L h z_0}_{1/f\text{ : time dependent focus}} x_0\,,}$$

$$\boxed{\beta/f(\pm\sigma_{z0}) = \pm k^2 L \beta h \sigma_{z0} = \pm a,}$$

Solenoid conditioner generates same undesirable RF-quad effect as TM_{210}-type conditioner. Is there some fundamental connection?

Numerical Example

FEL and conditioner parameters for the LCLS [2] and VISA [9].

parameter	**symbol**	**LCLS**	**VISA**	**units**
electron energy/mc^2	γ_u	28000	140	
undulator period	λ_u	3	1.8	cm
radiation wavelength	λ_r	1.5	8500	Å
und. beta-function (natural focusing)	β_u	72	0.6	m
final rms bunch length	σ_{z_f}	24	100	μm
conditioning coefficient (one phase)	a	33	1.8	

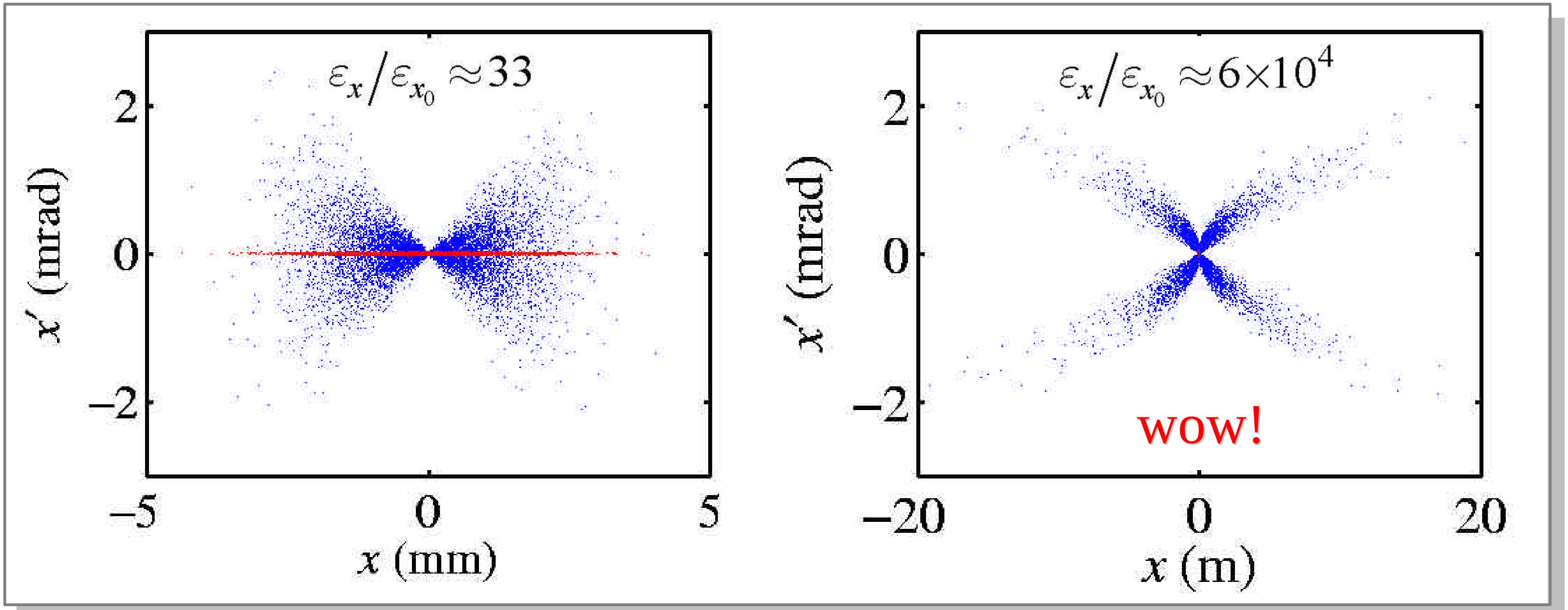
For LCLS using natural focusing ($b_u \approx 72$ m)… $\boxed{\epsilon_x/\epsilon_{x0} \approx 33.}$

A “two-phase” conditioner is **<u>much</u>** worse.

Particle Tracking Through Solenoid System

With $k = \dfrac{k_0}{1+\delta}$

After Solenoid-1 After Solenoid-2



If this is a general result, then conditioning a short wavelength FEL looks **impossible.**

Conditioning and Symplecticity

Assume that the conditioner does not introduce coupling between the vertical and horizontal planes, and consider only the horizontal plane with the initial values of coordinates $(x_0,\, x_0')$ at the entrance, and the final values $(x,\, x')$ at the exit.

Instead of using variables x_0, x_0' and x, x', introduce new variables ξ_0, ξ_0', and ξ, ξ'

$$\begin{pmatrix} \xi_0 \\ \xi_0' \end{pmatrix} = Q_0 \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}, \qquad \begin{pmatrix} \xi \\ \xi' \end{pmatrix} = Q \begin{pmatrix} x \\ x' \end{pmatrix},$$

$$Q_0 = \begin{pmatrix} \frac{1}{\sqrt{\beta_0}} & 0 \\ \frac{\alpha_0}{\sqrt{\beta_0}} & \sqrt{\beta_0} \end{pmatrix}, \qquad Q = \begin{pmatrix} \frac{1}{\sqrt{\beta}} & 0 \\ \frac{\alpha}{\sqrt{\beta}} & \sqrt{\beta} \end{pmatrix},$$

with β_0, α_0 and β, α the Twiss parameters.

The map from ξ_0, ξ_0', z_0, δ_0 to ξ, ξ, z, δ is symplectic. In linear approximation

$$\begin{pmatrix} \xi \\ \xi' \end{pmatrix} = A \begin{pmatrix} \xi_0 \\ \xi_0' \end{pmatrix} ,$$

where

$$A = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} ,$$

with ψ the betatron phase advance.

“One-Phase” Conditioner

Contribution $x_0^2/(\beta\epsilon_0)$ of the x-coordinate to the parameter r^2 is equal to ξ_0^2/ϵ_0,

$$r^2 \to \frac{\xi_0^2}{\epsilon_0}\,.$$

Conditioning requires

$$\delta = \delta_0 + \frac{1}{2}\frac{a}{\sigma_{z0}}\xi_0^2\,.$$

Symplecticity and Generating Function

Symplecticity means that ξ_0, ξ_0', z_0, δ_0 and ξ, ξ', z, δ are related via a canonical transformation.

We use a generating function which depends on old coordinates ξ_0 and z_0 and new momenta ξ' and δ, $F(\xi_0, z_0, \xi', \delta)$.

$$\xi_0' = \frac{\partial F}{\partial \xi_0}\,, \qquad \delta_0 = \frac{\partial F}{\partial z_0}\,, \qquad \xi = \frac{\partial F}{\partial \xi'}\,, \qquad z = \frac{\partial F}{\partial \delta}\,.$$

In paraxial approximation, all coordinates and momenta are considered small and we can expand F in Taylor series:

$$F \approx F_2 + F_3 + \ldots\,,$$

where F_2 is a quadratic, and F_3 is a cubic function of the coordinates and momenta.

We need F_2 to generate a linear map for ξ and ξ' with a unit transformation for z and δ. I assume $\psi = 2\pi n$ (integer number of betatron oscillations)

$$F_2 = \xi_0 \xi' + \delta z_0 \,.$$

This generating function gives

$$\begin{aligned} \xi_0' &= \xi' \,, \\ \xi &= \xi_0 \,, \\ z &= z_0 \,, \\ \delta_0 &= \delta \,. \end{aligned}$$

The function F_3 involves 2^{nd}-order abberations in the system. We chose only the term responsible for the conditioning:

$$F_3 = -\frac{1}{2}\frac{a}{\sigma_{z0}} z_0 \xi_0^2 \,.$$

We find

$$\delta_0 = \delta - \frac{1}{2}\frac{a}{\sigma_{z0}} \xi_0^2 \,,$$

hence

$$\delta = \delta_0 + \frac{1}{2}\frac{a}{\sigma_{z0}} \xi_0^2 \,.$$

We also have from the generating function $F_2 + F_3$

$$
\begin{aligned}
\xi &= \xi_0 \,, \\
\xi' &= \xi_0' + \frac{a}{\sigma_{z0}} z_0 \xi_0 \,, \\
z &= z_0 \,.
\end{aligned}
$$

For the single phase solenoid conditioner $\psi = 2\pi n$, $\beta_0 = \beta$, $\alpha_0 = \alpha = 0$, and this equation agrees with equations for "one-phase" conditioner (in the limit $h \to \infty$).

Calculate the projected emittance increase of the beam due to the conditioning:

$$\epsilon_x^2 = \langle \xi^2 \rangle \langle \xi'^2 \rangle - \langle \xi \xi' \rangle^2$$

where the averaging is

$$\langle \ldots \rangle = \int \frac{dz_0}{\sqrt{2\pi}\sigma_{z0}} e^{-z_0^2/2\sigma_{z0}^2} \int \frac{d\xi_0 \, d\xi_0'}{2\pi\epsilon_{x0}} e^{-(\xi_0^2+\xi_0'^2)/2\epsilon_{x0}} \ldots$$

Result

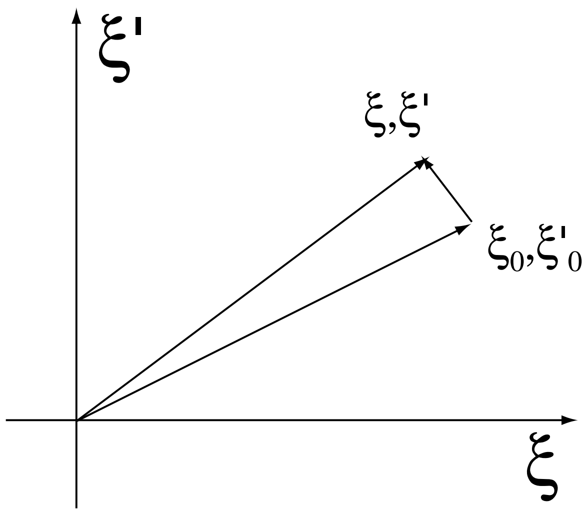
$$\epsilon_x^2 = \epsilon_{x0}^2 (1 + a^2) \, .$$

For large a

$$\frac{\epsilon_x}{\epsilon_{x0}} \approx a$$

The standard approach in the beam optics uses Taylor expansion, assuming that $F_3 \ll F_2$. This is true only for $a \ll 1$. However, if we allow $F_3 \gg F_2$ and set $F = F_2 + F_3$, then the map is symplectic, and the model is still valid.

Gentle two-phase conditioning results in phase rotation



$$F_3 = -\frac{1}{2}\frac{a}{\sigma_{z0}} z_0 (\xi_0^2 + \xi'^2)\,.$$

$$\delta = \delta_0 + \frac{1}{2}\frac{a}{\sigma_{z0}} (\xi_0^2 + \xi'^2)\,.$$

$$\xi = \xi_0 - \frac{a}{\sigma_{z0}} z_0 \xi_0'\,, \qquad \xi' = \xi_0' + \frac{a}{\sigma_{z0}} z_0 \xi_0\,.$$

For small a, this is a rotation in the phase space by angle az_0/σ_{z0} (A. Wolski)

Idea: large-a conditioning without associated emittance growth can be achieved as a sequence of small-a steps with a phase advance between them.

Conclusions

- We analyzed a beam conditioner that uses two strong solenoids and found that a side effect of conditioning is a strong differential focusing of the beam which results in the emittance growth that is directly related to the conditioning parameter a. The effect is so strong for LCLS parameters, that it would ruin the linear optics and result in the loss of the beam.

- We showed that the differential focusing is related to conditioning via the symplecticity of the map. Previously, we thought that this side effect is inevitable, but recently A. Wolski showed that it can be avoided.

- I still think that the associated with conditioning emittance growth imposes an important constraint on possible choice of conditioner design—one cannot do strong conditioning on a short distance...