X-ray Optics for the APS

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Outline

I. Total External Reflection and Mirrors

II. Diffraction by Perfect Single Crystals (Dynamical Diffraction)

A. Monochromators and phase space manipulation

B. Sagittal Focusing with crystals

III. Non-crystal Optical Components

Total External Reflection and X-ray Mirrors

The index of refraction for x-rays can be calculated using a simple model for the polarizability of the material.

The index of refraction, n, is related to the dielectric constant, k, for the material and can be written as:

$$\kappa^{1/2} = n = 1 - \delta - i\beta$$

where $\delta = (n_e r_e/2\pi)\lambda^2$ with n_e the number of electrons per unit volume and $\beta = \lambda\mu/4\pi$, with μ the linear absorption coefficient.

Let an x-ray (in vacuum, where $n_1 = 1$) impinge on a material with index of refraction n_2. From Snell's Law, it is clear that $\phi_2 > \phi_1$, since $n_2 < 1$. When $\phi_2 = 90°$, we have:

$$\sin(\phi_c) = \cos(\theta_c) = n_2\cos(0); \quad (\theta_c = 90° - \phi_c)$$

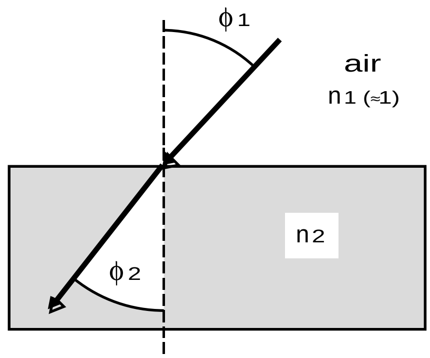
or

$$1 - (\theta_c)^2/2 = 1 - \delta$$

So

$$\theta_c = (2\delta)^{1/2} = C\,\lambda\,\sqrt{\rho}$$

θ_c is the so-called critical angle, the angle at which there is total external reflection.

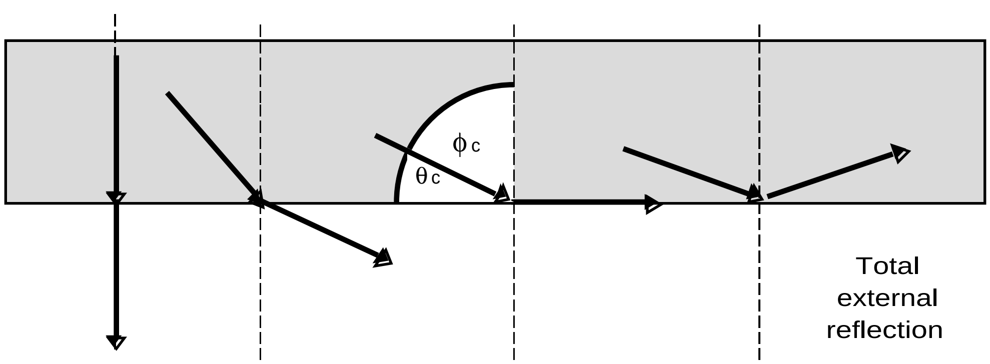


Snell's Law says:

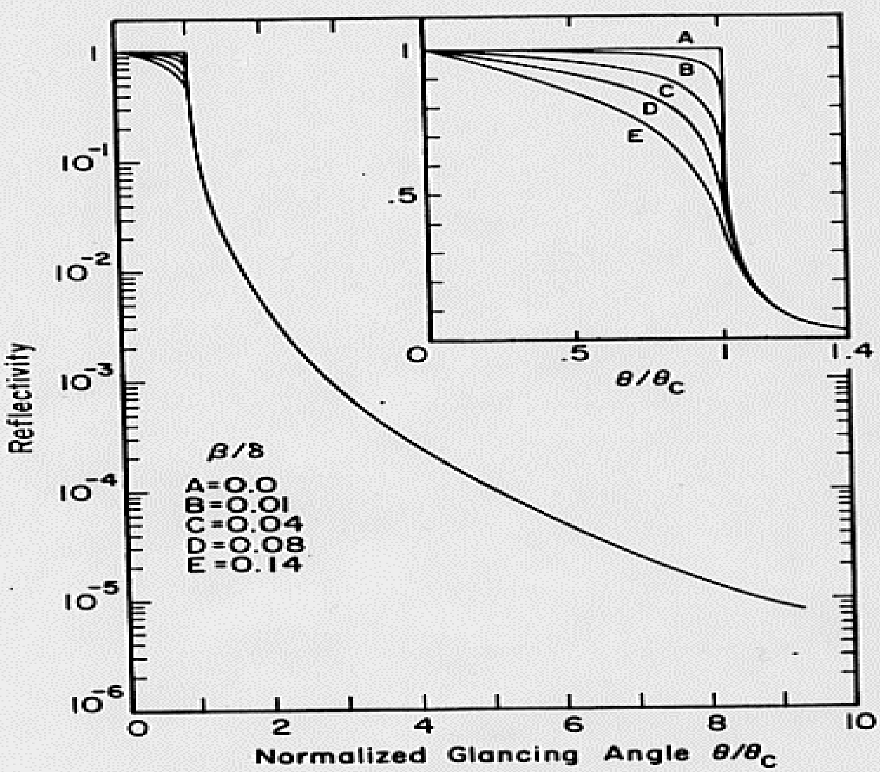
$n_1 \sin(\phi_1) = n_2 \sin(\phi_2)$

Typical values for n_2 are (@ 1 Å):

$1 - 10^{-5}$ to 10^{-6}



$\sin(\phi_c) = n_2/n_1$



Bilderback, D.H., 1981, Proc. Soc. Photo-Opt. Instrum. Eng. **315**, 90.

Uses of Mirrors

Separation of branch lines

Low pass filters

One-dimensional focusing, collimating, etc. (cylinders and ellipses)

Two-dimensional focusing (toroids and ellipses)

The relationship between the focal length, f, the source-to-mirror distance, F_1, and the mirror-to-image distance, F_2, the radius of curvature, R_m (for meridional radius) and the grazing incidence angle is given by the "lens" equation:

$$1/f = 1/F_1 + 1/F_2$$

then

$$R_m = [2/\sin\theta]\,[F_1\,F_2/(F_1 + F_2)]$$

$$R_s = R_m \sin^2\theta$$

Example: A bent toroidal mirror is installed on the bending magnet beamline at the APS following a DCM.

Given:

$F_1 = F_2 = 30\text{m}$

$\Theta = (2.5 \text{ mrad})$

$\phi_v \approx 1/\gamma = 73$ microradians; $\phi_H = 1$ mrad

$\sigma_x = 100$ microns; $\sigma_y = 40$ microns

The calculated parameters for this situation are:

$R_m = 12$ km $R_s = 0.075$ m (7.5 cm)

$L_{vert} = 0.88$ m $L_{hor} = 0.60$ m

$L_m = L_1 + L_2 \approx 1.5$ m

$\phi_{vertical}^{\ extremal} = 100$ microradians

$A_{cly} = 140$ microns

Figure and finish

The mirror figure is often defined as the overall (macroscopic) shape of the mirror while the finish describes (microscopic) roughness.

With radiation opening angles on the order of 10 microradians or less, the current requirement for the slope errors for an x-ray mirror is a few microradians rms. The 1 microradian rms level over a mirror of length 1 meter is at the state-of-the-art in polishing capabilities.

Mirror finish is also technically challenging for fabricators with specified rms roughnesses on the order of several Ångstroms. State-of-the-art mirror polishing techniques can result in mirror surfaces with sub-Ångstrom rms roughness and sub-microradian rms slope errors over small areas.

Diffraction by Perfect Crystals

In Darwin's 1914 analysis which took into account multiple scattering, (i.e. he self-consistantly solved Maxwell's equations in a medium with a periodic dielectric constant), he discovered several important points:

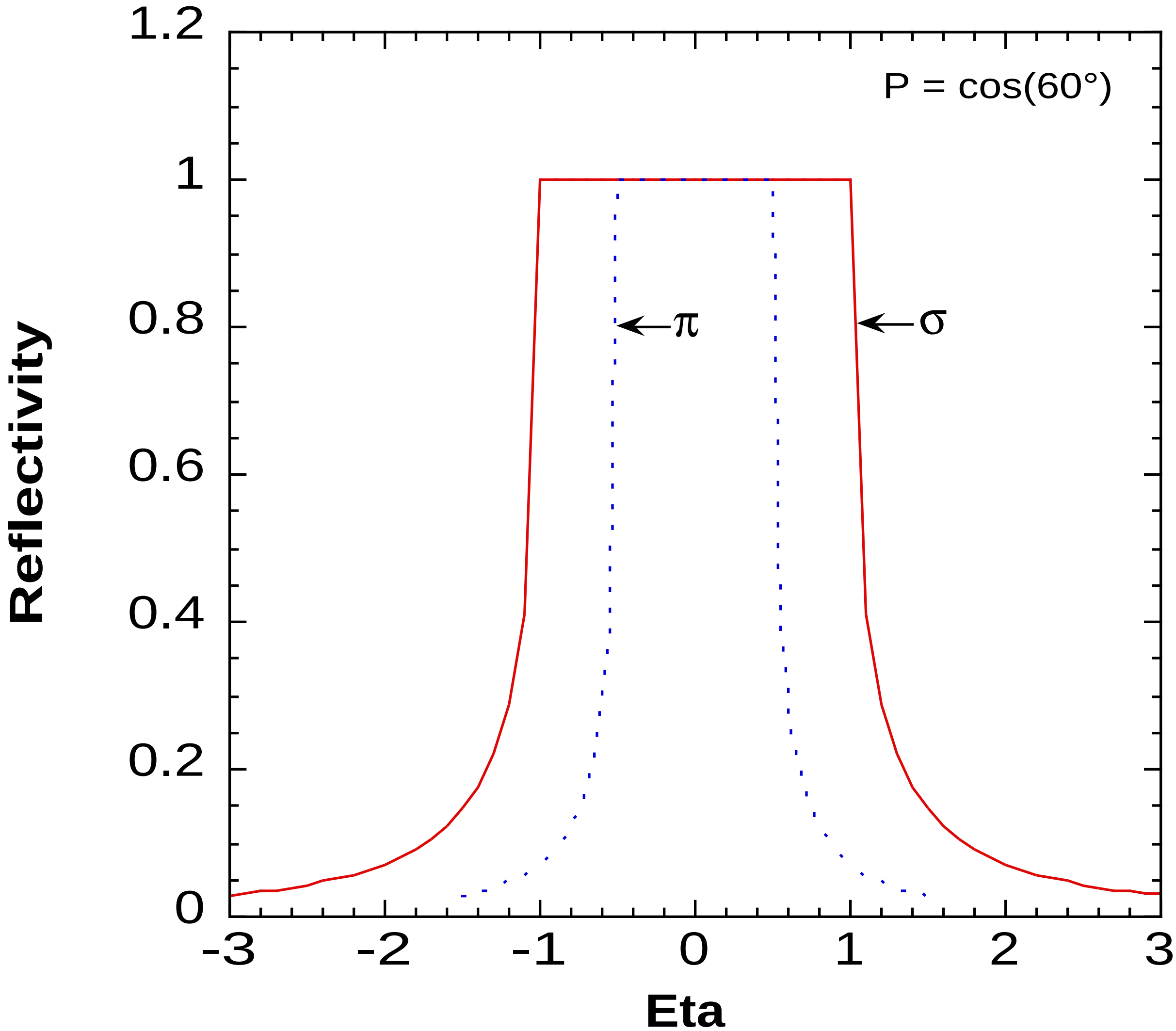
The resultant waves inside the crystal travel with a group velocity slightly greater than the speed of light (i.e. the index of refraction must be slightly less than unity) resulting in a slight shift in the Bragg angle (as defined by Bragg's Law).

At that corrected Bragg angle the reflectivity is 100% (neglecting absorption) and the reflectivity curve has a finite angular width (what we now call the Darwin width).

Diffraction by Perfect Crystals

The Darwin width is proportional to the structure factor and hence the integrated intensity must be proportional to F (and not F^2 as is the case in kinematical theory).

Due to extinction (the attenuation of the incident beam due to the scattering process) the effective volume of the scattering medium that participates in the diffraction process decreases significantly within the Darwin width.


P = cos(60°)
Reflectivity
Eta
←π
←σ
1.2
1
0.8
0.6
0.4
0.2
0
-3
-2
-1
0
1
2
3

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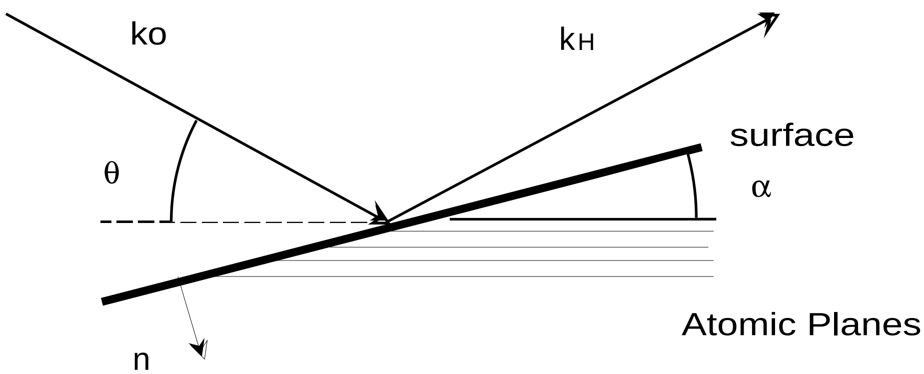
Phase Space Manipulation with Perfect Crystals



Sagittal Focusing with Perfect Crystals

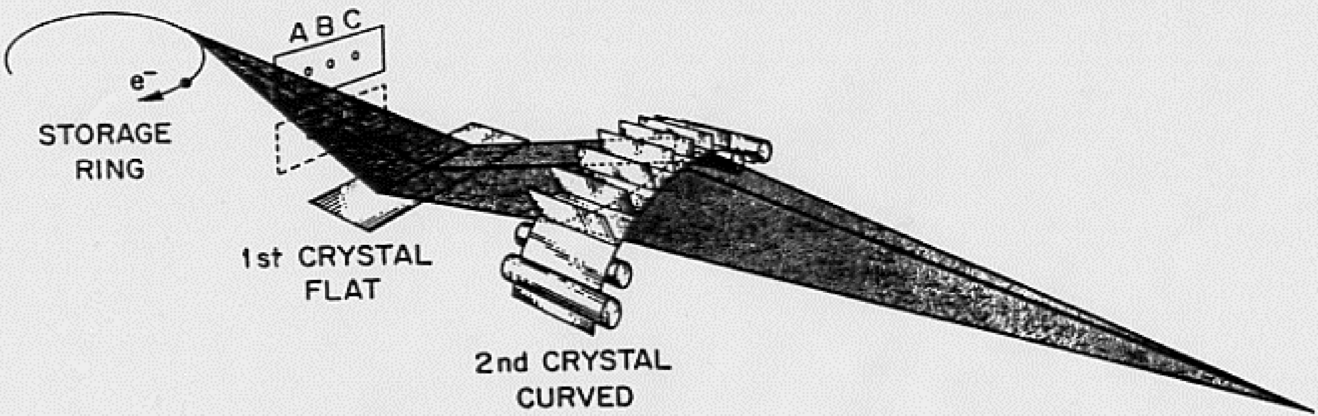
Recall that sagittal focusing is focusing out-of-the-plane of diffraction.

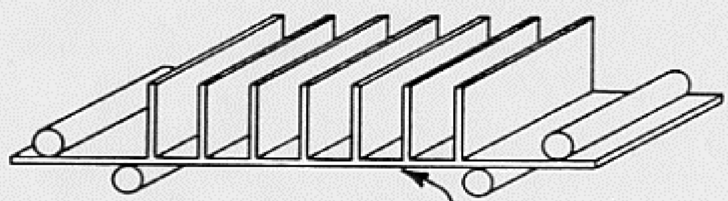
The ideal geometry for point-to-point focusing is one in which the reflecting crystal planes are curved to an ellipsoidal shape.

However we will continue to use a cylindrical shape in which case the sagittal radius, R_s, is given by:

$$R_s = 2\sin\theta\,[F_1 F_2/(F_1 + F_2)].$$

One of the reasons cylindrical shapes are so appealing is that they can (relatively) easily be produced by bending flat plates to cylinders.





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SAGITTAL FOCUSING OF SYNCHROTRON X-RADIATION WITH CURVED CRYSTALS *

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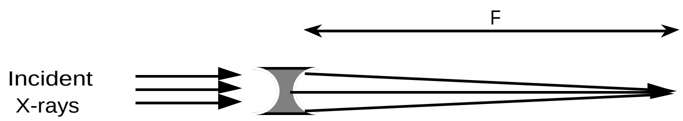
Compound Refractive Lenses

X-ray lenses have been considered for many years. Kirkpatrick and Baez pointed out that:

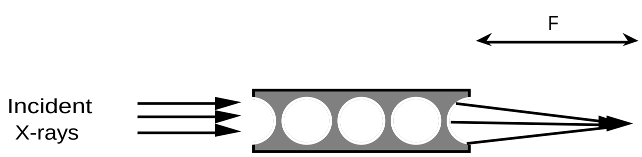
> Roentgen's first experiments convinced him that x-rays could not be concentrated by lenses; thirty years later his successors understood why. It may readily be shown that the focal length f of a single refracting surface of radius R is approximately R/δ. For several surfaces in series, arranged cooperatively, we have $1/f = \delta\,(1/R_1 + 1/R_2 + \text{etc.})$. To make a successful lens we require a large δ and slight absorption.

Unfortunately materials of large δ are also strong absorbers, the absorption coefficient increasing much more rapidly than δ with increasing atomic number. An element of low atomic number, such as beryllium, is indicated.

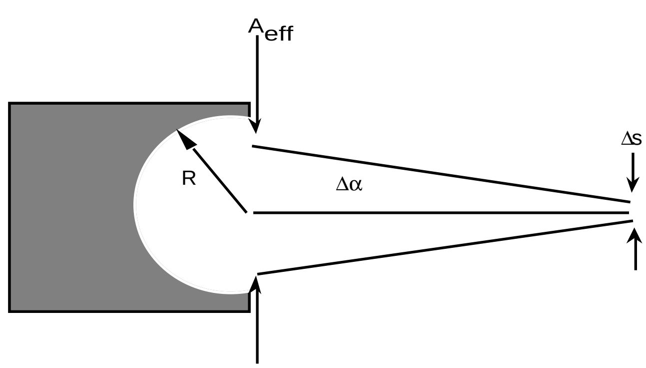
Compound Refractive Lens



Single Refractive Ler



Compound Refractive Len:



What has happened over the last several years that now makes the compound lens a possibility? The answer is high brilliance synchrotron radiation beams.

Consider a cylindrical hole of radius R machined into some material with index of refraction given by $n = 1-\delta$. This will act as a lens for x-rays with a focal length, f, given by:

$$1/f = \delta(1/R + 1/R)$$

or

$$f = R/2\delta$$

If a linear array of N holes are fabricated the focal length is given by:

$$f = R/2N\delta$$

Plugging in some numbers, suppose that R = 1 mm (see below), $\delta \approx 10^{-6}$, and N = 50 then the focal length, f, would be at 10 m. These lenses focus at rather larger distances and might be well adapted to the scale of synchrotron radiation beamlines.

These x-ray lenses behaves as a conventional lens and we can use the thin lens formula to describe its properties:

$$1/f = 1/F_1 + 1/F_2$$

where F_1 (F_2) is the source-to-lens (lens-to-image) distance.

Fresnel Zone Plates

Zone plates in the optical region of the spectrum have been well known for many years and in Baez 1952 suggested that zoneplates might be useful in the VUV region of the spectrum.

Again, due to their small size (typically considerably less than 1 mm in diameter) they only became practical with the availability of highly collimated synchrotron radiation.

Today, the use of zone plates for the two-dimensional focusing of x-rays is well established in the synchrotron community, especially in the soft x-ray microscopy community and becoming increasingly so in the hard x-ray regime.

Zone plates are circular diffraction gratings; that is structures composed of alternating concentric zones of two materials with different (complex) refractive indices.

The focusing capability is based on constructive interference of the wavefront modified by passage through the zone plate.

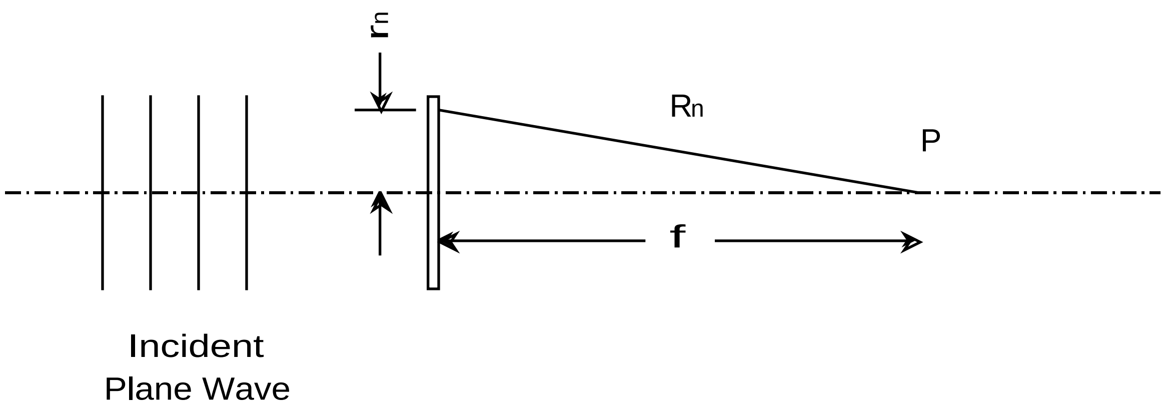
The wave that emerges from the zone plate is the superposition of spherical waves, one from each of the zones.

The wavefront modification is obtained through the introduction of a relative change in amplitude or phase in the beams emerging from two neighboring zones.

A zone plate is called an amplitude zone plate if the focusing results because waves of the "wrong" phase are removed by opaque (high absorption) zones.

It is called a phase zone plate if the phase change upon transmission through a zone is the mechanism for the focusing.

Consider the figure below where R_n is the distance from P to the outer boundary of the n^{th} "zone", which has as a radius r_n, and f is the focal length.



For constructive interference, the pathlength from the n^{th} zone and the $(n+2)^{th}$ zone should differ by a (multiple m of the) wavelength, or the pathlength of adjacent zones should differ by a (multiple m of the) half wavelength, i.e.,

$$R_n = f + m(\lambda/2)n \qquad n = 1, 2, 3.... ,$$

The radius, $r_{n,}$ of the n^{th} zone can be written as (assuming $f >> n\lambda$):

$$r_n = (mf\lambda n)^{1/2}$$

The focal length for the m^{th} order can be written as:

$$f_m = r_n^2/ m\lambda n$$

In general, the size of the focal spot from the zone plate is determined by the width of the outermost ring, Δr_{out}, and is given by:

$$\Delta x = 1.22\, \Delta r_{out}/m\ .$$

State of the art zone plate masks are fabricated with e-beam writers and then reproduced with x-ray lithographic techniques.

Zones plates with outermost ring widths of less than 30 nanometers can currently be fabricated.

If illuminated with an x-ray beam whose spatial coherence length is equal to or greater than the size of the zone plate, a diffraction-limited focus can be obtained.