

## Software Assessment of Accelerator Structure Quality with Bead-pull Technique

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### ABSTRACT

Software program to evaluate the quality of traveling-wave accelerator structure at Argonne National Laboratory was developed. In order to understand the extent to which the structure was tuned and thus determine the amount of tuning required, local reflection coefficient  $\Gamma_n$  of each cell in the structure must be measured. The program acquires the perturbed and unperturbed global reflection coefficients,  $S_{11p}$  and  $S_{11u}$ , with a network analyzer and uses the data to compute  $\Gamma_n$  based on Steele's perturbation theory. We verified the accuracy of the software program by testing on a pre-tuned 42-regular-cell accelerator structure operating at  $2\pi/3$  mode. The results give us cell-to-cell phase advance error of less than  $0.5^\circ$ , and the absolute value of the average local reflection coefficient is better than the standard value in the literature.

**Keywords:** Traveling wave RF structure, Steele perturbation theory, bead-pull technique, unwrapped phase, LabVIEW, global reflection coefficient, local reflection coefficient

### INTRODUCTION

The Advanced Photon Source (APS) at the Argonne National Laboratory provides brilliant X-ray beams for world-class research in engineering, physics, biology, chemistry, material science, and environmental science. In order to produce X-ray through particle radiation, electrons are accelerated to  $>99.999\%$  of the speed of light. This can be achieved when high-voltage radio-frequency field in a traveling wave is applied to electrons inside a linear accelerator, and that the electric field

frequency is precisely adjusted to the correct mode for the most effective acceleration.

The  $2\pi/3$  mode has the best compromise between structure length, group velocity, overall dissipation, and efficiency [1][2][3]. Therefore, our accelerator structure is designed such that under  $113^\circ\text{F}$  and at the operating frequency of 2856 MHz, the cell-to-cell phase advance is  $2\pi/3$ , or 120 degrees. However, the technology used to produce the structure has a limited precision, so fine-tuning of each of the cells is still required.

The local reflection coefficient  $\Gamma_n$  of an

accelerator structure cell is a useful parameter that can determine the extent to which deformation of accelerator structure geometry, and thus tuning of the resonant frequency  $f_{res}$ , of each cell is required. Though measurement of  $\Gamma_n$  is not easily achievable, the perturbed and unperturbed global reflection coefficients in the bead-pull method,  $S_{11p}$  and  $S_{11u}$ , can be measured easily with a network analyzer. Therefore, the goal is to calculate  $\Gamma_n$  based on information from  $S_{11p}$  and  $S_{11u}$ , along with Steele's equation and the reasonable assumption that the energy dissipation is minimal.

## THEORY

For accelerator structure with  $N$  cells, the regular cells are those numbered  $n = 2 \dots N-1$  that are neither the input coupler nor the output coupler at the edges. These regular cells can be described as a two-port network with scattering matrix  $S_n$ . When each cell is assumed to be lossless in energy and not containing any nonreciprocal material,  $S_n$  is unitary and symmetrical, and thus can be written as

$$S_n = \begin{bmatrix} \Gamma_n & \sqrt{1-|\Gamma_n|^2} e^{-j\varphi} \\ \sqrt{1-|\Gamma_n|^2} e^{-j\varphi} & \Gamma_n \end{bmatrix} \quad (1)$$

where the matrix element  $\Gamma_n$  is the local reflection coefficient and  $\varphi$  is the phase advance between adjacent cells [1][4]. Suppose that the progressive and regressive waves inside a lossless traveling wave structure have longitudinal electric field amplitudes of  $A_n$  and  $B_n$  at cell  $n$ , the total field will simply be

$$I_n = A_n + B_n \quad (2)$$

and the following relation between adjacent cells must be obeyed [1][4].

$$\begin{bmatrix} B_n \\ A_{n+1} \end{bmatrix} = S_n \begin{bmatrix} A_n \\ B_{n+1} \end{bmatrix} \quad (3)$$

by equations (1), (2), (3), and the assumption that the accelerator structure is ideal with only small  $\Gamma_n$ , we end up with an expression for the local reflection coefficient

$$\Gamma_n = \frac{-I_{n-1} + 2I_n \cos \varphi - I_{n+1}}{I_{n-1} - I_n \exp(-j\varphi)} \quad (4)$$

therefore, in order to obtain the local reflection coefficient at cell  $n$ , we need to obtain the electric fields at cells  $n-1$ ,  $n$ , and  $n+1$ . To achieve this, we implement the bead-pull technique based on the fact that placing a small perturbing object inside the structure will be manifested in terms of a variation of global reflection coefficient measured at the input port. The amount of this variation is dependent upon the electric field at the position of the object before the perturbation. More precisely, the relation between electric field and the global reflection coefficients measured is described by Steele's perturbation theory [5]

$$2P_i(S_{11p} - S_{11u}) = -j\omega k I^2 \quad (5)$$

where  $P_i$  is the input power,  $S_{11p}$  the reflection coefficient in the presence of a perturbing object,  $S_{11u}$  the reflection coefficient in the absence of the perturbing object,  $\omega$  the angular frequency of the electric field,  $k$  a constant that depends on geometry of the object, and  $I$  the electric field at the position of the perturbing object. During a measurement, the values of  $P_i$  and  $\omega$  are kept as constant and do not have any effect on  $\Gamma_n$  in equation (4) due to the division. Hence, we can find the local reflection coefficient  $\Gamma_n$  at each cell just by knowing the global reflection coefficients,  $S_{11p}$  and  $S_{11u}$ .

Also, the resonant frequency  $f_{res}$  and the local reflection coefficient  $\Gamma_n$  of the cell are related by

$$\Gamma_n = jQ'_{0,n} \frac{\Delta f}{f_{RF}} = jQ'_{0,n} \frac{f_{res} - f_{RF}}{f_{RF}} \quad (6)$$

where  $f_{RF}$  is the working frequency,  $f_{res}$  is the resonant frequency of the cell, and  $Q'_{0,n} = c\varphi_n/v_{g,n}$  with  $c$  the speed of light and  $v_{g,n}$  the group velocity of the electric field at the  $n^{\text{th}}$  cell. Therefore, we can tune the resonant frequency by deforming each cell until we observe the smallest magnitude of the reflection coefficient. That is, we look for  $f_{res}$  that results in the smallest  $|\Gamma_n|$ . Typically, this corresponds to  $|\Gamma_n| < 0.005$  [1].

To find a value for the phase advance  $\varphi$  in equation (4), instead of looking for the phase difference between cells  $n-1$  and  $n$ , a more realistic algorithm taking into account cells  $n-1$ ,  $n$ , and  $n+1$  was used to find the phase advance at cell  $n$

$$\varphi = \cos^{-1}\left(\frac{I_{n-1} + I_{n+1}}{2I_n}\right) \quad (7)$$

the imaginary part of the argument inside arccosine vanishes in the ideal case, but the assumptions made behind the algorithm are not entirely valid in the reality [6]. Fortunately, the magnitude of the imaginary part is usually negligible compared to that of the real part.

Note that the previous equations to calculate the phase advance and local reflection coefficient at cell  $n$  depend on the electric field measured at cells  $n-1$ ,  $n$  and  $n+1$ . Therefore, a separate formula to compute the local reflection coefficient of the output coupler cell  $\Gamma_N$  is desired. Once all the regular cells and the output coupler cell are tuned, we simply tune the input coupler cell such that  $S_{11u}$  becomes the minimum. Assuming that the output coupler can be considered as an ensemble of cells  $N-1$  and  $N$ , the local reflection coefficient of the output coupler cell  $\Gamma_N$  can be expressed as [1]

$$\Gamma_N = -j \frac{\alpha}{\sin(2\varphi)} \quad (8)$$

where  $\alpha$  is the real part of the average local reflection coefficient of cell  $N-1$

$$\bar{\Gamma}_{N-1} = \frac{\frac{1}{M} \sum_2^{M+1} B_{N-i} e^{j(i-1)\varphi}}{A_{N-1}} \quad (9)$$

for better accuracy, the numerator was obtained by taking the average of the reflected waves, which could be deduced based on the reflections at other cells corrected by the phase differences. In our case, we chose the averaging number  $M$  to be 41, taking into account all the regular cells except for cell 43 (cell  $N-1$ ) that we were trying to calculate in equation (9).

## METHODS

We tested the LabVIEW program with an accelerator structure having 42 regular cells designed to work at the nominal frequency of 2856 MHz and phase advance of  $2\pi/3$  per cell, where each cell has length of 34.989 mm. During the tuning process, the external surface, and thus the geometry, of each cell can be deformed by the tuners. There are irises coupled between cells and both ends of the cavity are connected to an input or output matching coupler, whose purpose were to convert between the rectangular waveguide mode and the traveling wave accelerating mode inside the accelerator structure.

The perturbing object is a hollow cylindrical metallic bead steadily positioned at the center of the accelerator structure by a nylon line that is oriented parallel to the axis of the structure. The bead has 2 mm outer diameter, 0.4 mm inner diameter, and 5 mm length. The value of the unperturbed global reflection coefficient,  $S_{11u}$ , was measured by Agilent Technologies' Network Analyzer E8362B and captured by the LabVIEW program when the bead

was outside of the structure. The value of the perturbed global reflection coefficient,  $S_{11p}$ , was acquired by the same equipment and program while the bead moved along the structure. During the measurement of  $S_{11p}$ , the Oriental Motor's PK264-E2.0A stepping motor was controlled by LabVIEW to move in step size of 1 mm. Therefore, the nylon line that was connected to the motor, and thus the perturbing bead on the nylon line, also moved in step size of 1 mm. Starting from outside of the input coupler, the bead walked over 1660 mm and taking 1660 data points until it passed the output coupler on the other end of the structure. Based on equations (4) and (5), algorithms were written in LabVIEW to find values of  $\Gamma_n$ . The geometry of the cell, and thus its resonant frequency, can then be tuned by looking for the minimum value of  $|\Gamma_n|$  with a tuner.

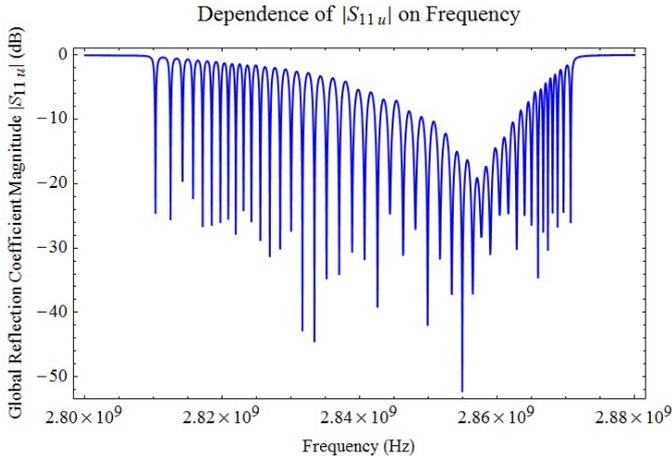
A challenge in the LabVIEW algorithm was to compute the phase advances using equation (7), where electric fields associated with positions apart from each other by a cell length were required. In other words, electric fields corresponding to the same position relative to each cell were desired. Since the electric field magnitude in the complex plane is a periodic function with cell length as the period, our goal can be achieved by including the algorithm that seeks the electric field within each cell that has a maximum magnitude [7]. These electric fields are then associated with positions separated from each other by a cell length and represent  $I_n$ 's in equation (4).

Another issue has to do with the network analyzer, which cannot distinguish angles between  $\theta$  or  $\theta \pm 360 \cdot x$  degrees, with  $x$  being some integer. However, we know that the phase of the electric field, and thus that of the  $S_{11p}$ - $S_{11u}$  in equation (5),

on an accelerator mode decreases from the input coupler toward the output coupler [1][2][6][7]. On the complex plane, this meant that  $S_{11p}$ - $S_{11u}$  should rotate in the clockwise direction down the structure. Therefore, we employed the "unwrap phase" algorithm that subtracts the phase of  $S_{11p}$ - $S_{11u}$  by an additional 360 degrees whenever the phase moves clockwise from the first quadrant to the fourth quadrant. To address the situation where data points cross the border of first/fourth quadrant back and forth for a few times due to small randomness, 360 degrees is added back whenever the phase has a sudden change from the fourth quadrant back to the first quadrant. Eventually, a net angle subtraction of 360 degrees is involved when the number of border-crossings, including both directions, is an odd number. Based on equation (5), we can then obtain the phase of the electric field by halving the unwrapped phase of  $S_{11p}$ - $S_{11u}$  added by 90 degrees, where the addition of 90 degrees is due to multiplication of the imaginary unit  $j$ .

## RESULTS

A traveling wave was composed by several identical cells strongly coupled to each other. When the magnitude of the unperturbed global reflection coefficient  $|S_{11u}|$  was measured at the accelerator structure input cell, several modes of the structure indicated by local minima of  $|S_{11u}|$  at various frequencies were shown on the network analyzer. This was because at a mode of the structure, most of the energy was accumulated inside such that the reflection was minimal. Since this structure was designed and pre-tuned to operate at 2856 MHz, the  $|S_{11u}|$  plot shown in **Figure 1** with the lowest modes around 2856 MHz was expected.

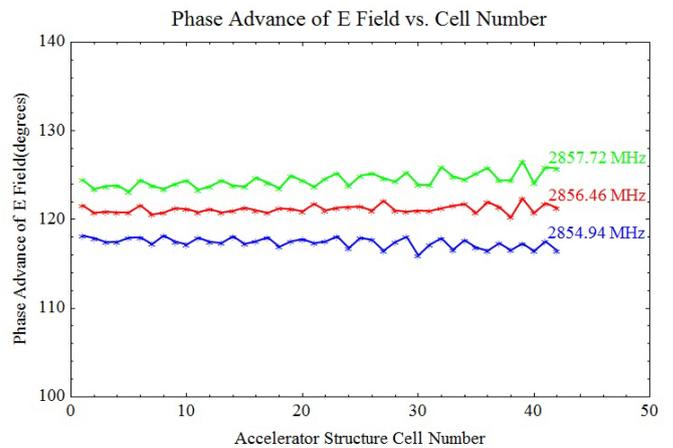


**Figure 1.** Magnitudes of global reflection coefficients measured at different frequencies, with temperature fixed at 77°F. This accelerator structure is designed such that at 113°F,  $2\pi/3$  exists when the frequency is at 2856 MHz. However, our experiment was done at 77°F, so the  $2\pi/3$  mode frequency shifted to 2856.46 MHz.

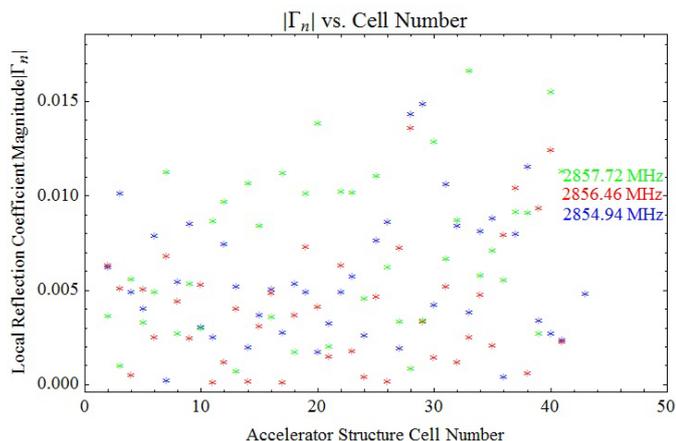
The accelerator structure is designed such that the  $2\pi/3$  mode exists when the traveling radio-frequency wave has frequency near 2856 MHz at 113°F. However, our experiment was done under room temperature, 77°F, so the frequency corresponding to the  $2\pi/3$  mode was shifted up slightly [1]. On the network analyzer, seven modes that had frequencies near 2856 MHz were selected (2851.68 MHz, 2853.40 MHz, 2854.94 MHz, 2856.46 MHz, 2857.72 MHz, 2859.00 MHz, and 2860.38 MHz) and their corresponding  $S_{11p}$  and  $S_{11u}$  were acquired. By equations (4)(5)(7), as well as the algorithms to find the same position at each cell and unwrap the phases mentioned earlier, our LabVIEW program then plotted the phase advance of the electric field and magnitude of the local reflection coefficient at each cell of the structure. The three modes corresponding to phase advances closest to  $2\pi/3$  and their corresponding magnitudes of local reflection coefficients  $|\Gamma_n|$  are shown in

**Figure 2** and **Figure 3**. In these plots, the 42 regular cells of the accelerator structure are indexed from 1 to 42. The input coupler and output coupler cells, indexed 0 and 43, are not shown here since equations (4) and (7) for local reflection coefficient and phase advance calculation take into account not only the electric field of the current cell, but also those of the previous and the next cells.

In **Figure 2**, the average phase advances of the modes at 2854.94 MHz, 2856.46 MHz, and 2857.72 MHz were  $117.401^\circ$ ,  $121.190^\circ$ ,  $124.445^\circ$ . Therefore, the best candidate for the  $2\pi/3$  mode is at 2856.46 MHz, which has about  $\pm 0.500^\circ$  of phase advance error from the average value and corresponds to  $|S_{11u}|$  of -37 dB and lowest average  $|\Gamma_n|$  of 0.00420932 in **Figure 3**. Also, the local reflection coefficient magnitude of the output coupler cell,  $|\Gamma_M|$ , was found to be 0.0344910.



**Figure 2.** Phase advance of E field at each cell of the accelerator structure. The plot shows three modes closest to the  $2\pi/3$  mode, with 2856.46 MHz giving the average phase advance of 121.190 degrees.



**Figure 3.** Local reflection coefficient magnitude  $|\Gamma_n|$  at each cell of the accelerator structure. The plot shows  $|\Gamma_n|$  of three modes closest to the  $2\pi/3$  mode, with 2856.46 MHz giving the lowest average  $|\Gamma_n|$  of 0.00420932.

These results in **Figure 2** and **Figure 3** showed that the accelerator structure was tuned fairly well for all cells, in consistent with the design of this 42-regular-cell structure. Therefore, the experiment also demonstrated the success and effectiveness of the LabVIEW program to verify the design frequency and phase advance of the structure. In the future, the LabVIEW code will be used on accelerator structures that are mistuned to identify the exact cells that need to be tuned. For instance, a sudden variation in phase advance or a large value in local reflection coefficient magnitude may indicate that the corresponding cell is troublesome. Also, based on the local reflection coefficient,  $\Gamma_n$ , we can understand the extent to which tuning of the resonant frequency is required and fix the structure. These structures include the Linear Accelerator Cavity for electron acceleration and the deflection cavity used for characterization of electron gun performance. Both cavities are crucial for the Advanced Photon Source of Argonne National Laboratory to provide the top-notch X-ray beams

for users around the world in disciplines of materials science, physics, biology, chemistry, and environmental sciences.

## CONCLUSION

When the 42-regular-cell accelerator structure was operating at  $2\pi/3$  mode at frequency of 2856.46 MHz and temperature of 77°F, the unperturbed and perturbed global reflection coefficients were measured by the network analyzer. This data allowed us to compute the electric field phase, phase advance, and the local reflection coefficient at each cell. At the desired mode, the average phase advance  $\varphi$  of our accelerator structure was calculated to be  $121.190^\circ$ , with small variation of about  $\pm 0.500^\circ$ . The average magnitude of local reflection coefficient  $|\Gamma_n|$  was 0.00420932, which was better than the standard value suggested by the literature, 0.005.

A LabVIEW program for efficient and effective accelerator structure tuning was also developed. In the future, this program will be used on other accelerator structure to test the extent to which structures are mistuned and perform the required tuning. These structures include the Advanced Photon Source's Linear Accelerator for electron acceleration and the deflection cavity for characterization of the electron gun performance at the Advanced Photon Source, Argonne National Laboratory.

## REFERENCE

- [1] D. Alesini et al. "Tuning procedure for traveling wave structures and its application to the C-Band cavities for SPARC photo injector energy upgrade." *Journal of Instrument*, vol. 8, Oct. 2013.
- [2] J. Shi et al. "Tuning of Clic accelerating

structure prototypes at CERN,” in *Proc. LINAC*, 2010, pp. 97-99.

[3] T. Shintake, et al. “Experiment of nanometer spot size monitor at FFTB using laser interferometry,” in *Proc. Particle Accelerator Conference*, 1995.

[4] R. E. Collin. *Foundations for microwave engineering*. Columbus, OH: McGraw-Hill, 1992.

[5] C. W. Steele. “A Nonresonant Perturbation Theory.” *IEEE Trans. on Microwave Theory and Techniques*, vol. 14, Feb. 1966, pp. 70-74.

[6] N. M. Kroll, et al. “Applications of Time Domain Simulation to Coupler Design for Periodic Structures,” in *International Linac Conference*, 2000.

[7] S. H. Wang. Lecture, Topic: “RF electron LINAC.” OCPA 4, Yangzhou, Jiangsu, China, Jul. 27 – Aug. 5, 2006.

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### Author Contributions

L-. C. Tai developed the LabVIEW code used to obtain the results and performed the experiments. G. Waldschmidt designed the project and the original version of the LabVIEW code, provided simulation results, and supervised the experiments. T. L. Smith and T. D. Jonasson set up the bead-pull motor and accelerator structure temperature-control systems. The paper was written by L-. C. Tai, with inputs based on discussions with all of the co-authors.

### Additional information

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