# Using the Booster to Accelerate a Low-Emittance Beam

SRFEL-003

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Steve Milton proposed using the APS booster to accelerate a low emittance beam from a photoinjector. The beam would be extracted before the emittance degraded due to quantum excitation. This note assesses the feasibility of this idea.

## 1.0 Emittance Evolution

The differential equation for the emittance in the booster is

$$\frac{d\varepsilon}{dt} = \frac{Q}{2} - \frac{2\varepsilon}{\tau} - \frac{\varepsilon}{E}\frac{dE}{dt}$$

where Q is the quantum excitation rate, $\varepsilon$ is the emittance, $\tau$ is the damping time, and E is the energy. In order, the three terms on the right hand side give the contributions of quantum excitation, radiation damping, and adiabatic damping due to acceleration. One can compute an upper bound on the emittance by ignoring the damping terms and integrating over time from 0 to T, corresponding to acceleration from $E_1$ to $E_2$.:

$$\varepsilon < \varepsilon_{in} + \int_0^T \frac{Q}{2} dt = \varepsilon_{in} + \frac{1.079 \cdot 10^{-19} \cdot I_5}{3L \cdot \frac{d\gamma}{dt}} \left( \gamma^6 \Big|_{\gamma_1}^{\gamma_2} \right)$$

where L is the length of the accelerator, $\gamma$ is the relativistic factor, and $I_5$ is the fifth radiation integral (note that this integral is proportional to the number of cells). For the normal booster lattice, $I_5 = 3.45 \cdot 10^{-4}$, whereas for the low-emittance lattice (discussed in a previous note), $I_5 = 1.75 \cdot 10^{-5}$.

The booster acceleration rate is 33 keV/turn, or $5.25 \cdot 10^4 s^{-1}$.. Assuming an injected beam normalized emittance of 2nm per Steve's suggestion, one obtains the graph in figure 1. Unfortunately, for the normal booster lattice one sees that the quantum excitation effect is considerable and that the emittance does not, therefore, follow the adiabatic damping curve.

Another simple way to look at this is to compare the quantum excitation rate to the adiabatic damping rate. For the scheme to work, one wants the former much less than the latter. This type of consideration might provide a design tool if one considered building a machine based on this principle. The equation is

$$\frac{\varepsilon_{eq}}{\tau} \ll \frac{1}{2}\frac{\varepsilon_{in}}{E}\frac{dE}{dt}$$

The LHS of this equation is proportional to $E^5$, so things get pretty difficult (as we already knew) as the final energy increases. Accelerating at a higher rate is important, but it is hard to improve that by more than a factor of 30 or so over our present booster. It seems pretty clear that a low-emittance booster lattice is essential if this idea is to work.

A more complicated analysis involves integrating the emittance equation numerically. This gives the results in figure 2. This shows that the emittance “upper bound” is not a bad estimate of the actual emittance for our case.

It is worth asking how the transient emittance compares to the equilibrium emittance of the lattice. Figure 3 addresses this question. We see that below 2.1GeV, the equilibrium emittance is better than the transient emittance. Of course, when intra-beam scattering comes into play, it may be that the transient performance will look more attractive at low energies. It depends on the IBS scattering rate, which is beyond my scope here.

I also used **elegant** to track with synchrotron radiation effects. I assumed that the equilibrium emittance was half the normal values due to coupling. Results are shown in figures 4 through 6. Agreement is as expected given that I’m assuming full coupling in the **elegant** runs and that the emittance numbers from **elegant** also include an energy-spread contribution to the beam size. In order to keep the bunch short, I assumed a constant RF voltage of 9MV in the simulation of the low-emittance lattice.

## 2.0 FEL Optimization

I used Ming Xie’s formulae for SASE FEL performance (as incorporated into **sddssasefel**) to assess the potential of this scheme, using Steve’s parameters of a 1nC beam and an undulator having K=1.879 with a 2cm period.As one can see from figure 7, the results are promising. The oscillation in the results is presumably a result of the oscillations in the energy spread and bunch length from tracking.

## 3.0 Future Calculations

Several things will be explored in future notes:

1. Add intra-beam scattering to see if the beam emittance and energy spread are maintained.

2. Add impedances to see if the beam is stable. The difficulty here is to get a reasonable impedance model.
3. Since the equilibrium emittance is better below 2.1GeV, why not run the machine in a non-transient mode? What can we get this way?

**FIGURE 1. Upper bound on geometric emittances due to quantum excitation in a ramped booster**

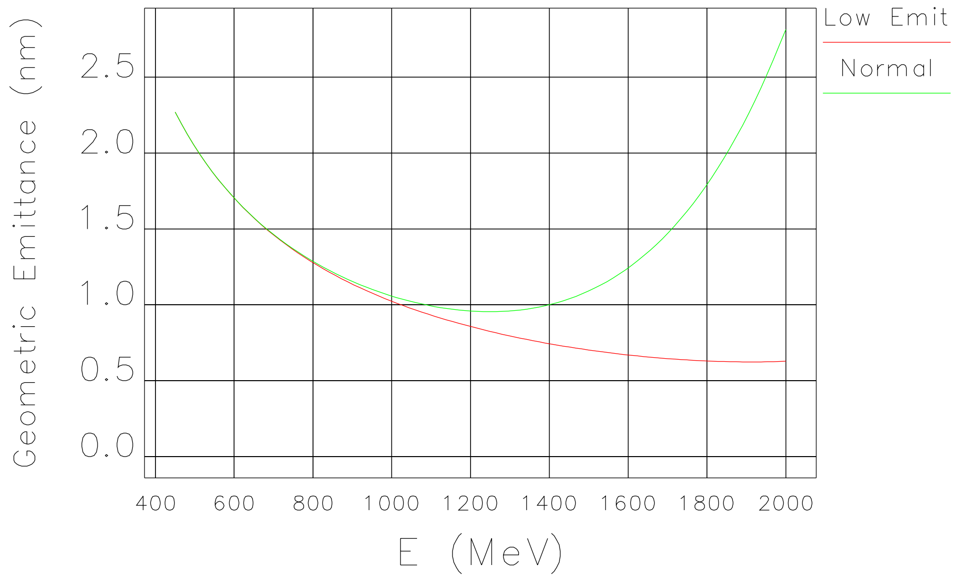


**FIGURE 2. Results of numerical integration of the emittance equation for a low-emittance injected beam**

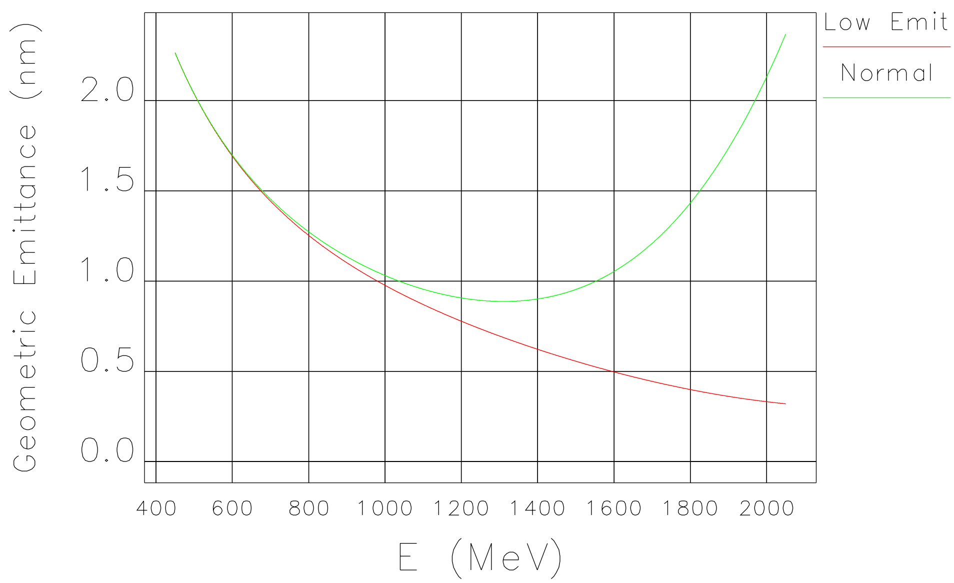

**FIGURE 3. Results of numerical integration for the low emittance lattice, compared to the equilibrium emittance.  Note that this graph goes up to 4GeV, unlike the others.**

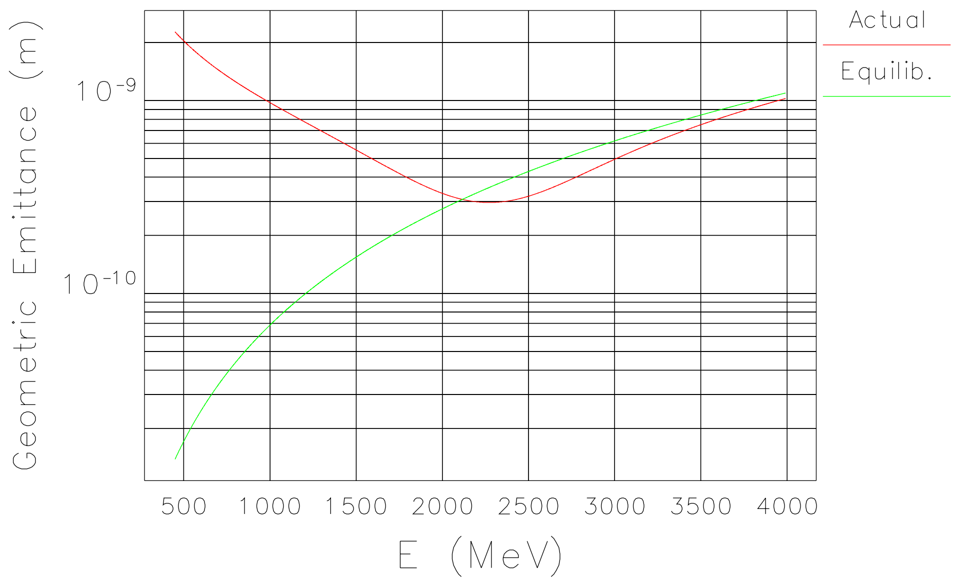


**FIGURE 4. Results of tracking with elegant assuming equal equilibrium emittances in x and y planes**

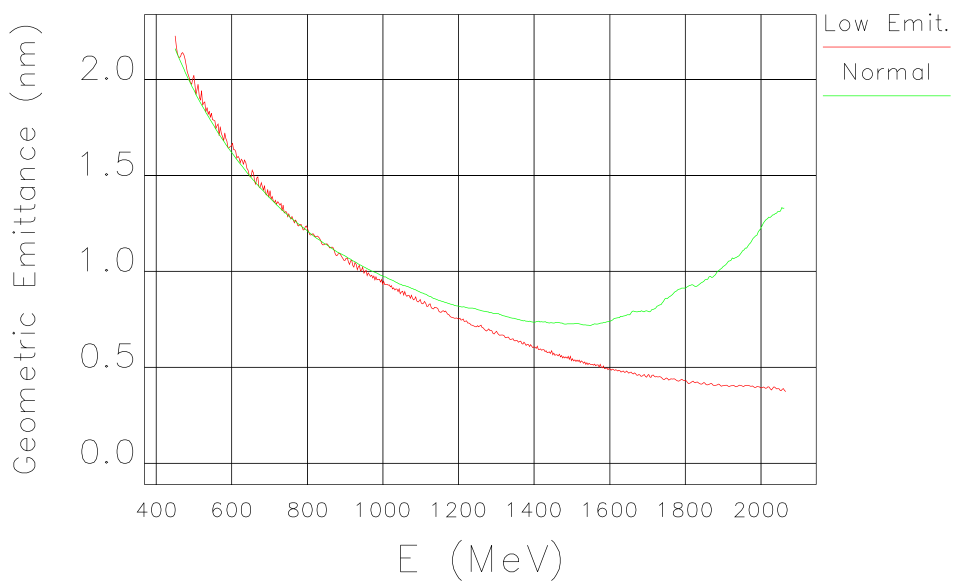

**FIGURE 5. Results for the fractional energy spread from tracking with elegant for the low-emittance lattice only. Normal lattice is similar.**

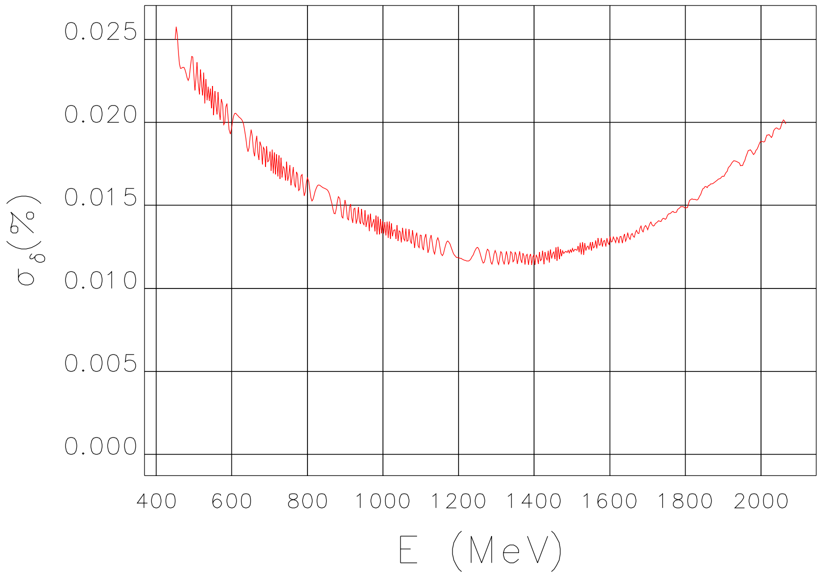


**FIGURE 6. Results for bunch length from tracking, to go with Figure 5.**

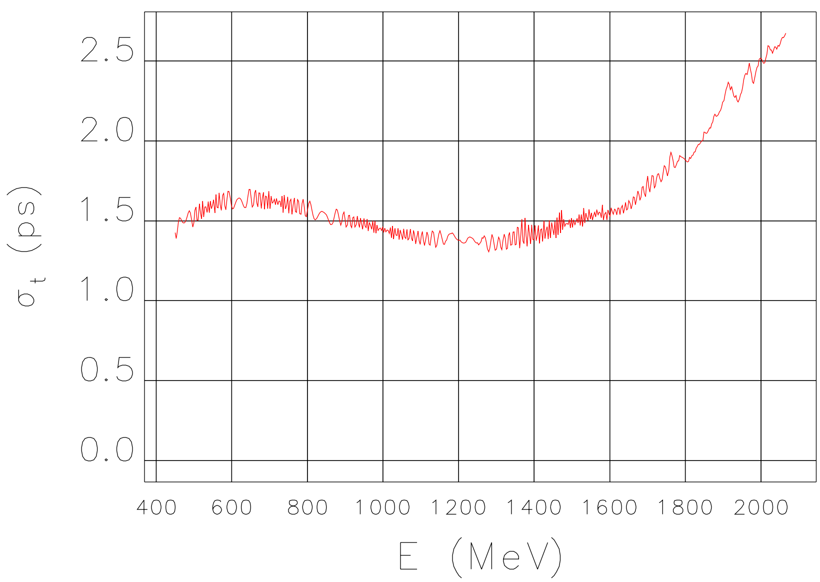

**FIGURE 7. SASE FEL performance for the low emittance booster plus photo-injector**