



17<sup>TH</sup> ADVANCED BEAM DYNAMICS WORKSHOP ON

**FUTURE LIGHT SOURCES**

# Microwave Instability and Impedances

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# IMPEDANCE BUDGET

## Impedance not easy to model

- shorter and shorter bunches (very high beam spectrum)  
3D computer codes require mesh sizes  $\approx 1$  cm
- complex geometries

## Lot of precautions are taken

- flanges  $\rightarrow$  vacuum join + rf contact
- screens of vacuum ports
- shielding of bellows
- smooth tapers

$\Rightarrow$  Mostly inductive with  $Z/n \ll 1 \Omega$

But ...

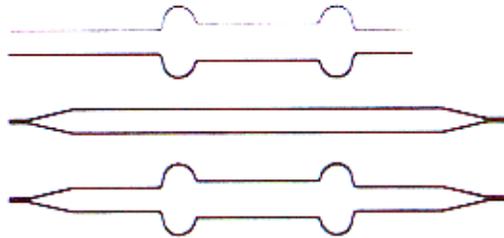
High frequency tail of rf cavity impedance

Trapped modes produced by slots, BPMs ... (enlargements of the beam pipe)

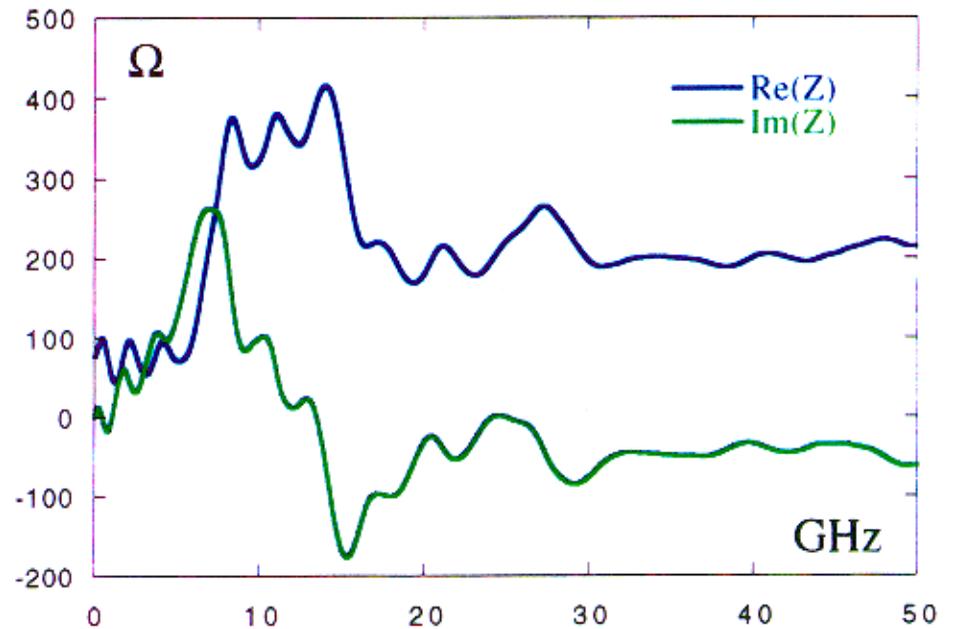
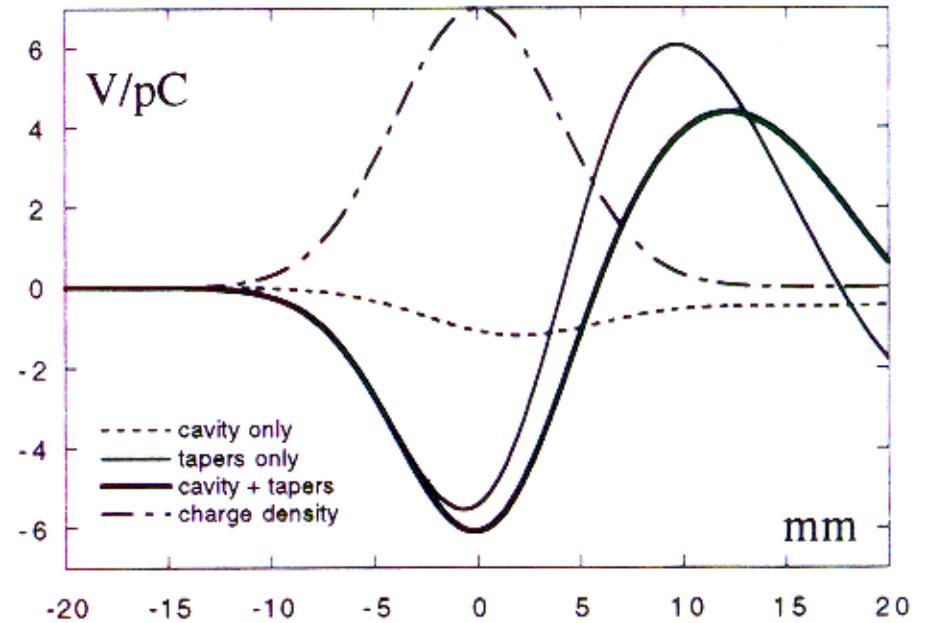
$\Rightarrow$  resonances

## Ex. SOLEIL cavity

$$\sigma_z = 4 \text{ mm}$$



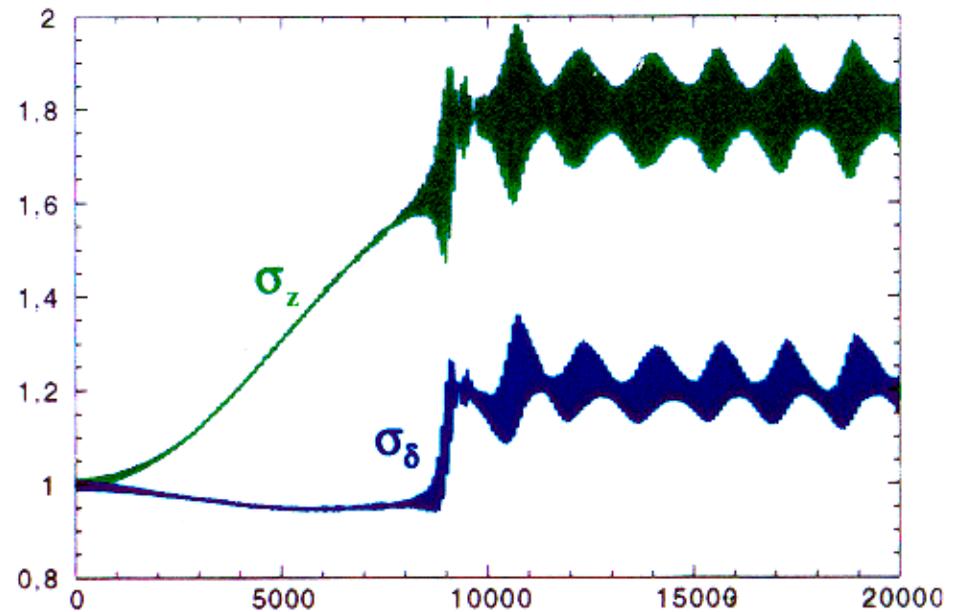
⇒ large broadband resonance  
around 10 GHz



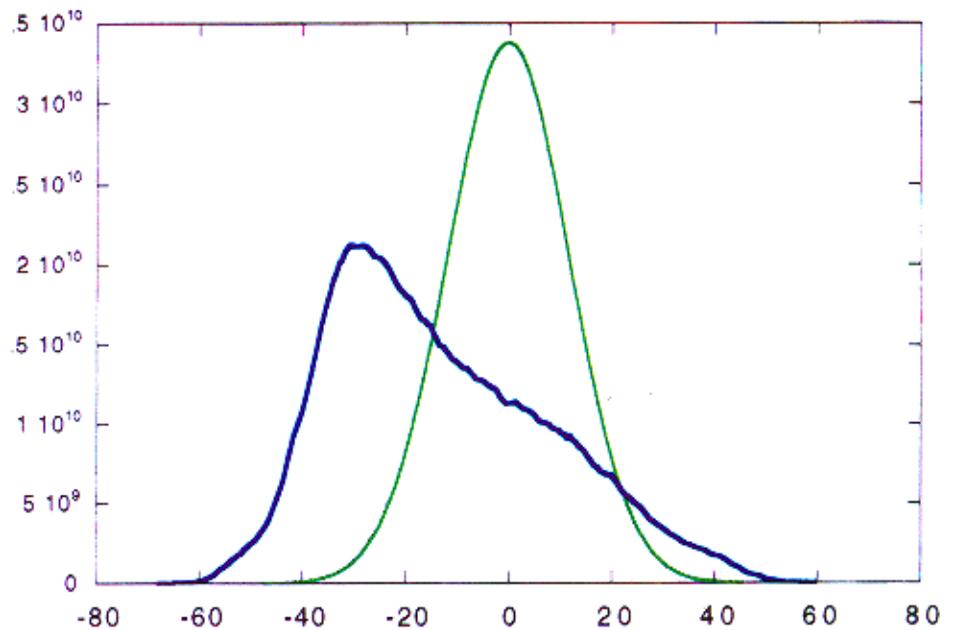
Cavities + tapers only

Tracking simulations ...

$\Rightarrow I_{th} \leq 50$  mA



Initial & final  
charge densities :



# MICROWAVE INSTABILITY & IMPEDANCE MODEL

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Tracking simulations, with the aim of studying the microwave regime with short and intense bunches, suggest different instability mechanisms, according to the impedance model. In order to get a better insight of the source of the instability, i.e. azimuthal or radial mode coupling, we chose to follow the Vlasov-Sacherer approach to investigate the stability of the stationary solution. The generalized Sacherer's integral, including mode coupling and potential well distortion, was then solved by using the "step function technique" for the expansion of the radial function, as proposed by Oide and Yokoya.

For illustration, the effect of the resonant frequency of a broadband resonator in the SOLEIL storage ring was studied. When the resonator frequency is much higher than the bunch spectrum width, azimuthal mode coupling can occur before radial mode coupling.

When the resonator frequency is lower, radial mode coupling comes usually first, but two or more bunchlets are produced at relatively low current. The diffusion process between the bunchlets, which leads to the well-known "saw-tooth" behaviour, originates actually from a fast growing microwave instability.

Lastly, the beneficial effect of an harmonic cavity on the microwave instability is estimated and discussed.

Sacherer's integral, including mode coupling :

$$(\Omega - m \omega_s) R_m(r) = -m \omega_s \psi_o(r) k \sum_{m'} \int G_{mm'}(r, r') R_{m'}(r') r' dr'$$

with the Kernel  $G_{mm'}(r, r') = \int \frac{Z(\omega)}{i \omega} J_m(\omega \sigma_\tau r) J_{m'}(\omega \sigma_\tau r') d\omega / \sigma_\tau$

→ Generalized Sacherer's integral,  
including mode coupling & potential well distortion :

$$(\Omega - m \omega(J)) R_m(J) = -m \omega(J) \psi_o(J) k \sum_{m'} \int G_{mm'}(J, J') R_{m'}(J') dJ'$$

with the Kernel  $G_{mm'}(J, J') = \int \frac{Z(\omega)}{i \omega} K_m^*(\omega, J) K_{m'}(\omega, J') d\omega / \sigma_\tau$

integral equation solved by expanding the radial function  $R_m(J) = \sum_n C_{mn} h_n(J)$   
according to the “mesh technique”, as suggested by Oide and Yokoya\*

⇒ eigenvalue's problem

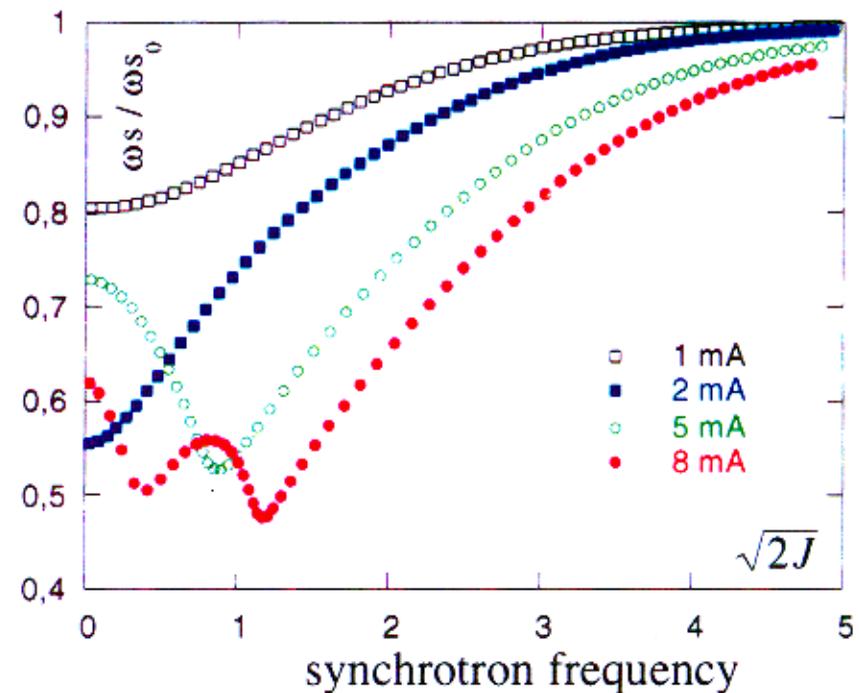
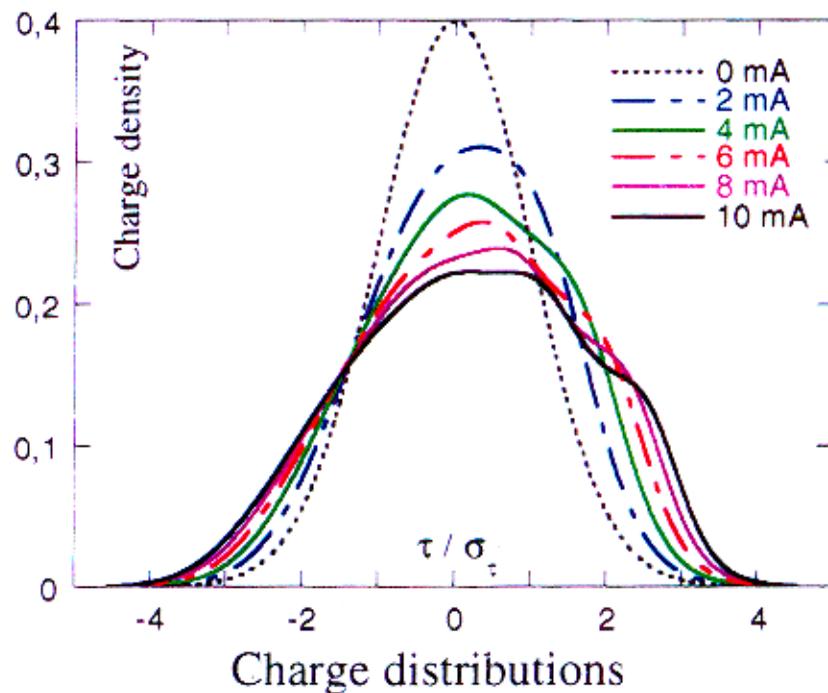
\* K. Oide and K. Yokoya, “Longitudinal single-bunch instability in electron storage rings”,  
KEK Preprint 90-10, April 1990

## For illustration: SOLEIL storage ring + broadband resonator

$R_s$  fixed to  $3.6 \text{ k}\Omega$ , but since the feature of the impedance, as seen by the beam, changes with the resonant frequency, we resolved to vary the resonator frequency on a wide frequency range, from 10 to 30 GHz, in order to study the effect on the phase space topology and, above threshold, on the origin of the microwave instability.

## High Resonant Frequency (30 GHz)

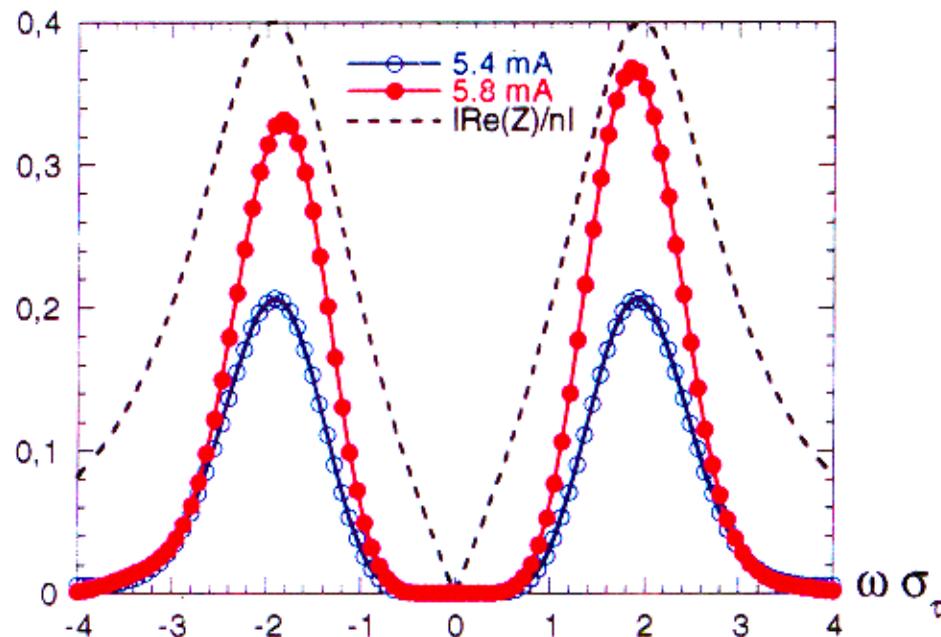
potential well distortion ...



# Stability of the stationary distribution ...

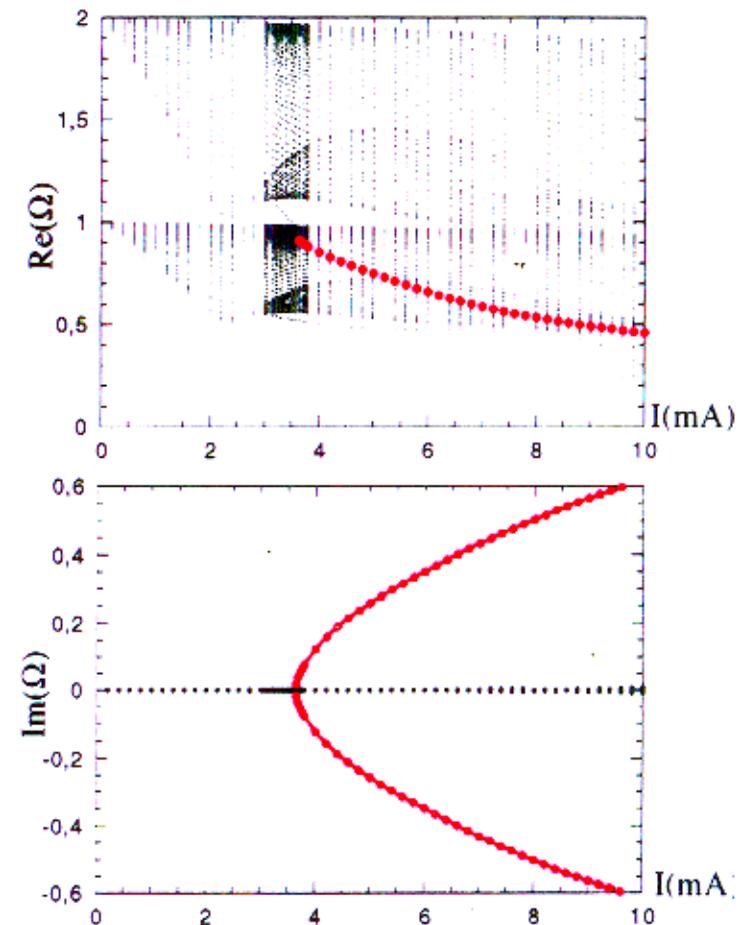
**Radial mode coupling:** eigenvalues calculated for each mode  $m$ , separately  
 $\Rightarrow$  modes  $m=3$  to 5 unstable above 5 mA Ex.

Power spectra of the  $m=3$  mode just before and after the instability threshold (5.5 mA)



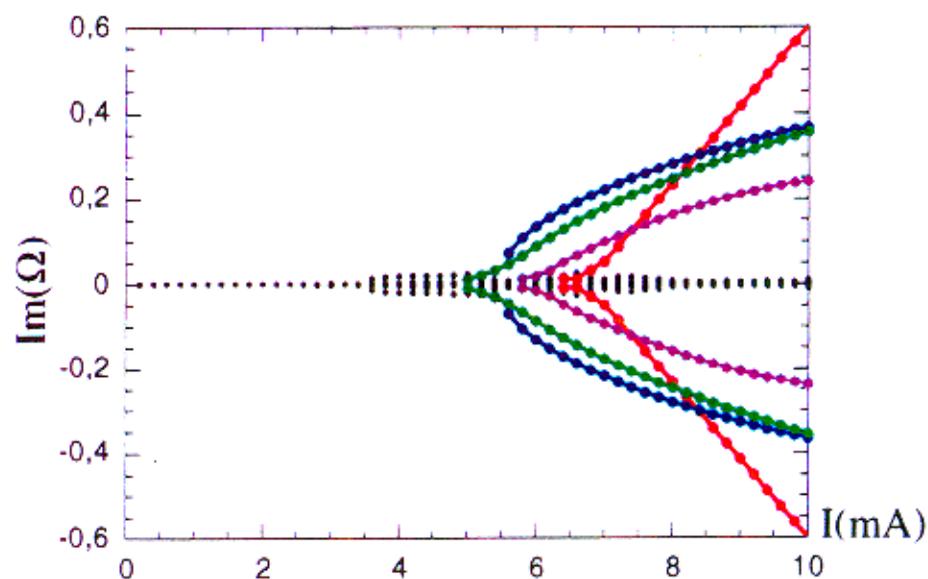
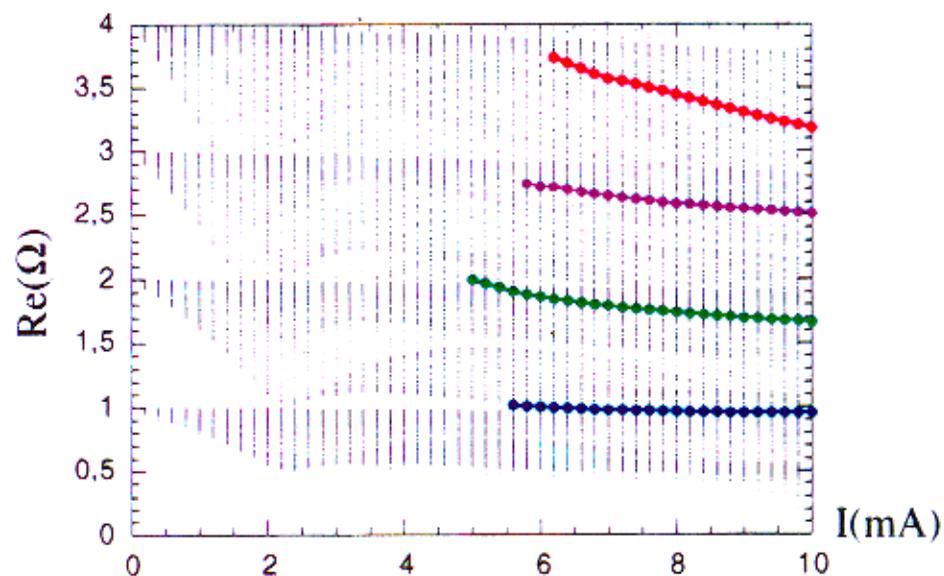
$\Rightarrow$  overlap of resistance with power density higher at freq.  $> 0$  than at freq.  $< 0$  above 5.5 mA, confirming the emergence of radial mode coupling

**Azimuthal mode coupling:** eigenvalues calculated for a pair of mode  $m$   
 $\Rightarrow$  coupling between modes  $m=1$  &  $m=2$  at relatively low intensity

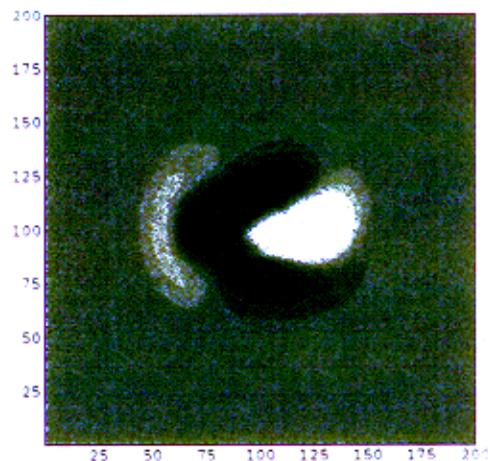


when more azimuthal modes than enough are taken into account ( $m=1$  to 6) :

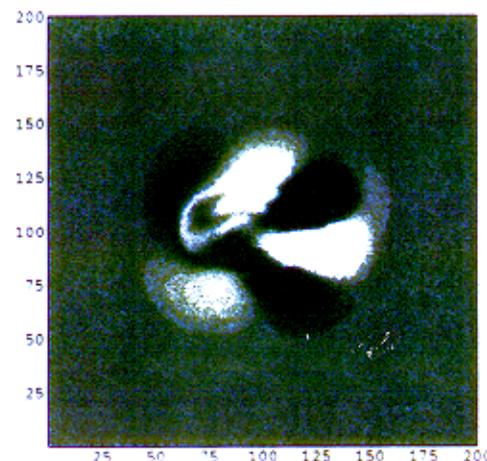
- complete mixing at relatively low current, after a rapid spread
- growth rate increases dramatically around 5 mA (= onset of the instability)
- at higher intensity, the growth rate is larger than the radiation damping rate of the SOLEIL ring
- several types of instability (identified by solid circles) develop simultaneously
- the nature of the most unstable modes changes with the intensity : at threshold 5 mA, microwave instability induced by radial coupling of sextupole mode and coupling of dipole and quadrupole modes; instabilities finally overtaken by the radial  $m=5$  mode coupling above 8 mA; one octupole mode can be also identified, but with a smaller growth rate.



Density-plot of the highest growth rate modes in the physical phase-space (6 mA)  
coupling of  $m=1,2$  modes



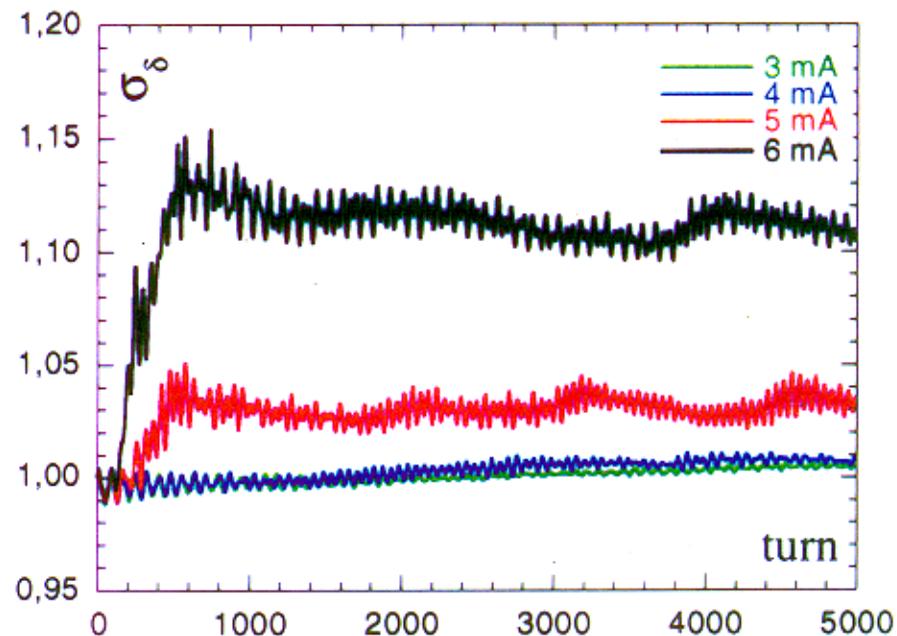
radial coupling of  $m=3$  mode



## Tracking simulations

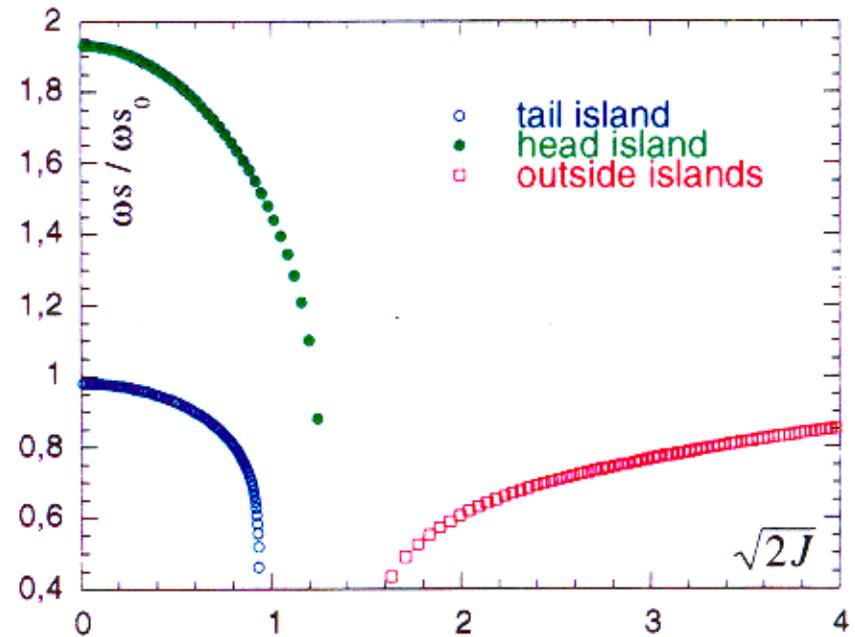
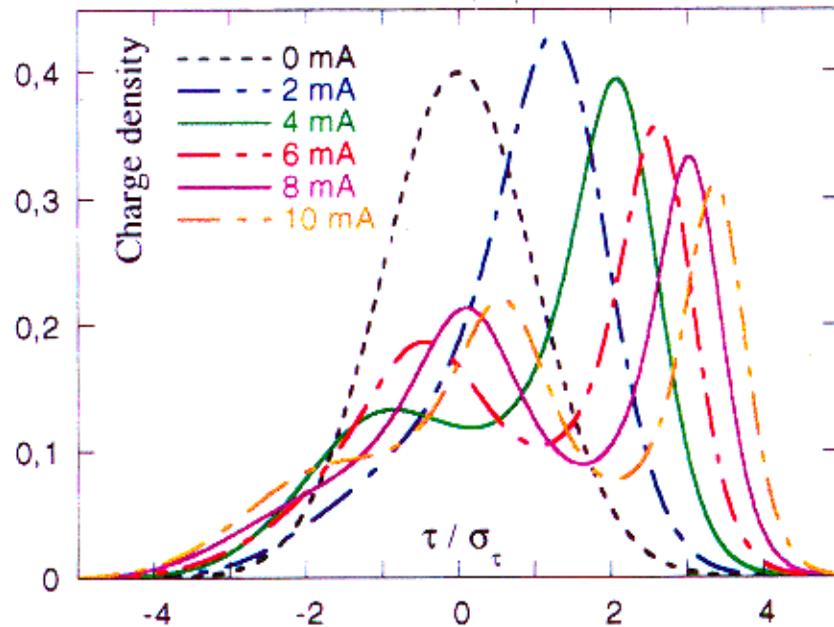
bunch with correct longitudinal density, as computed from the Haissinski's equation is first injected into the ring and the evolution of the distribution in phase-space, bunch length and energy spread is then inspected as a function of the number of revolutions.

threshold = 5 mA confirmed  
+ tiny fluctuations for current slightly < threshold, correspond to weak positive growth rates, close to the synchrotron radiation damping rate, as predicted.

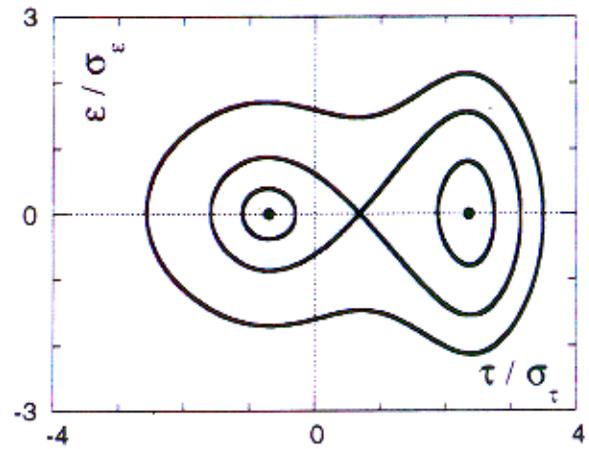
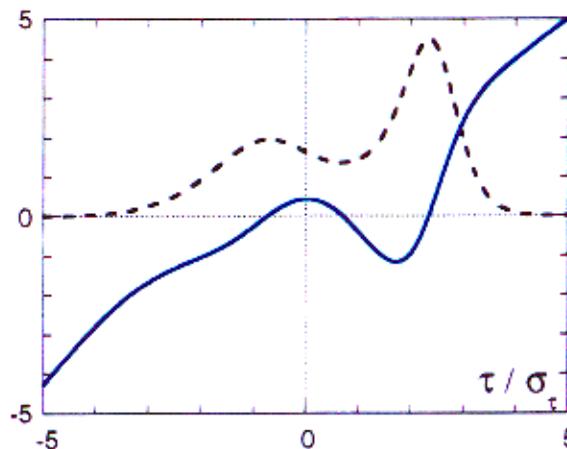


# Low Resonant Frequency (11 GHz)

## potential well distortion ...



bunch much more distorted than before and two peaks appear above 3.5 mA, as soon as there are two or more stable fixed points, forming distinct islands in the phase space



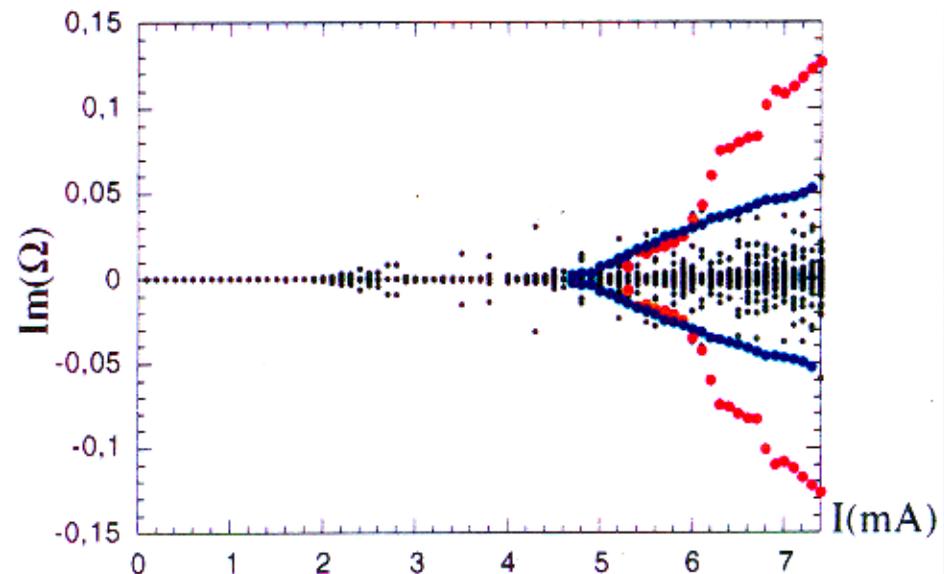
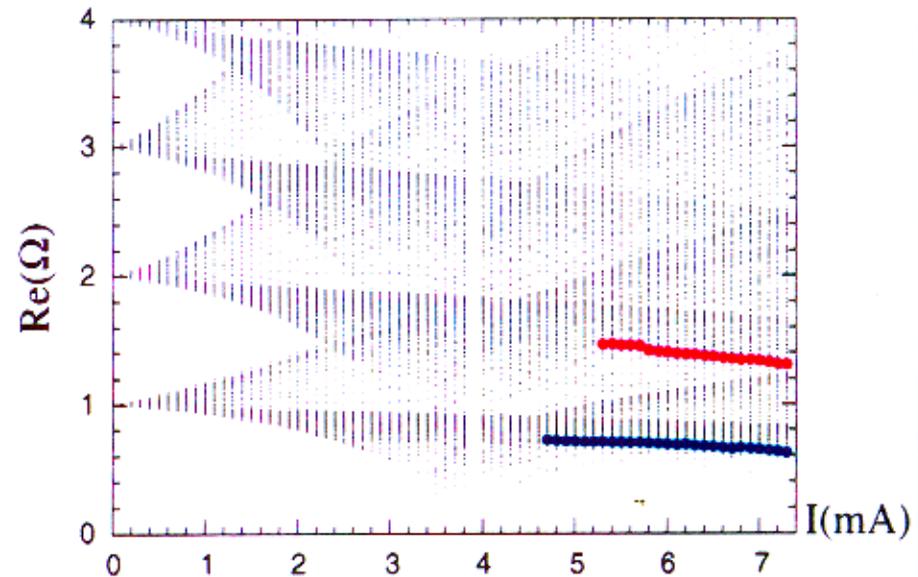
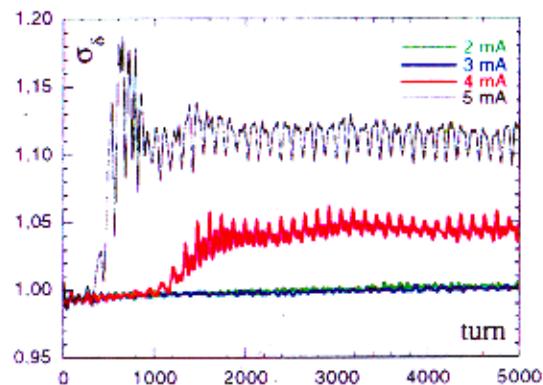
## Stability of the stationary distribution ...

growth rate looks more chaotic than for the higher frequency resonator, because of the rapid change of the topology of the phase space, perturbed by the formation of two or more bunchlets.

Above 4 mA, which can be considered as a threshold, two mode families with regular growth rate increase (identified by solid circles), stand out nevertheless.

It is worth noting the sudden change of behaviour at a current of 6 mA.

## Tracking simulations ...

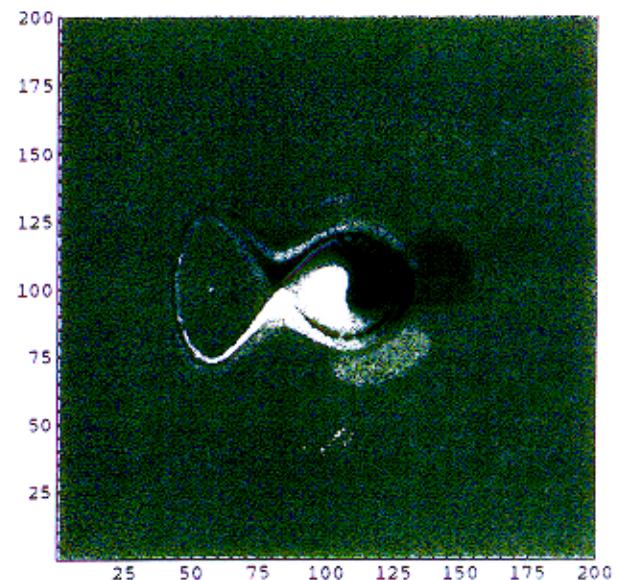
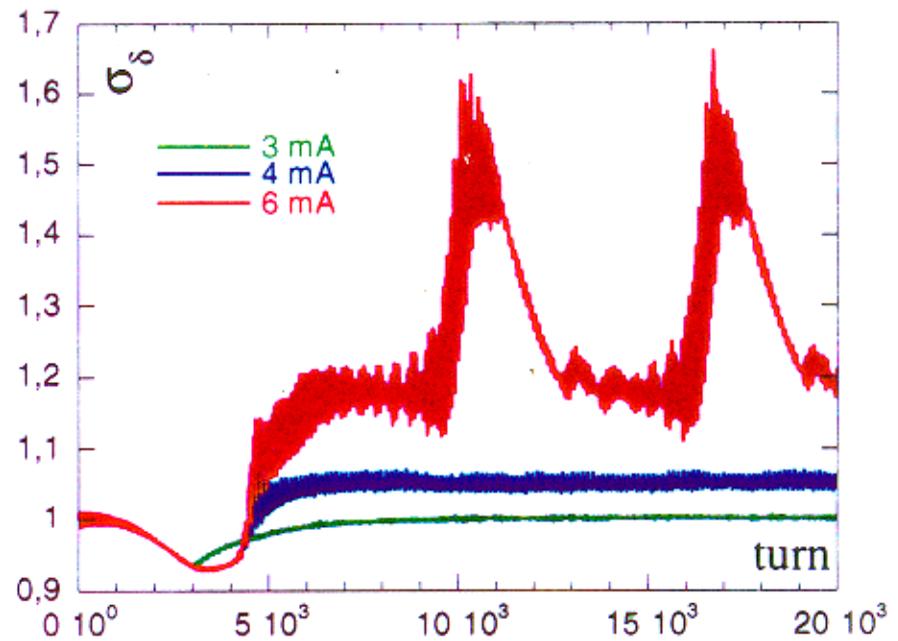


## Saw-tooth instability

the so-called sawtooth instability, already observed in existing machines appears suddenly at 6 mA. Tracking results show a quick increase of both energy spread and bunch length, followed by a slower decrease, with recurrence of about 150 Hz.

**density-plot** of the most unstable mode at the limit of emergence of the sawtooth behaviour (6 mA)

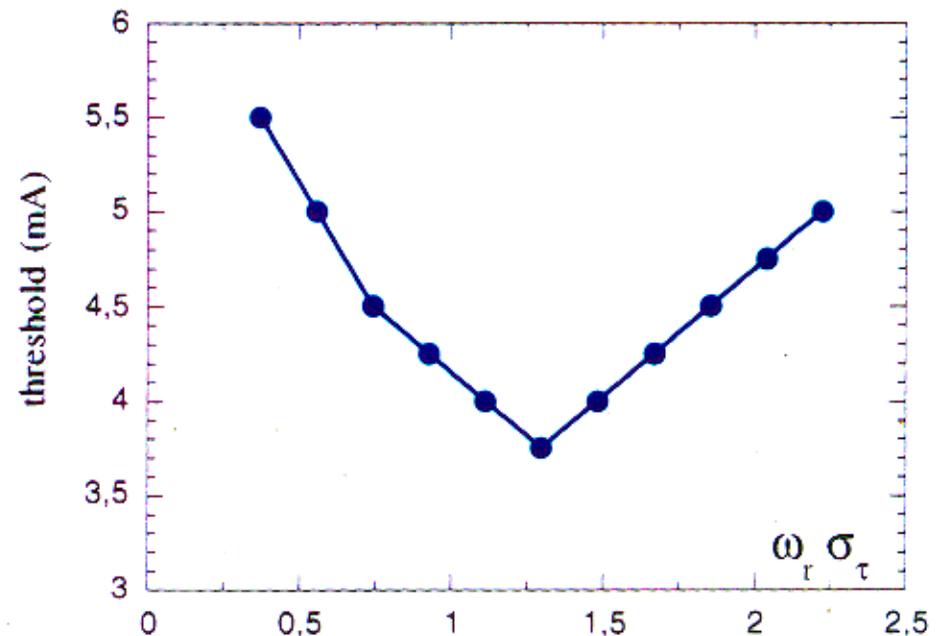
- azimuthal pattern: reveals a pure dipole mode inside the trailing bunchlet
- this unstable dipole mode widens so far as to reach the separatrix of the tail island  $\Rightarrow$  Particles can diffuse through the unstable fixed point and populate the head bunchlet, leading to relaxation oscillations. A phenomenological description of the sawtooth behaviour was suggested in [6], but the diffusion process was assumed to originate from random emission of radiation, instead of strong instability



## Conclusion

The threshold of the microwave instability has been estimated over a wide frequency range of the broadband resonator. Although the source of the instability, radial or azimuthal mode coupling, is changing and although the azimuthal mode number is differing greatly (from  $m=1$  or 2 at low frequency to  $m=5$  or 6 at high frequency), the onset of the instability does not change a lot from 5 GHz to 30 GHz.

The threshold is not the only criterion and simulations showed, in particular, that lower frequency resonators are more harmful since they can induce dipole or quadrupole oscillations of large amplitude. In addition, sawtooth type instabilities can develop, owing to the formation of micro-bunches.



Bunch current, for which the growth rate is just above the radiation damping rate, as a function of the frequency  $\omega_r \sigma_\tau$ .

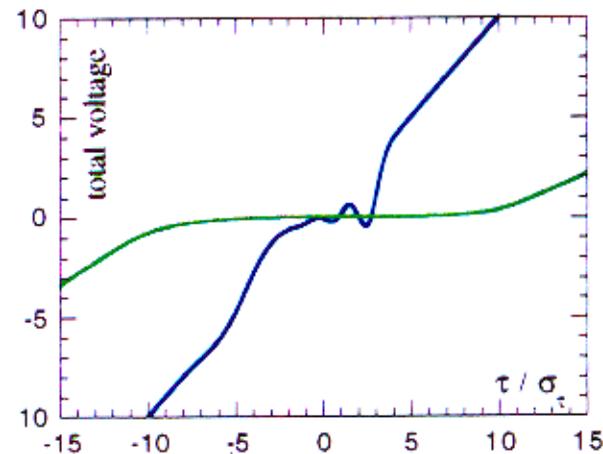
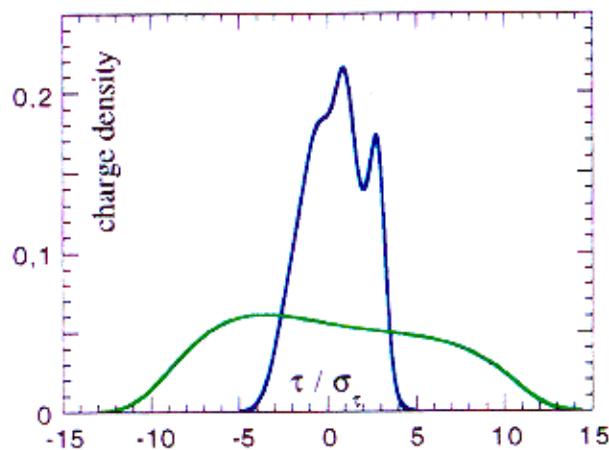
## Harmonic Cavity

Although the primary goal of an harmonic cavity, operating in the bunchlengthening mode, is to increase beam lifetime in Synchrotron Light Sources, it has also a beneficial effect on the microwave instability.

Energy gain : 
$$\Delta E = eV_{rf} [\sin(\phi_s - \phi) + K \sin(\phi_n - n\phi')]$$

Induced voltage (idle cavity) : 
$$k V_{rf} = 2 R I_0 \cos \psi$$

As an example, we look at the effect on the microwave instability, driven by the resonator of intermediate frequency 20 GHz

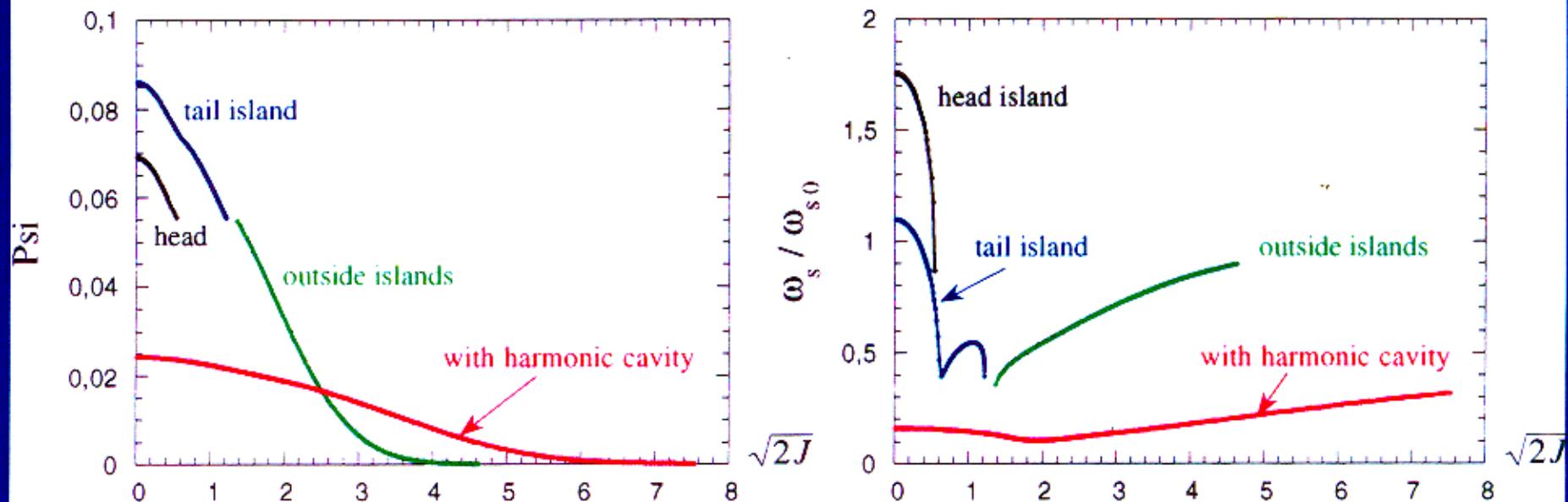


charge distributions (left) and net voltage experienced by the bunch (right) with the sole fundamental cavity and with the additional harmonic cavity.

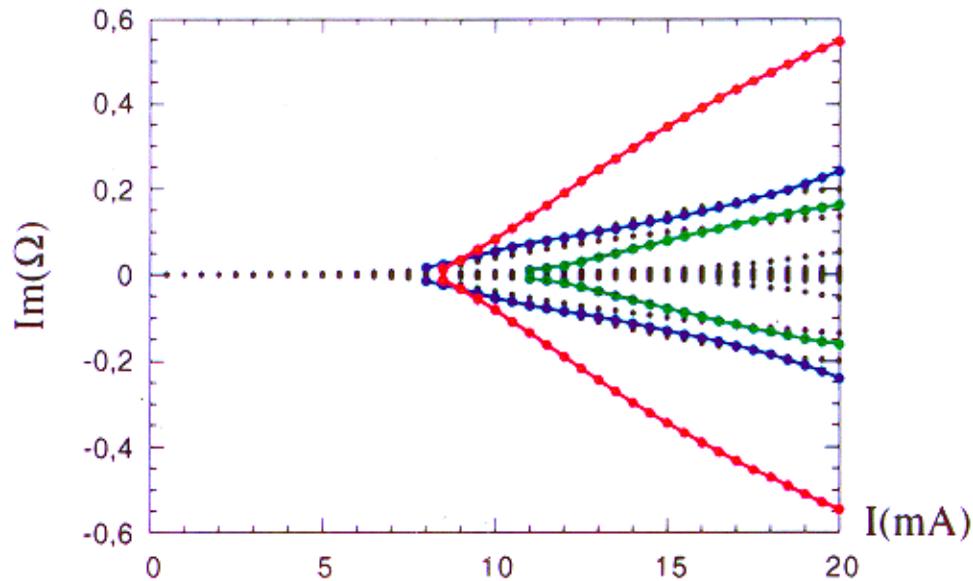
for  $k= 0.328$ , the external focusing is nearly zero around the bunch center and the rms bunchlength is increased by a factor slightly lower than 3.

As the use of the harmonic cavity reduces strongly the **peak current**, we could expect a large increase of the instability threshold.

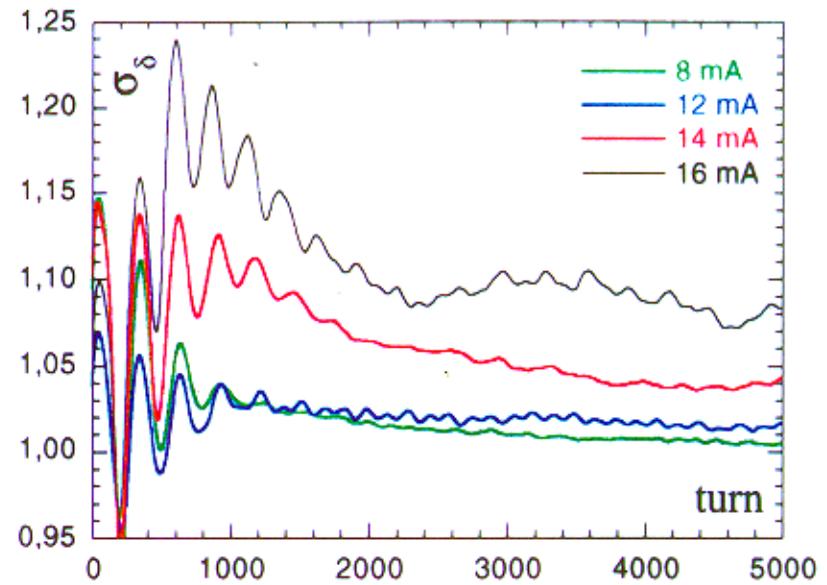
Besides, since the final voltage, including the wake potential, is smoothed off, it will suppress multiple bunchlets, which would appear at relatively low current. However, we found that, even though the particle density is divided by a factor of about 4, the instability threshold enhancement is only a factor two.



This efficiency loss can be explained by the **lower** synchrotron frequency spread due to a lower potential well distortion. In case of short bunches, the non-linearity and then the Landau damping effect of an harmonic cavity, even operating at the third harmonic, is much smaller than the wakefield's one.

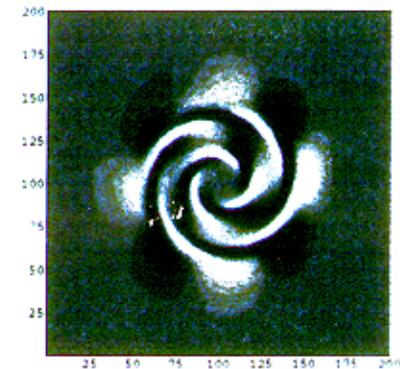
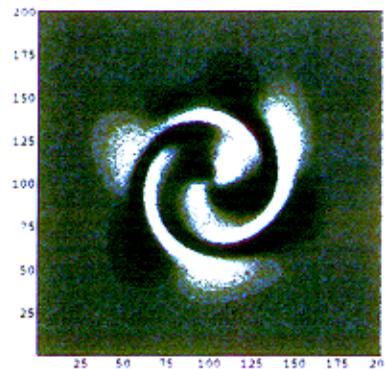
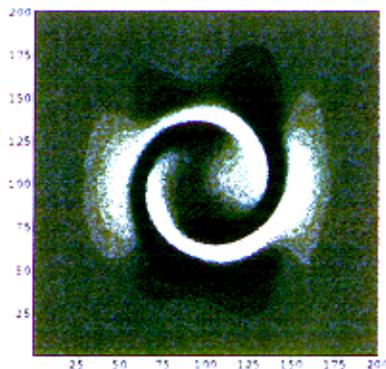


Vlasov-Sacherer method



tracking results

As the bunches are much longer, modes of high azimuthal periodicity can be easily excited. Different periodicities (quad., sext., ...) appear at center and at periphery of the bunch



Distributions of the most unstable three modes (15 mA)