Measurement of Bunch Length Using Spectral Analysis of Incoherent Fluctuations

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Need for speed

Next linear collider projects require 1 ps bunch length

X-ray Free-Electron lasers require high peak current to achieve lasing, which leads to 100 fs or less bunches

LCLS is going to use 200 fs FWHM beam

SPPS can already produce sub-100fs beams

Short bunch diagnostics

- High power rf deflecting structures
- Electro-optic crystal diagnostics
- Coherent synchrotron radiation in THz region

Fluctuations of incoherent radiation

- The method was proposed by M. Zolotorev and G. Stupakov and also by E. Saldin, E. Schneidmiller and M. Yurkov
- Emission can be produced by any kind of incoherent radiation: synchrotron radiation in a bend or wiggler, transition, Cerenkov, etc.
- The method does not set any conditions on the bandwidth of the radiation

Theory

- Each particle in the bunch radiates an electromagnetic pulse ***e(t)***

- Total radiated field is $E(t) = \sum_{k=1}^{N} e(t - t_k)$

- Fourier transform of the field is $\hat{E}(\omega) = \hat{e}(\omega) \sum_{k=1}^{N} e^{i\omega t_k}$

- Power spectrum of the radiation is

$$P(\omega) = \hat{e}(\omega)\hat{e}^*(\omega) \sum_{k=1}^{N} e^{i\omega t_k} \sum_{m=1}^{N} e^{-i\omega t_m} = \left|\hat{e}(\omega)\right|^2 \sum_{k,m=1}^{N} e^{i\omega(t_k - t_m)}$$

Average power

Average power is $\langle P(\omega)\rangle = \left|\hat{e}(\omega)\right|^2 \left(N + N^2 \left|\hat{f}(\omega)\right|^2 \right)$

incoherent radiation → N

coherent radiation → $N^2 \left|\hat{f}(\omega)\right|^2$

The coherent radiation term carries information about the distribution of the beam at low frequencies of the order of $\omega \approx \sigma^{-1}$

Difference between coherent and incoherent power is huge

- Coherent to incoherent ratio $R(\omega) = N\left|\hat{f}(\omega)\right|^2$

- consider a Gaussian beam with σ_t=1 ps and total charge of 1 nC (approximately 10^{10} electrons)

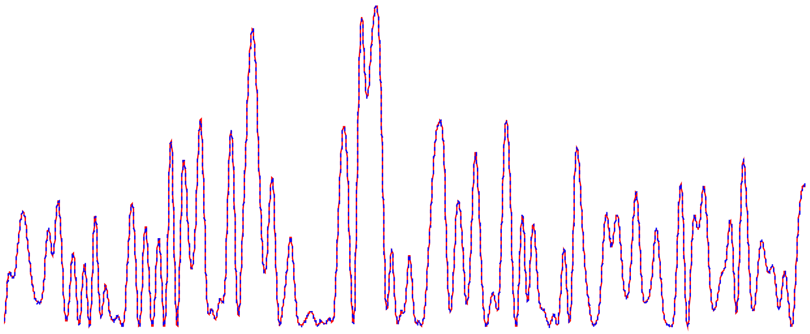
$$f(t) = \frac{1}{\sqrt{2\pi}\sigma_t} e^{-\frac{t^2}{2\sigma_t^2}} \quad \text{and} \quad \hat{f}(\omega) = e^{-\frac{\omega^2\sigma_t^2}{2}}$$

At 1 THz: $R \approx 10^{10}$

At 10 THz: $R \approx 10^{-34}$

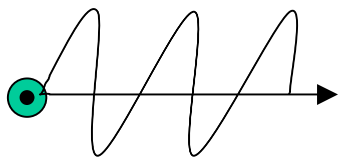
High frequencies still contain information

- Power spectrum before averaging: $$P(\omega) = \left|\hat{e}(\omega)\right|^2 \sum_{k,m=1}^{N} e^{i\omega(t_k - t_m)}$$
- Each separate term of the summation oscillates with the period $\Delta\omega = 2\pi/(t_k - t_m) \sim 2\pi/\sigma_t$
- Because of the random distribution of particles in the bunch, the summation fluctuates randomly as a function of frequency ω.

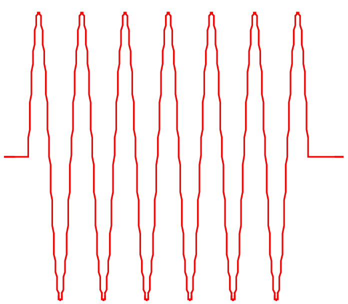


Example: incoherent radiation in a wiggler

Single electron

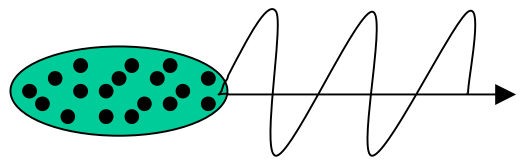


$e(t)$

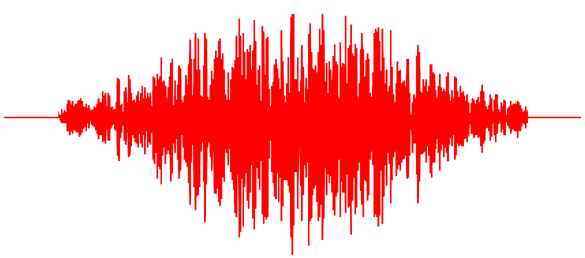


~10 *fs*

Electron bunch

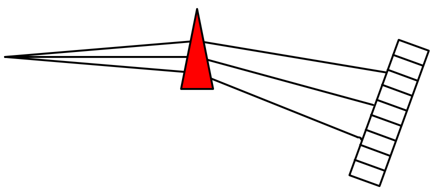


$$E(t) = \sum_{k=1}^{N} e(t - t_k)$$

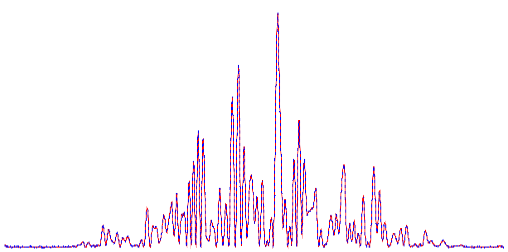


~1 *ps*

Spectrometer



$$E(\omega) = e(\omega) \cdot \sum_{k=1}^{N} e^{i\omega t_k}$$



Spike width is inversely proportional to the bunch length

Quantitative analysis

We can calculate autocorrelation of the spectrum:

$$\left\langle P(\omega)P(\omega')\right\rangle = \left|\hat{e}(\omega)\right|^2 \left|\hat{e}(\omega')\right|^2 \left\langle \sum_{k,m,p,q=1}^{N} e^{i\omega(t_k - t_m) + i\omega'(t_p - t_q)} \right\rangle$$

$$\left\langle P(\omega)P(\omega')\right\rangle = N^2 \left|\hat{e}(\omega)\right|^2 \left|\hat{e}(\omega')\right|^2 \left(1 + \left|\hat{f}(\omega - \omega')\right|^2\right)$$

$$g(\Omega) = \frac{1}{2}\left(1 + \left|\hat{f}(\Omega)\right|^2\right)$$

Variance of the Fourier transform of the spectrum

Fourier transform of the spectrum:

$$G(\tau) = \int_{-\infty}^{\infty} P(\omega) e^{i\omega\tau} d\omega$$

Its variance:

$$D(\tau) = \left\langle \left| G(\tau) - \left\langle G(\tau) \right\rangle \right|^2 \right\rangle$$

It can be shown, that the variance is related to the convolution function of the particle distribution:

$$D(\tau) = A \int_{-\infty}^{\infty} f(t) f(t-\tau) dt$$

Some of the limitations

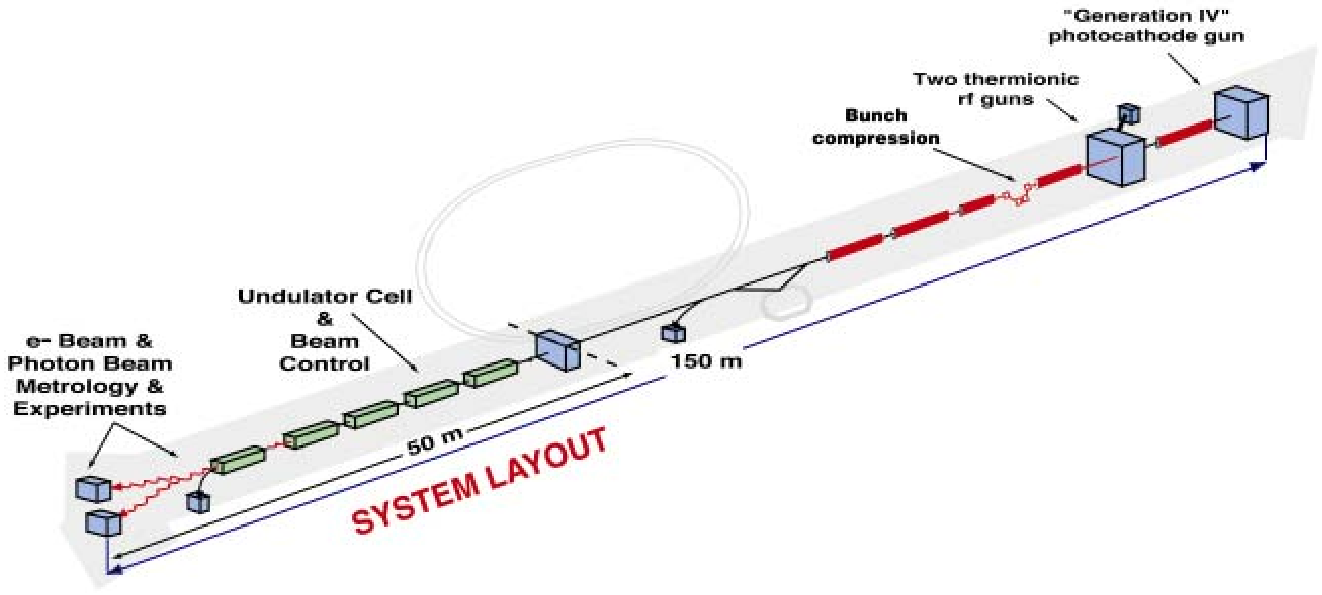
- Bandwidth of the radiation has to be larger than the spike width
- Transverse bunch size – radiation has to be fully coherent to observe 100% intensity fluctuations

$$\lambda_{rad} / 2\pi\sigma_{\theta}$$

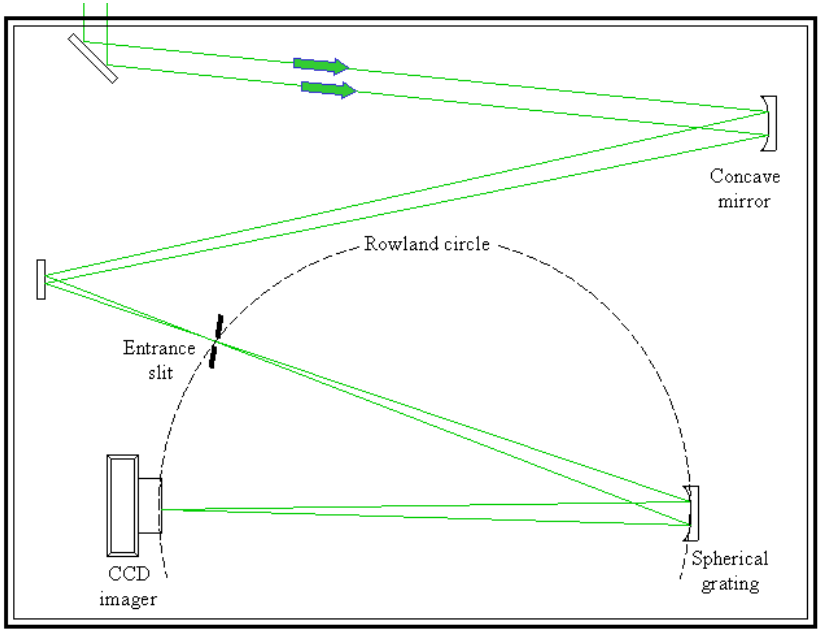
- In order to neglect quantum fluctuations, number of photons has to be large

$$\langle n_{ph} \rangle \approx \frac{1}{2}\alpha N_e \frac{\Delta\omega}{\omega}$$

LEUTL at APS


"Generation IV"
photocathode gun
Two thermionic
rf guns
Bunch
compression
Undulator Cell
&
Beam
Control
e- Beam &
Photon Beam
Metrology &
Experiments
150 m
50 m
SYSTEM LAYOUT

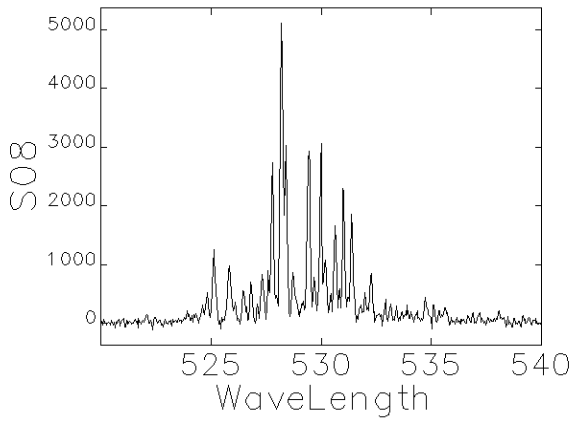
Spectrometer

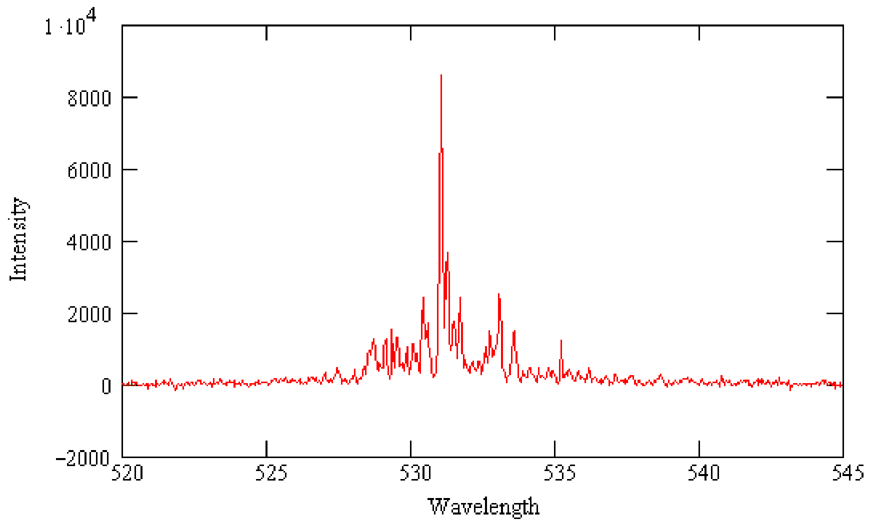


Grating	Grooves/mm	600
	Curv. radius [mm]	1000
	Blaze wavelength[nm]	482
CCD camera	Number of pixels	1100×330
	Pixel size [μm]	24
Concave mirror curv. radius [mm]		4000
Spectral resolution [Å]		0.4
Bandpass [nm]		44
Resolving power at 530 nm		10000
Wavelength range [nm]		250 – 1100

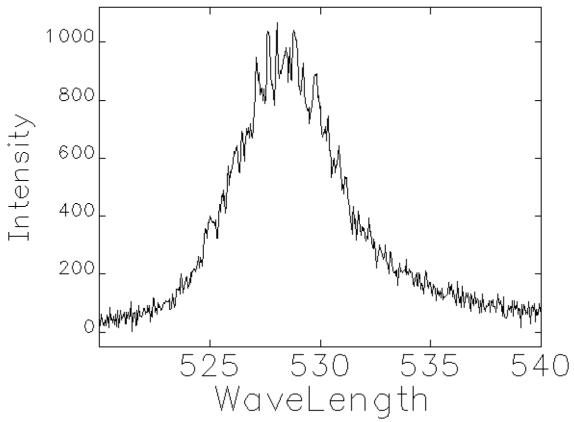
Single shot spectrum

Typical single-shot spectrum



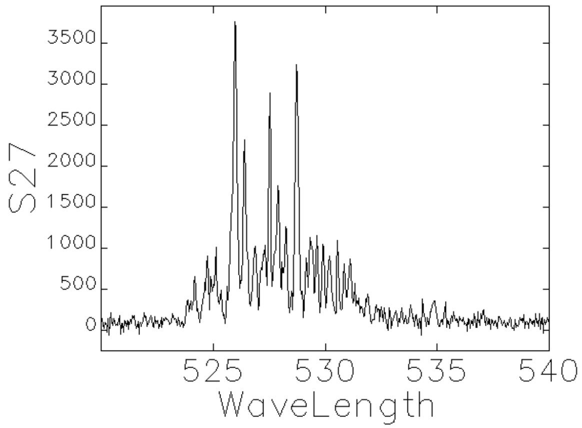


Average spectrum

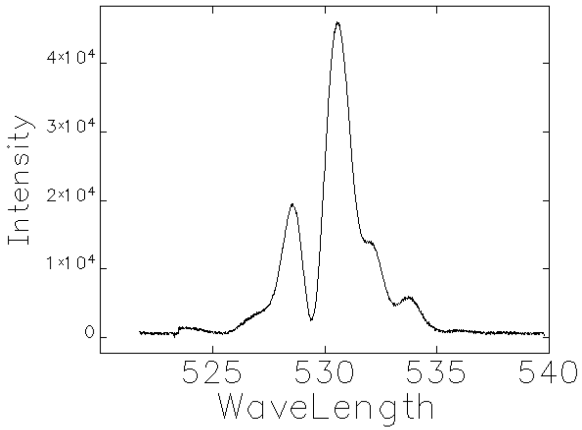


Spectrum for different bunch length

Long 2-ps rms bunch



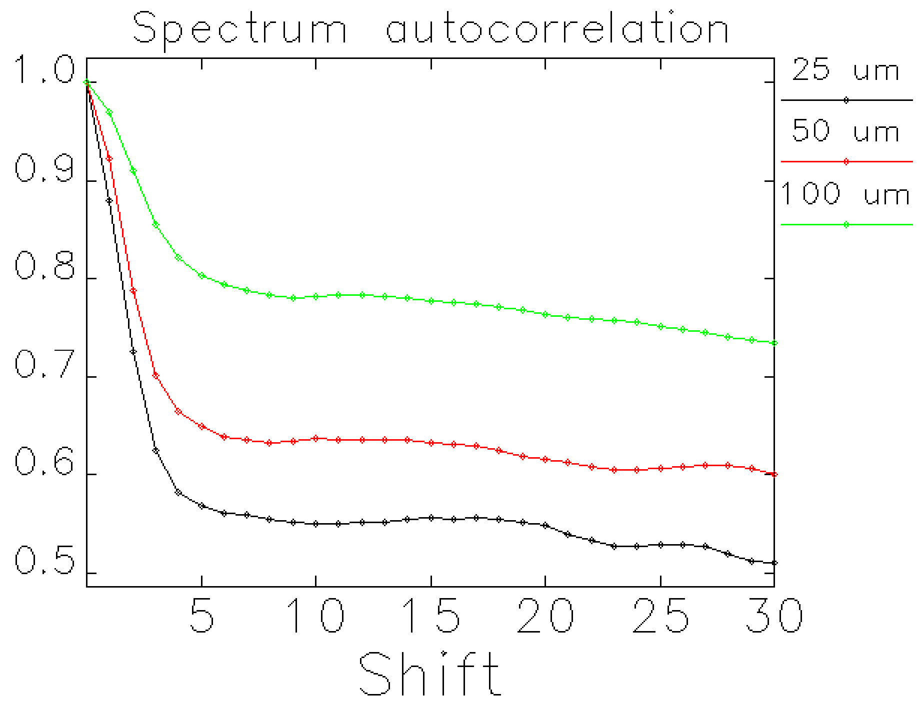
Short 0.4-ps rms bunch



Note: Total spectrum width (defined by the number of poles in the wiggler) is not wide enough for the short bunch.

Spectrum correlation

$$C_n = \left\langle \sum_i P(\omega_i) P(\omega_{i+n}) \right\rangle / \left\langle \sum_i P(\omega_i)^2 \right\rangle$$



Bunch length

From the plot the correlation width is 2 pixels.

Frequency step corresponding to one pixel is $2.4 \cdot 10^{11}$ rad/s.

Assuming the beam to be Gaussian, from equation

$$g(\Omega) = \frac{1}{2}\left(1 + \left|\hat{f}(\Omega)\right|^2\right)$$

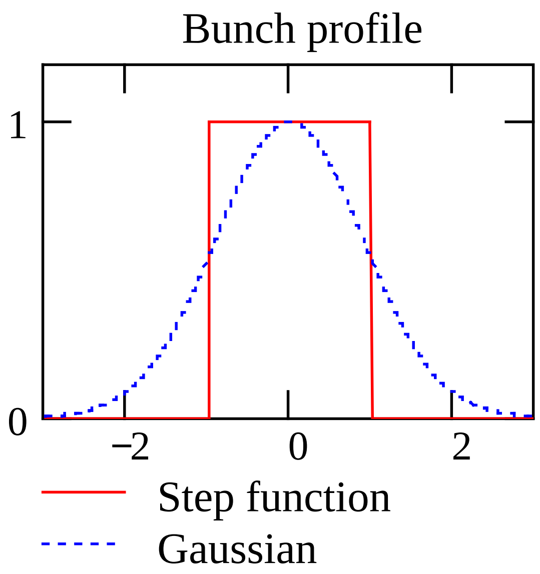
we get

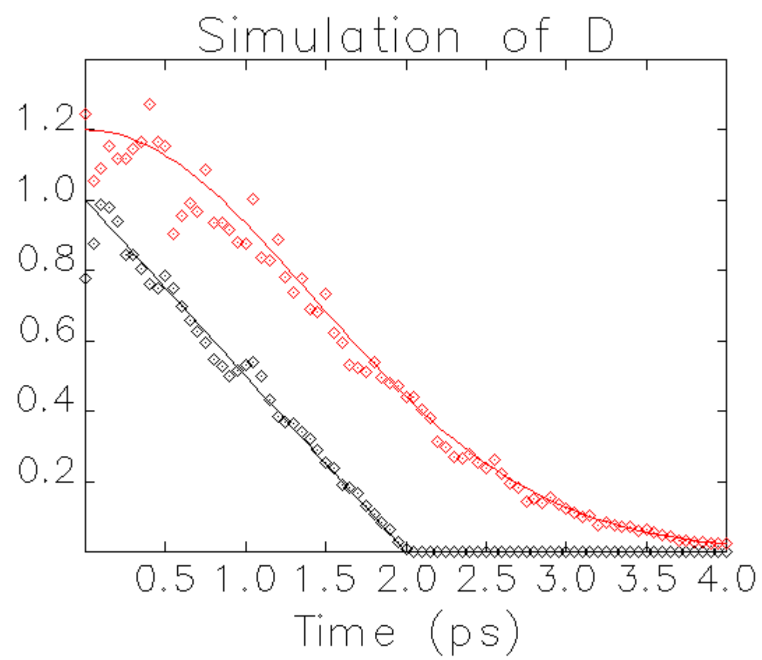
$$\tau_b \approx \frac{1}{n \cdot \delta\omega} \approx 2\,ps$$

Convolution of the bunch profile

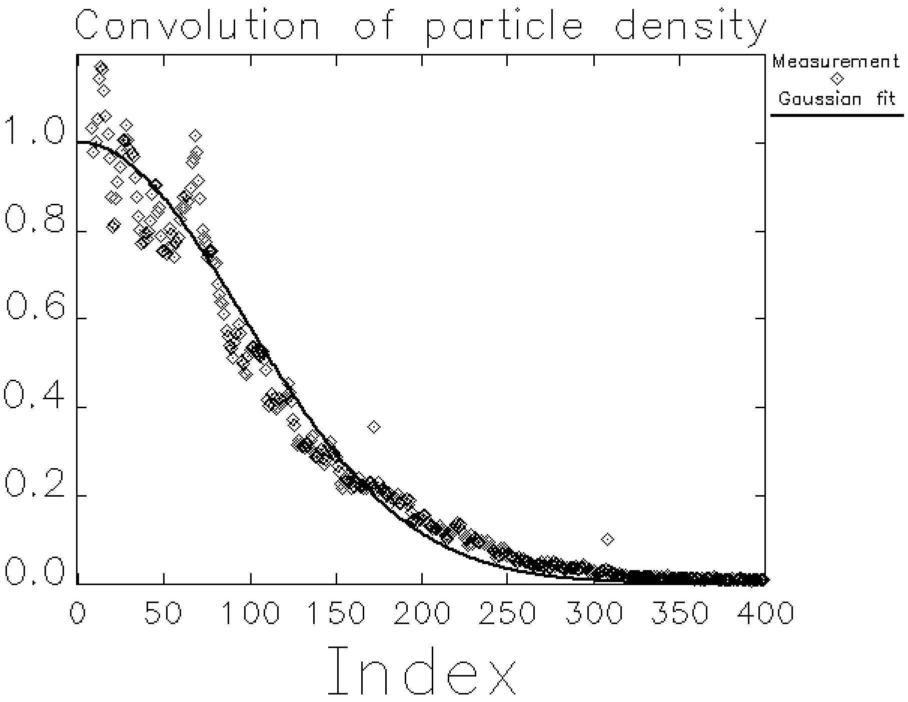
$$G_{k,n} = \sum_{m=1}^{N_{ch}} P_{m,n} e^{2\pi imk/N_{ch}}$$

$$D_k = \sum_{m=1}^{N_p} \left| G_{k,m} - \frac{1}{N_p} \sum_{n=1}^{N_p} G_{k,n} \right|^2$$





Convolution recovered from the measurements



Convolution of the Gaussian is also a Gaussian with

$$\sigma = \sqrt{2} \cdot \sigma_t$$

The Gaussian fit gives us

$$\tau_b = 1.8 ps$$

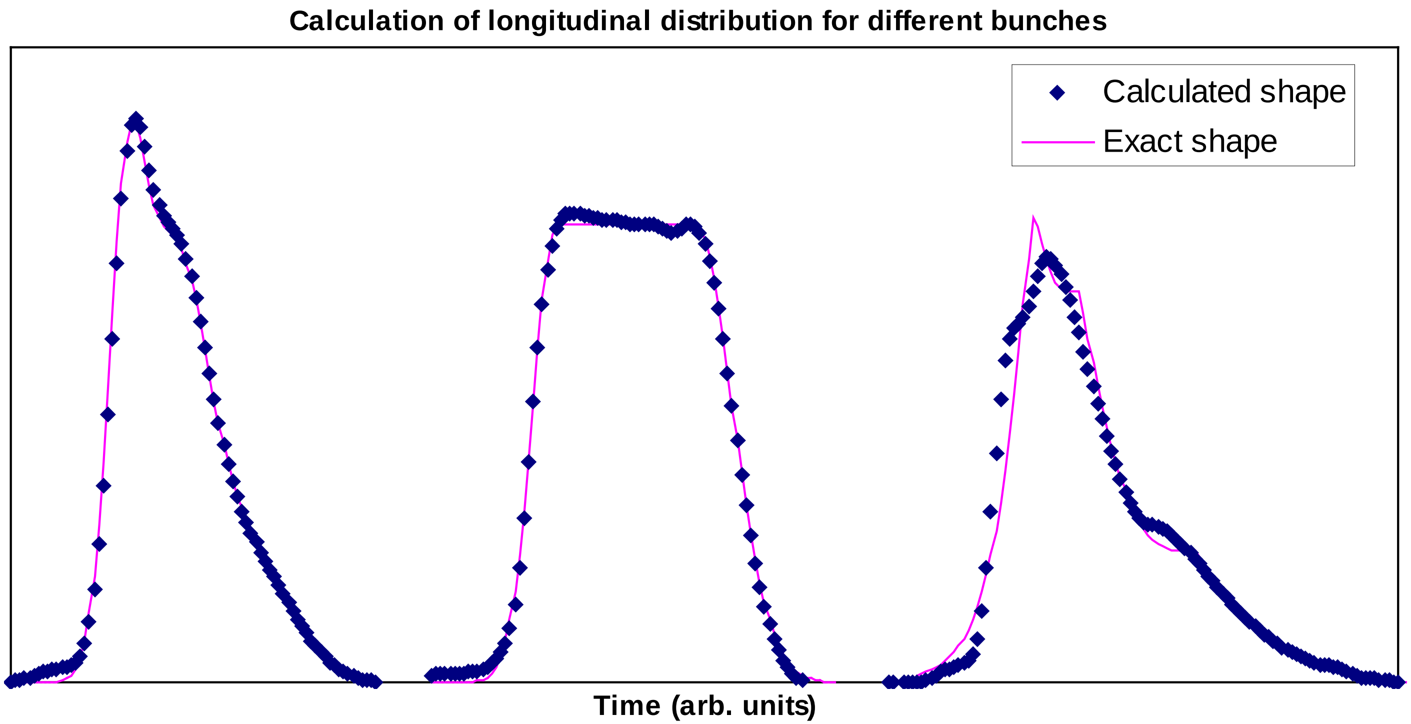
Phase retrieval

The amplitude and the phase information of the radiation source can be recovered by applying a Kramers-Kronig relation to the convolution function in combination with the minimal phase approach.

$$\psi_m(\omega) = -\frac{2\omega}{\pi} P\int_0^\infty dx \frac{\ln[\rho(x)/\rho(\omega)]}{x^2-\omega^2}$$

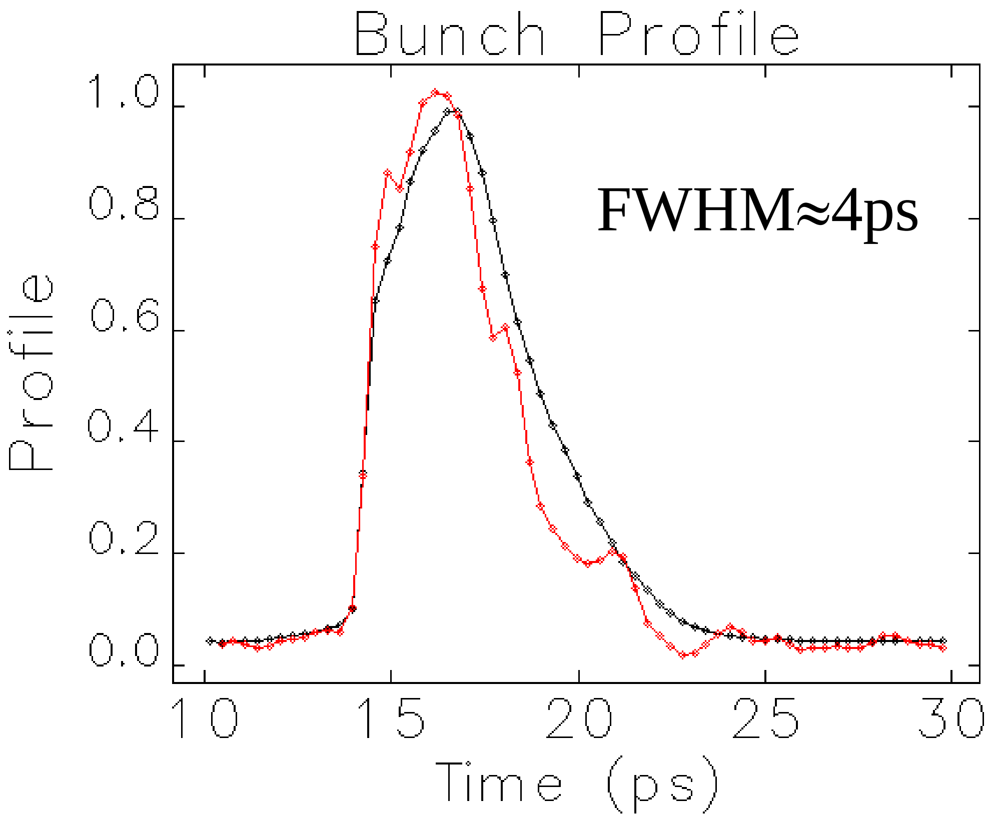
$$S(z) = \frac{1}{\pi c}\int_0^\infty d\omega \cdot \rho(\omega) \cdot \cos\left(\psi_m(\omega) - \frac{\omega z}{c}\right)$$

Phase retrieval example



Bunch profile

Two different measurements (two sets of 100 single-shot spectra)



Conclusions

- A technique for recovering a longitudinal bunch profile from spectral fluctuations of incoherent radiation has been implemented. It allows one to build the convolution of the bunch profile, then the Kramers-Kronig relation is used to recover the bunch shape. Although we used synchrotron radiation, the nature of the radiation is not important.
- Typically, analysis of many single shots is required, however one can perform statistical analysis over wide spectral intervals in a single pulse

Conclusions

- An important feature of the method is that it can be used for bunches with lengths varying from a centimeter to tens of microns (30 ps – 30 fs)
- There are several important conditions for this technique. In order to be able to measure a bunch of length σ_t, the spectral resolution of the spectrometer should be better than $1/\sigma_t$. Also, the spectral width of the radiation and the spectrometer must be much larger than the inverse bunch length