Measurement of Incoherent Radiation Fluctuations and Bunch Profile Recovery

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Theory

- Each particle in the bunch radiates an electromagnetic pulse ***e(t)***

- Total radiated field is $$E(t) = \sum_{k=1}^{N} e(t - t_k)$$

- Fourier transform of the field is $$\hat{E}(\omega) = \hat{e}(\omega) \sum_{k=1}^{N} e^{i\omega t_k}$$

- Power spectrum of the radiation is

$$P(\omega) = \hat{e}(\omega)\hat{e}^*(\omega) \sum_{k=1}^{N} e^{i\omega t_k} \sum_{m=1}^{N} e^{-i\omega t_m} = \left|\hat{e}(\omega)\right|^2 \sum_{k,m=1}^{N} e^{i\omega(t_k - t_m)}$$

Average power

Average power is $\langle P(\omega) \rangle = \left| \hat{e}(\omega) \right|^2 \left(N + N^2 \left| \hat{f}(\omega) \right|^2 \right)$

incoherent radiation (N) | coherent radiation ($N^2 \left| \hat{f}(\omega) \right|^2$)

The coherent radiation term carries information about the distribution of the beam at low frequencies of the order of $\omega \approx \sigma^{-1}$

Difference between coherent and incoherent power is huge

- Coherent to incoherent ratio $R(\omega) = N\left|\hat{f}(\omega)\right|^2$

- consider a Gaussian beam with σ_t=1 ps and total charge of 1 nC (approximately 10^{10} electrons)

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma_t} e^{-\frac{t^2}{2\sigma_t^2}} \quad \text{and} \quad \hat{f}(\omega) = e^{-\frac{\omega^2\sigma_t^2}{2}}$$

At 1 THz: $R \approx 10^{10}$

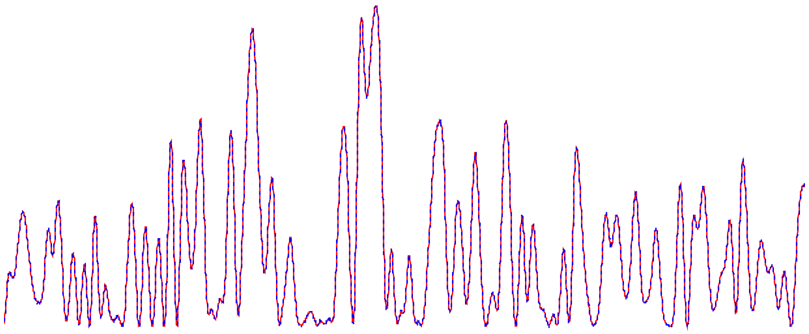
At 10 THz: $R \approx 10^{-34}$

High frequencies still contain information

- Power spectrum before averaging: $P(\omega) = \left|\hat{e}(\omega)\right|^2 \sum_{k,m=1}^{N} e^{i\omega(t_k - t_m)}$

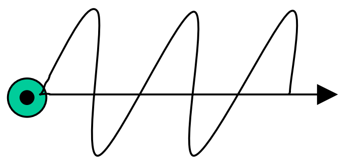
- Each separate term of the summation oscillates with the period $\Delta\omega=2\pi/(t_k-t_m)\sim 2\pi/\sigma_t$

- Because of the random distribution of particles in the bunch, the summation fluctuates randomly as a function of frequency ω.

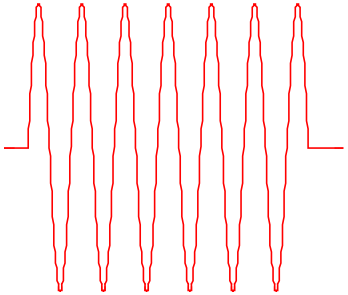


Example: incoherent radiation in a wiggler

Single electron

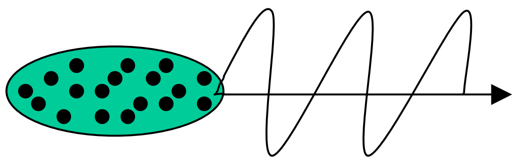


$$e(t)$$

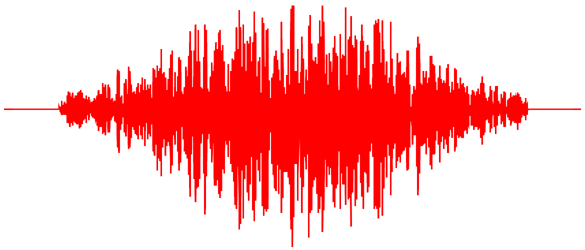


~10 *fs*

Electron bunch

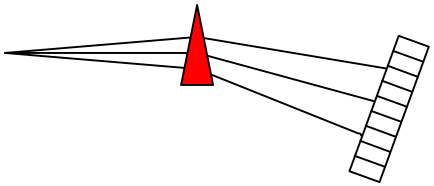


$$E(t)=\sum_{k=1}^{N} e(t-t_k)$$

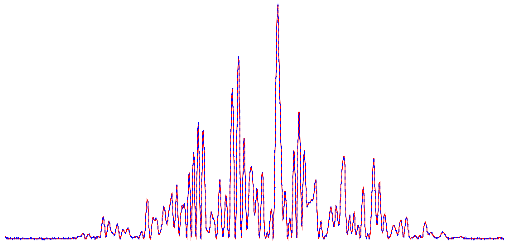


~1 *ps*

Spectrometer



$$E(\omega)=e(\omega)\cdot\sum_{k=1}^{N} e^{i\omega t_k}$$



Spike width is inversely proportional to the bunch length

Bunch profile measurements using fluctuations of incoherent radiation

- The method was proposed by M. Zolotorev and G. Stupakov and also by E. Saldin, E. Schneidmiller and M. Yurkov
- Emission can be produced by any kind of incoherent radiation: synchrotron radiation in a bend or wiggler, transition, Cerenkov, etc.
- The method does not set any conditions on the bandwidth of the radiation

Quantitative analysis

We can calculate autocorrelation of the spectrum:

$$\left\langle P(\omega)P(\omega')\right\rangle = \left|\hat{e}(\omega)\right|^2 \left|\hat{e}(\omega')\right|^2 \left\langle \sum_{k,m,p,q=1}^{N} e^{i\omega(t_k - t_m) + i\omega'(t_p - t_q)} \right\rangle$$

$$\left\langle P(\omega)P(\omega')\right\rangle = N^2 \left|\hat{e}(\omega)\right|^2 \left|\hat{e}(\omega')\right|^2 \left(1 + \left|\hat{f}(\omega - \omega')\right|^2\right)$$

$$g(\Omega) = \frac{1}{2}\left(1 + \left|\hat{f}(\Omega)\right|^2\right)$$

Variance of the Fourier transform of the spectrum

Fourier transform of the spectrum:

$$G(\tau) = \int_{-\infty}^{\infty} P(\omega) e^{i\omega\tau} d\omega$$

Its variance:

$$D(\tau) = \left\langle \left| G(\tau) - \left\langle G(\tau) \right\rangle \right|^2 \right\rangle$$

It can be shown, that the variance is related to the convolution function of the particle distribution:

$$D(\tau) = A \int_{-\infty}^{\infty} f(t) f(t-\tau) dt$$

Some of the limitations

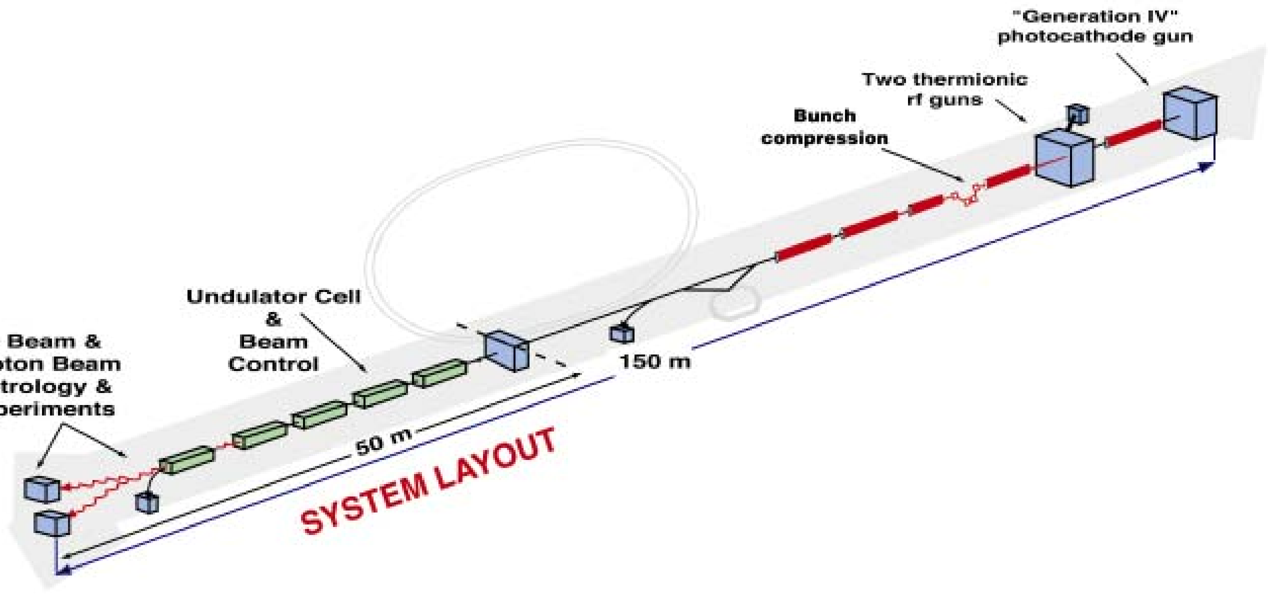
- Bandwidth of the radiation has to be larger than the spike width
- In order to neglect quantum fluctuations, number of photons has to be large

$$\langle n_{ph} \rangle \approx \frac{1}{2} \alpha N_e \frac{\Delta\omega}{\omega}$$

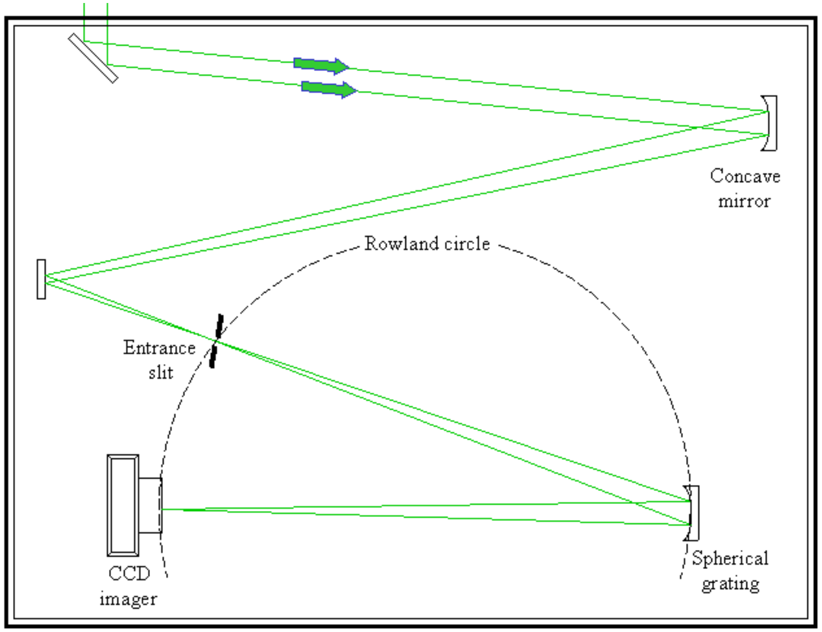
- Transverse bunch size – radiation has to be fully coherent to observe 100% intensity fluctuations

$$\lambda_{rad} \Big/ 2\pi\sigma_\theta$$

LEUTL at APS


"Generation IV"
photocathode gun
Two thermionic
rf guns
Bunch
compression
Undulator Cell
&
Beam
Control
e- Beam &
Photon Beam
Metrology &
Experiments
150 m
50 m
SYSTEM LAYOUT

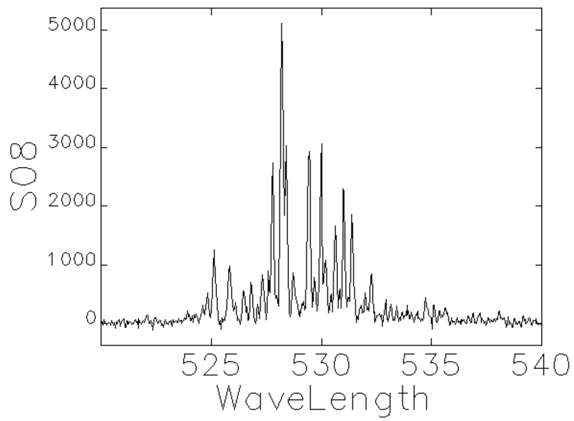
Spectrometer



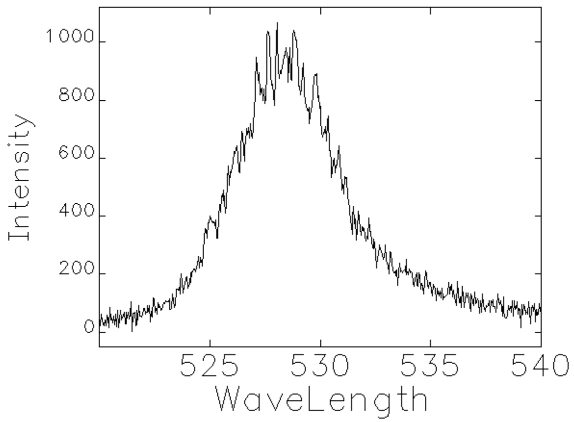
Grating	Grooves/mm	600
	Curv. radius [mm]	1000
	Blaze wavelength[nm]	482
CCD camera	Number of pixels	1100×330
	Pixel size [μm]	24
Concave mirror curv. radius [mm]		4000
Spectral resolution [Å]		0.4
Bandpass [nm]		44
Resolving power at 530 nm		10000
Wavelength range [nm]		250 – 1100

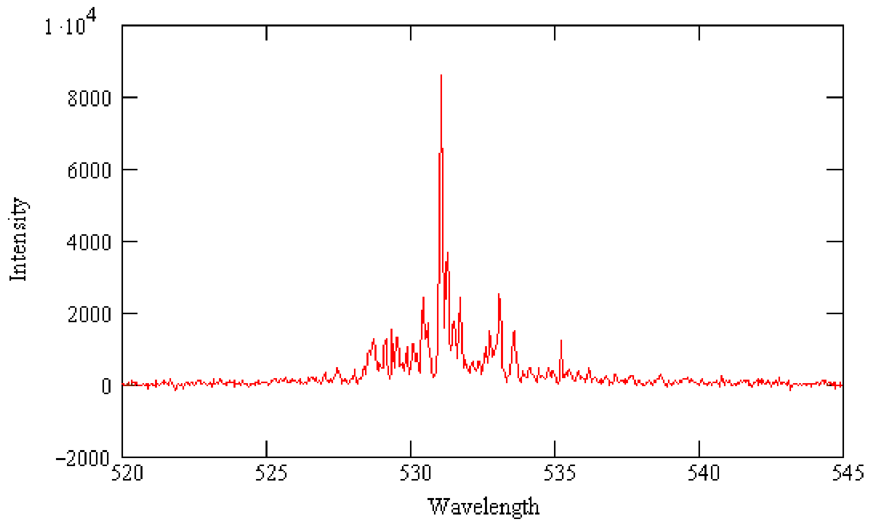
Single shot spectrum

Typical single-shot spectrum



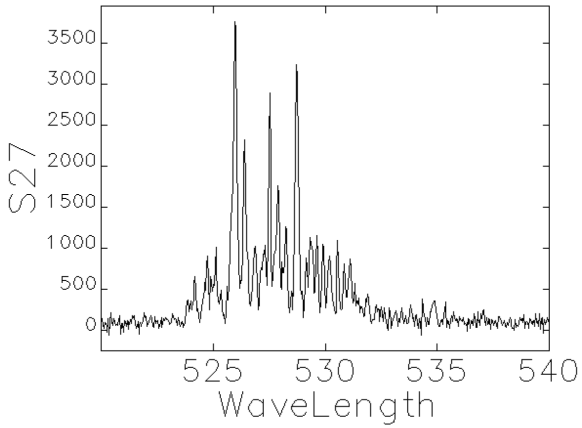
Average spectrum



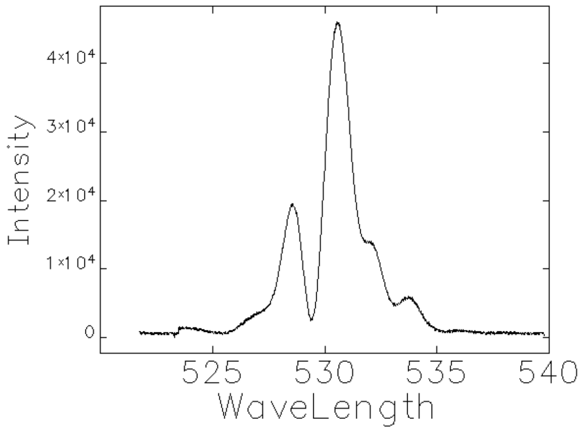


Spectrum for different bunch length

Long 2-ps rms bunch



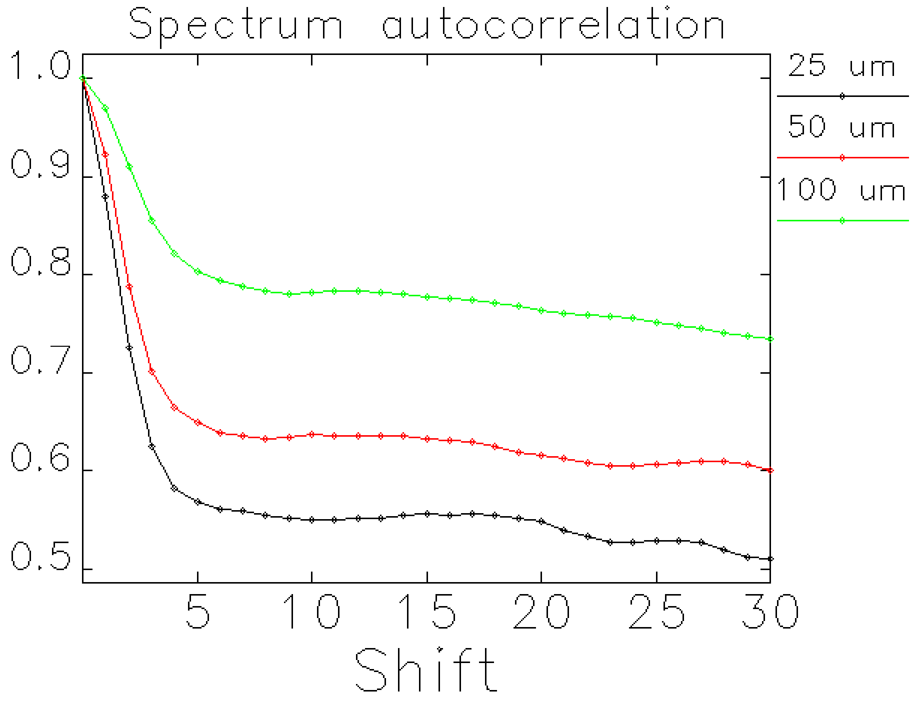
Short 0.4-ps rms bunch



Note: Total spectrum width (defined by the number of poles in the wiggler) is barely enough for the short bunch.

Spectrum correlation

$$C_n = \left\langle \sum_i P(\omega_i) P(\omega_{i+n}) \right\rangle / \left\langle \sum_i P(\omega_i)^2 \right\rangle$$



Bunch length

From the plot the correlation width is 2 pixels.

Frequency step corresponding to one pixel is $2.4 \cdot 10^{11}$ rad/s.

Assuming the beam to be Gaussian, from equation

$$g(\Omega) = \frac{1}{2}\left(1 + \left|\hat{f}(\Omega)\right|^2\right)$$

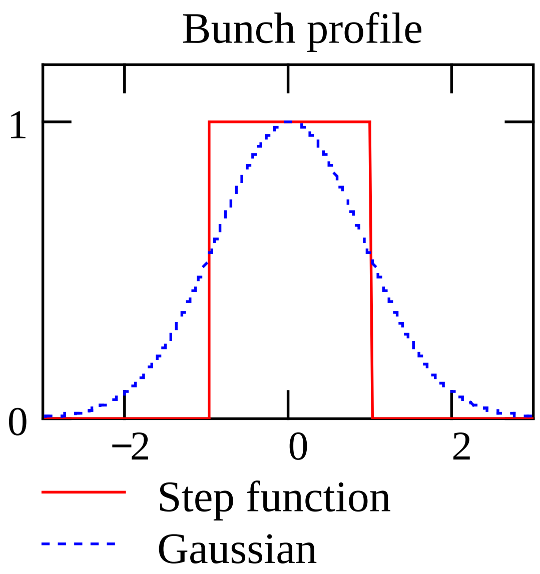
we get

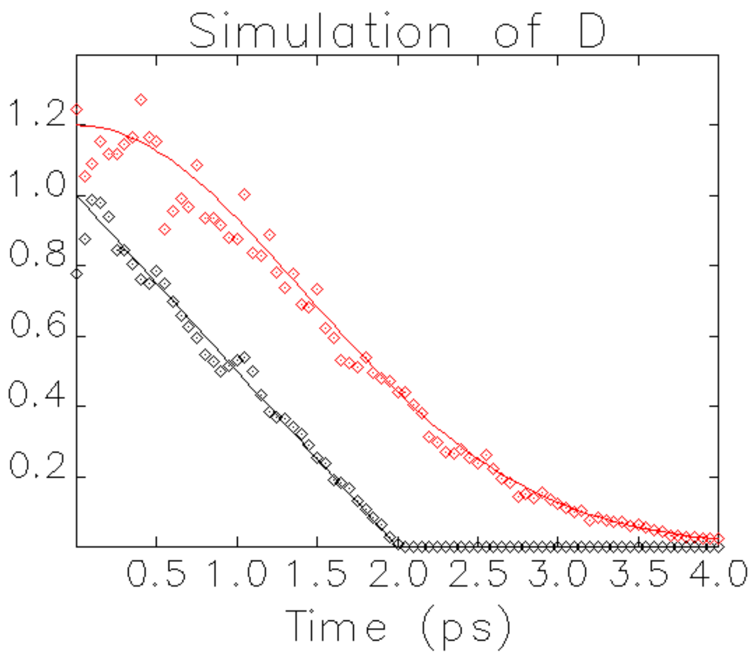
$$\tau_b \approx \frac{1}{n \cdot \delta\omega} \approx 2\,ps$$

Convolution of the bunch profile

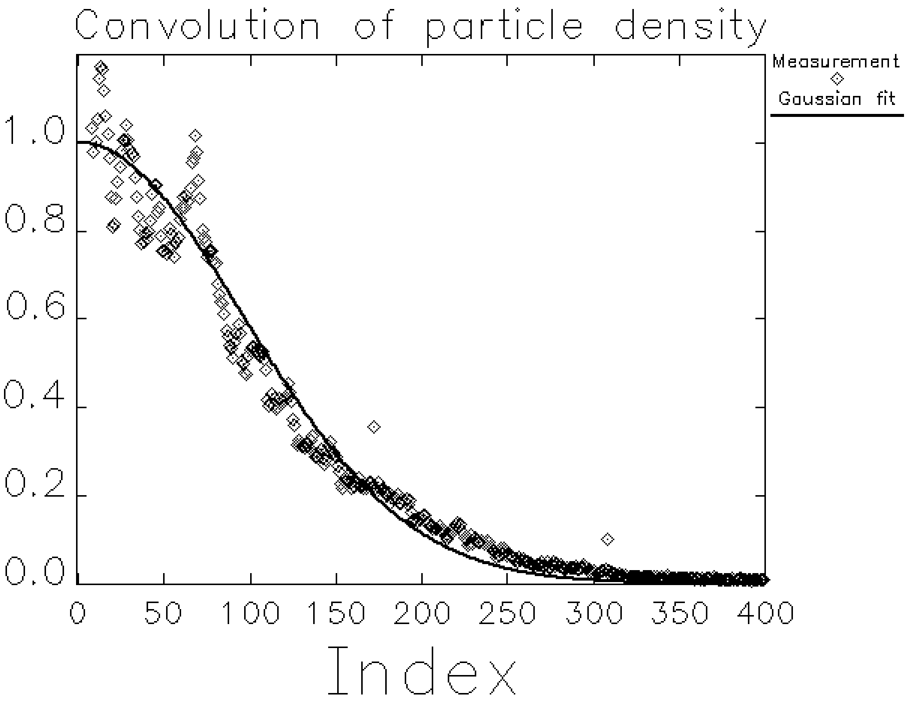
$$G_{k,n} = \sum_{m=1}^{N_{ch}} P_{m,n} e^{2\pi imk/N_{ch}}$$

$$D_k = \sum_{m=1}^{N_p} \left| G_{k,m} - \frac{1}{N_p} \sum_{n=1}^{N_p} G_{k,n} \right|^2$$





Convolution recovered from the measurements



Convolution of the Gaussian is also a Gaussian with

$$\sigma = \sqrt{2} \cdot \sigma_t$$

The Gaussian fit gives us

$$\tau_b = 1.8\,ps$$

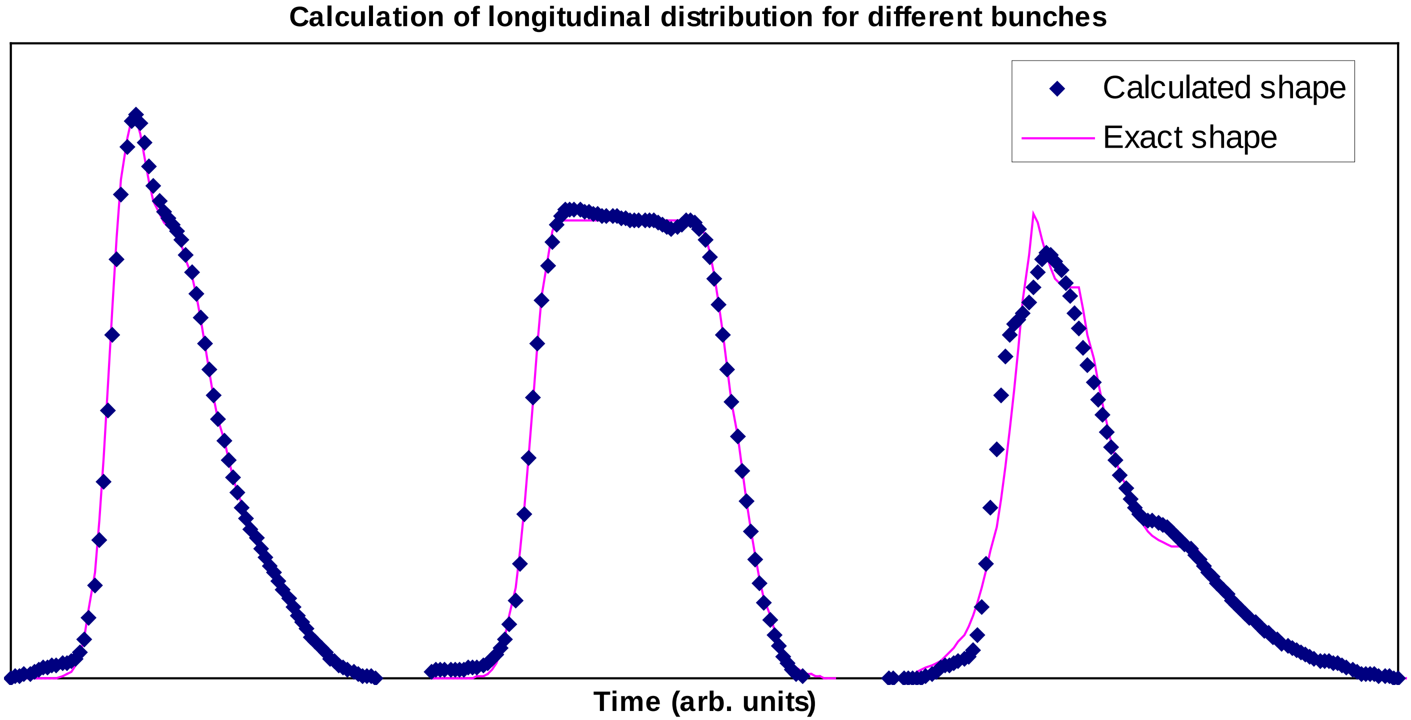
Phase retrieval

The amplitude and the phase information of the radiation source can be recovered by applying a Kramers-Kronig relation to the convolution function in combination with the minimal phase approach.

$$\psi_m(\omega) = -\frac{2\omega}{\pi} P\int_0^\infty dx \frac{\ln\left[\rho(x)/\rho(\omega)\right]}{x^2 - \omega^2}$$

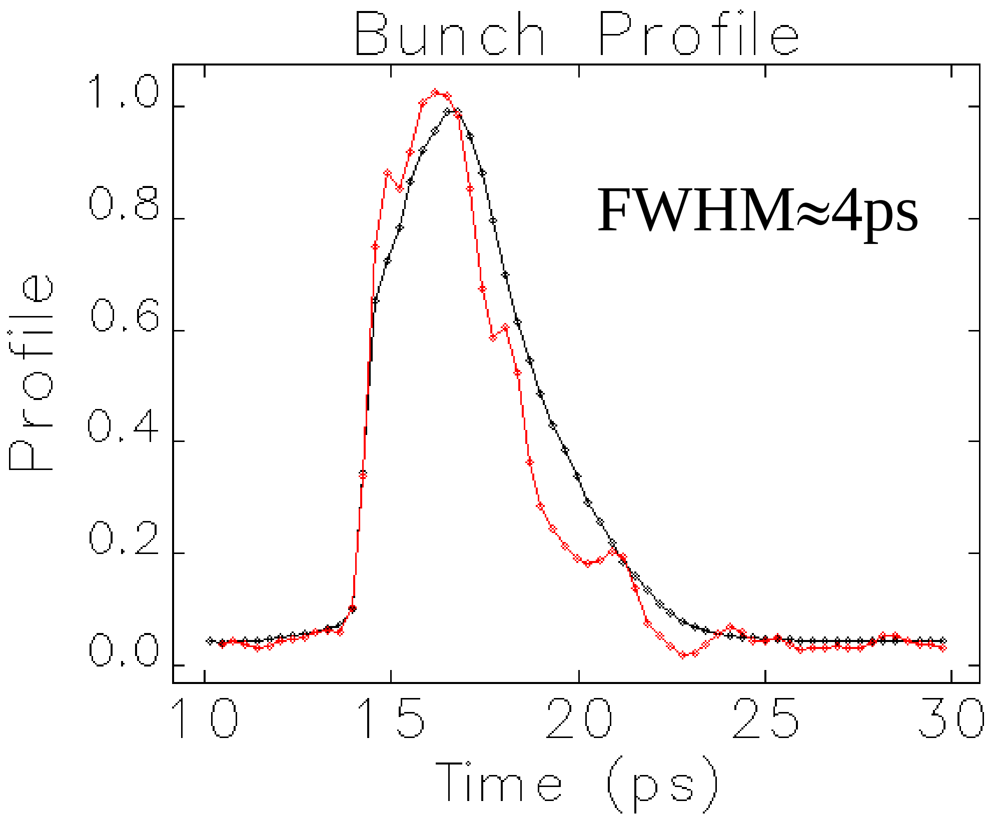
$$S(z) = \frac{1}{\pi c}\int_0^\infty d\omega \cdot \rho(\omega) \cdot \cos\left(\psi_m(\omega) - \frac{\omega z}{c}\right)$$

Phase retrieval example



Bunch profile

Two different measurements (two sets of 100 single-shot spectra)



Conclusions

- Measurements of incoherent radiation spectrum showing intensity fluctuations were done.
- A technique for recovering a longitudinal bunch profile from spectral fluctuations of incoherent radiation has been implemented. Although we used synchrotron radiation, the nature of the radiation is not important.
- Typically, analysis of many single shots is required, however one can perform statistical analysis over wide spectral intervals in a single pulse

Conclusions

- An important feature of the method is that it can be used for bunches with lengths varying from a centimeter to tens of microns (30 ps – 30 fs)
- There are several important conditions for this technique. In order to be able to measure a bunch of length σ_t, the spectral resolution of the spectrometer should be comparable with $1/\sigma_t$. Also, the spectral width of the radiation and the spectrometer must be larger than the inverse bunch length