FULLY COUPLED ANALYSIS OF ORBIT RESPONSE MATRICES AT THE FNAL TEVATRON*

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Abstract

Optics measurements have played an important role in improving the performance of the Fermi National Accelerator Laboratory (FNAL) Tevatron collider. Initial optics measurements were performed using a small number of differential orbits, which allowed us to carry out the first round of optics corrections. However, because of insufficient accuracy, it was decided to apply the response matrix analysis method for further optics improvements. The response matrix program developed at Argonne National Laboratory (ANL) has been expanded to include coupling – the essential feature required to describe the Tevatron optics. The results of the optics calibration are presented and compared to local beta function measurements.

INTRODUCTION

Tevatron is the largest proton-antiproton collider in the world. The commissioning of Tevatron Run II began in the spring of 2001 with the first luminosity seen in June. Since then, improving the linear optics model played an important role in the steady increase of luminosity. Until recently, the linear optics measurements were based on manual analysis of a few differential orbits [1], which neglected measurement inaccuracies, such as differences in beam position monitors (BPM) differential responses, BPM rolls, etc. This was a tedious procedure, which could not provide a full-scale optics determination; however, it did determine and correct major optics problems.

To completely determine the linear model of Tevatron, we have applied a response matrix fitting method based on the analysis of many differential orbits. This method creates the redundancy in the data that allows us to get a much more detailed understanding of the machine.

MEASUREMENTS AND DATA PROCESSING

Response matrix fitting is a well-known method of calibrating the machine optics. It was first suggested at SLAC [2] and then it was used at NSLS [3] for X-ray ring analysis. Today the method is widely used on many accelerators around the world.

Application of the response matrix fit method at the Advanced Photon Source (APS) at ANL allowed us to greatly improve our understanding of the storage ring model and to improve beam lifetime and injection efficiency [4]. The response matrix fitting software written at APS was used for Tevatron analysis.

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Software Modifications

The response matrix fitting program *SRLOCOFitting* [4] is written in Tcl/Tk, has an extensive graphical user interface, and uses SDDS toolkit [5] for data processing. For accelerator-related calculations, it uses `elegant` [6]. *SRLOCOFitting* was written to calibrate the APS model and to provide data for beta function correction. Coupling correction was not an issue at APS, therefore the calculations were limited to the noncoupled case.

On the contrary, coupling of horizontal and vertical motion is considered to be an important issue at Tevatron, therefore existing analyses had to be expanded to a fully coupled motion. Also, the lattice calculations for Tevatron at Fermilab are done with `OptiM` [7]. For the convenience of Fermilab collaborators, the decision was made to use `OptiM` as the code for response matrix calculations for Tevatron. Therefore, the software upgrade consisted of two major parts: the interface between the fitting software and `OptiM`, and implementation of coupled motion analysis.

To make the interface between the fitting program and accelerator codes simple, a separate program *calculateResponseMatrix* was written. This program runs `elegant` or `OptiM` to calculate the response matrix depending on accelerator name. It calculates response matrices using either beta functions or direct orbit calculations. It also converts `OptiM` output into SDDS format.

Coupled matrix fitting was implemented in two modes: coupling-only and full-matrix fitting. To explain this, we define the complete response matrix as

$$M = \begin{pmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{pmatrix},$$

where M_{xx} is horizontal response to horizontal correctors, M_{xy} is horizontal response to vertical correctors, etc. The fitting program has three analysis modes:

- No coupling – only diagonal matrices M_{xx} and M_{yy} are used. The following variables can be used for fitting: quadrupole gradients, corrector calibrations, BPM gains, and energy shift due to correctors.
- Coupling only – only off-diagonal matrices M_{yx} and M_{xy} are used. The variables used are: quadrupole tilts, corrector tilts, and BPM tilts.
- Full analysis – the complete matrix M is used. The variables are the combination of the previous two sets.

In order to explain why the coupling analysis was split into two different modes, we remind that one part of the response matrix analysis is building the derivative of the response matrix with respect to all variables used in the fitting and then inverting it. For the APS storage ring, the typical number of variables for "No coupling" or "Coupling only" modes is 1300, and the size of each M_{zz} in the response matrix is 40×400 (we usually limit the number correctors to 40 in each plane). This gives a derivative size of about 350 Mb. Such a matrix requires approximately 1.5 Gb of computer memory to invert. Full analysis for the APS would require a four times larger matrix and memory, which seems excessive for an ordinary computer. Also, the existing coupling at the APS storage ring is small, which makes the "Coupling only" mode a reasonable approach. On the other hand, Tevatron has fewer BPMs and therefore a smaller derivative of the response matrix. It also has larger coupling, which requires fully coupled analysis.

A dispersion fit was also added. The dispersion is treated as just one more column of the response matrix. The dispersion can also be used to calibrate average gain of BPMs, which otherwise would be a degenerated value. A number of other minor modifications have been made as well to ensure that the software could be used with other Fermilab accelerators.

Measurements and Response Matrix Fit

Tevatron has 110 correctors and 118 BPMs in each plane. The response matrix measurement procedure is as follows: each steering magnet is excited first with positive current and then with negative. At each current the orbit is measured 25 times. The total response to the steering magnet excitation is the average positive orbit minus average negative orbit. The output of the measurement program is an SDDS file containing average orbit responses and their RMS.

The dispersion measurement is done by scanning rf frequency, measuring orbit at five points and fitting a straight line at each BPM. This allows us to improve the measurement accuracy and to calculate the error bars of the measurement. Figure 1 gives an example of measured vertical dispersion.

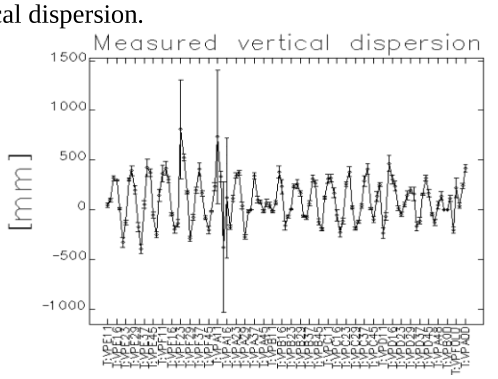

Figure 1: Measured vertical dispersion. The error bars are calculated from slope fitting.

The first full-scale response matrix measurements involving all steering magnets were done in August 2004. Here we will present the analyses of these measurements. The following variables were used in the fit: quadrupole gradient errors; corrector calibration errors; BPM gains; energy shift due to corrector changes; and quadrupole, BPM, and corrector rolls. The size of the full response matrix derivative is about 500 Mb and is too large to be inverted in a reasonable time. The response matrix was split into three approximately equal subsets, and each subset was analyzed separately. Figure 2 shows an example of measured and calculated coupled responses before and after fitting.

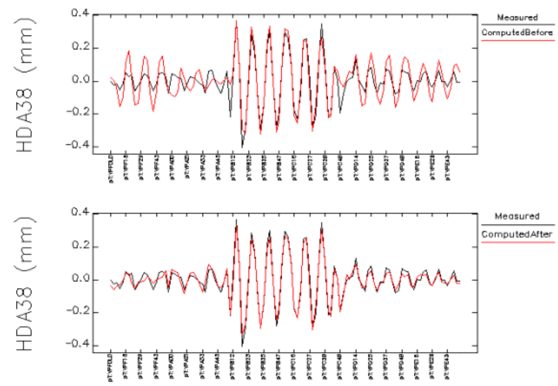

Figure 2: Typical measured and calculated responses before and after the fit. A vertical response to a horizontal corrector is shown.

A summary of the residual RMS errors after the fit is presented in Table 1. For all three sets of correctors the solution converged to approximately the same value of residual errors. This value also corresponds to the present accuracy of Tevatron BPMs, which means that the fitting is done to the best possible level and is limited by BPM accuracy only.

Table 1: RMS Difference between Calculated and Measured Response Matrices Before and After the Fit

	Before fit	Set 1	Set 2	Set 3
x-x (μm)	160	21	23	24
y-x (μm)	120	19	19	20
x-y (μm)	100	19	17	19
y-y (μm)	200	24	22	26
Hor. disp. (mm)	240	60	60	68
Ver. disp. (mm)	190	52	58	57

Accuracy of the Fit

The result of the fit is a set of variables used in the fit that makes model response matrix coincide with the measured response matrix within the accuracy of the measurements. Of those variables, the most important are quadrupole gradient errors and rolls that define beta functions and coupling of the machine. After the fit is done, one has to answer a very important question: is the accuracy of the measurement enough for the fit to produce a model that uniquely resembles the real storage ring?

As was mentioned before, the measured response matrix was split into three parts, and each part was analyzed separately. Comparing the results of separate fits, we can estimate the accuracy and uniqueness of the resulting model. Figure 3 (top) shows quadrupole gradient errors calculated in three different fits. The difference

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between these solutions is large, and this suggests that for present measurement accuracy the number of quadrupoles used in the fit is too large. Foreseeing this problem, we allowed only half of all quadrupoles to be varied for the coupling fitting. Figure 3 (bottom) shows skew quadrupole gradients that resulted from the fits. The variation between the solutions is much smaller than for the case of quadrupoles. This is an indication that if one wants to have a unique set of quadrupole errors that define the lattice, the number of quadrupoles in the fit has to be reduced.

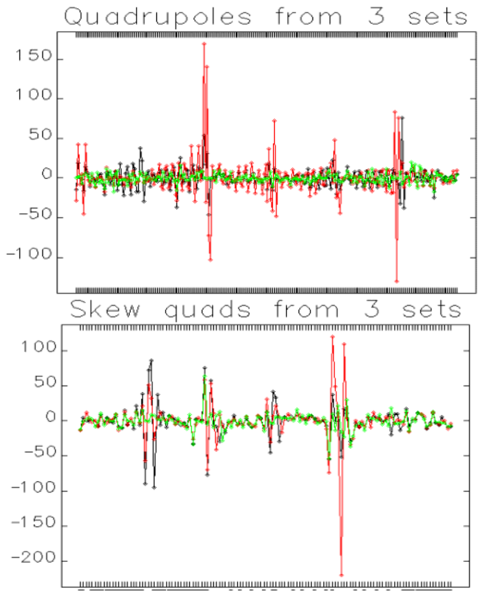

Figure 3: Top – quadrupole errors obtained by analyzing three different data sets; they show significant difference between different sets. Bottom – skew quadrupole errors for the same data sets; the variation between solutions is smaller due to smaller number of variables used in the fit.

As one can see with quadrupoles, variables can be degenerate. However, some of the variables, like BPM calibrations and rolls, have to be unique. Indeed, Fig. 4 presents calibration errors of horizontal BPMs for all three solutions, but there is little variation between solutions. The RMS difference of BPM gain errors between the solutions is 1.7% in the horizontal plane and 2.1% in the vertical plane.

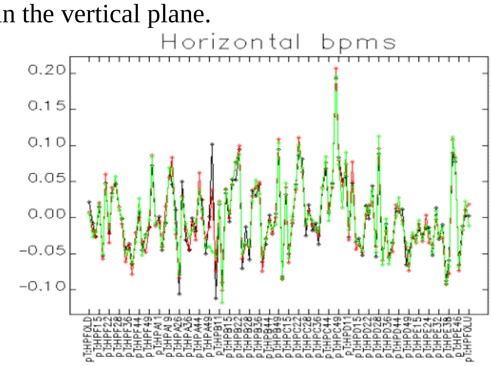

Figure 4: Horizontal BPM gains resulted from analysis of three different data sets. As expected, the differences between data sets are small.

Ambiguity of the quadrupole solutions may seem to be an important drawback of the fit; however, more important is whether the beta functions of the machine are reproduced in different solutions. Figure 5 demonstrates the relative difference between the horizontal beta function of one of the solutions and the average beta function based on all three sets. Two spikes on the plot correspond to interaction points where the beta function is very small. The total RMS differences between beta functions calculated using different quadrupole sets are 2.2% for horizontal, 3.1% for vertical beta functions, and 2.9% for horizontal dispersion.

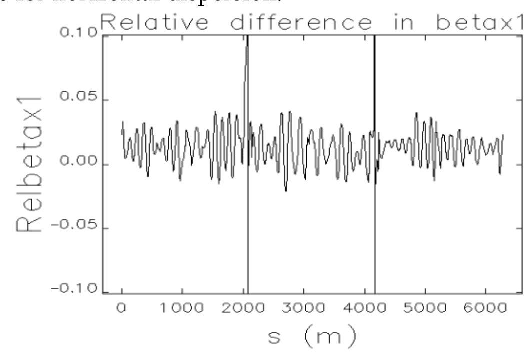

Figure 5: Relative difference in horizontal beta function between one of the data sets and the average beta function calculated based on all three solutions.

CONCLUSION

We have significantly improved the linear model of Tevatron with coupling using response matrix fitting. The accuracy of the measurements does not allow us to uniquely define quadrupole errors in the ring; however, the accuracy of the beta function determination is estimated to be below 3%. Independent measurements of beta functions using quadrupole scans confirm improvement of the model. We have also defined gain errors of all BPMs.

We anticipate that better knowledge of the model will help us to improve the operation of Tevatron in the near future.

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