Thermal buckling parametric analysis for a diamond disk for a x-ray beamline at the Advanced Photon Source

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A thermal buckling analysis for a diamond disk in the commissioning window assembly designed for x-ray beamlines at the Advanced Photon Source is presented. The analytical solution together with associated numerical analysis help to predict the critical temperature of the diamond disk before a thermal buckle occurs. *© 1996 American Institute of Physics.*

I. INTRODUCTION

A commissioning filter and window assembly has been designed[1] for the Advanced Photon Source (APS) insertion-device (undulator A) front ends. A multipurpose transmitting filter/beam position monitor (BPM) in this assembly is made of a 127-μm-thick, 1-inch-diameter diamond disk. The high heat flux from the insertion device deposited on the thin diamond disk induces high thermal stresses. Because diamond is a brittle material, thermal buckling is a critical issue. The temperature,[2] thermal stress, and thermal buckling closed-form solutions for a thin disk are presented in this paper. The heat flux from the x-ray is simulated as an axisymmetrical Gaussian-distributed heat flux. The finite element code ANSYS was also used to simulate the real boundary conditions.

II. NOMENCLATURE

Critical buckling temperature	T_{cr}	[°C]
Temperature	T	[°C]
Max. temperature	T_{max}	[°C]
Potential energy	V	[N m]
Thermal conductivity	k	[W/(m K)]
Max. heat flux	q_0	[W/m^2]
Thickness	t	[m]
Radius of the thin disk	a	[m]
Standard deviation	σ_0	[m]
Plate stiffness	D	[N m]
Off plane deflection	w	[m]
Radial deformation	u_{r0}	[m]
Radial stress before buckle	σ_{rr0}	[MPa]
Young's modulus	E	[MPa]
Thermal expansion coefficient	α	[1×10^{-6}/K]

III. CLOSED-FORM SOLUTION

Considering a thin disk subjected to an axisymmetrical Gaussian-type power heating (in Fig. 1), in-plane radial compressive stress will occur in the center. As a result, buckling will occur if the "critical load" is reached. It is convenient to define the so-called critical load, that is the minimum heat load needed before the material buckles. As to the thermo-driven load, the peak temperature will be more appropriate to be used as an index in determining when the buckle takes place.[3] In general, even though the circular disk is subjected to the axisymmetrical heat load, the buckling deflection w might not be axisymmetrical. In other words, "skirt wrapping" might occur on the periphery of the disk. In our problem, we assume the power distribution striking on the surface is much narrower than the disk itself, therefore the buckling will take place locally within the center region, and the skirt wrapping effect is assumed to be negligible in our problem. Thus the off-plane deflection w is assumed to be a function of r only. The potential energy equation is written as

$$V=\frac{D}{2}\int_0^a r\,dr\left[\left(\nabla^2 w\right)^2-2(1-\nu)\left(\frac{\partial^2 w}{\partial r^2}\frac{1}{r}\frac{\partial w}{\partial r}\right)\right]+\frac{t}{2}\int_0^a r\,dr\left(\sigma_{rr0}\left(\frac{\partial w}{\partial r}\right)^2\right). \tag{1}$$

The general solutions[4] for u_{r0} and σ_{rr0} are

$$u_{r0}=(1+\nu)\alpha\frac{1}{r}\int_0^r T(r')r'\,dr'+K_1 r+\frac{K_2}{r}, \tag{2}$$

$$\sigma_{rr0}=\frac{-\alpha E}{r^2}\int_0^r T(r')r'\,dr'+\frac{E}{1-\nu^2}\left[\frac{K_1(1+\nu)-K_2(1-\nu)}{r^2}\right]. \tag{3}$$

The boundary conditions are

$$u_{r0}=0 \text{ as } r\to 0 \text{ and } T=0,\ \sigma_{rr0}=0 \text{ as } r\to a.$$

The governing equation for a thin disk with an axisymmetrical, Gaussian-distributed heat flux is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{q_0}{kt}\exp\left(-\frac{r^2}{2\sigma_0^2}\right)=0.$$

The boundary conditions (in Fig. 1) are

T = 0 when r = a, and T = finite when r = 0.

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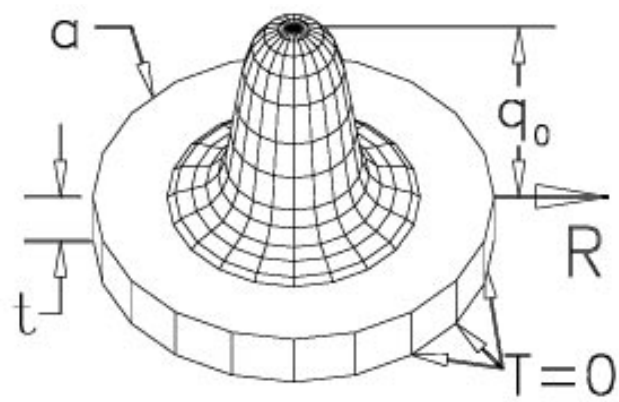

FIG. 1. The boundary conditions for a thin disk.

The temperature field $T(r)$,[2] is given as:

$$T(r) = \frac{q_0\sigma_0^2}{2kt}\left[E_1\left(\frac{r^2}{2\sigma_0^2}\right) - E_1\left(\frac{a^2}{2\sigma_0^2}\right) + \ln\left(\frac{r^2}{2\sigma_0^2}\right) - \ln\left(\frac{a^2}{2\sigma_0^2}\right)\right], \quad (4)$$

where $E_1(x)$ denotes *exponential function of the first kind.* The constants K_1 & K_2 in Eqs. (2) and (3) are found to be

$$K_1 = (1-\nu)\frac{\alpha}{a^2}\left(\theta(a) - \frac{q_0\sigma_0^4}{4kt}\right), \quad (5)$$

$$K_2 = -(1+\nu)\alpha\frac{q_0\sigma_0^4}{4kt}, \quad (6)$$

where

$$\theta(a) = \int Trdr = \frac{q_0\sigma_0^4}{2kt}\left\{\begin{array}{l}\frac{r^2}{2\sigma_0^2}E_1\left(\frac{a^2}{2\sigma_0^2}\right) + \frac{r^2}{2\sigma_0^2}\ln\left(\frac{a^2}{2\sigma_0^2}\right) - \\ \frac{r^2}{2\sigma_0^2}\left[\ln\left(\frac{a^2}{2\sigma_0^2}\right) - 1\right] + E_2\left(\frac{r^2}{2\sigma_0^2}\right)\end{array}\right\}, \quad (7)$$

and where $E_2(x)$ is the *exponential function of the second kind.* The maximum temperature $T_{\max}$ must be located at the center of the disk, which results in:

$$T_{\max} = \frac{-q_0\sigma_0^2}{2kt}\left[E_1\left(\frac{a^2}{2\sigma_0^2}\right) + \ln\left(\frac{a^2}{2\sigma_0^2}\right) + \gamma\right] = \frac{-q_0\sigma_0^2}{2kt}\varepsilon\left(\frac{a^2}{2\sigma_0^2}\right). \quad (8)$$

$\gamma = 0.57721\ldots$ is Euler's constant. And the temperature field in Eq. (4) can be rewritten as

$$T(r) = -\frac{T_{\max}}{\varepsilon\left(\frac{a^2}{2\sigma_0^2}\right)}\left[\begin{array}{l}E_1\left(\frac{r^2}{2\sigma_0^2}\right) - E_1\left(\frac{a^2}{2\sigma_0^2}\right) \\ + \ln\left(\frac{r^2}{2\sigma_0^2}\right) - \ln\left(\frac{a^2}{2\sigma_0^2}\right)\end{array}\right]. \quad (9)$$

By substituting Eq. (9) into Eq. (7), and then Eq. (7) into Eq.(5), σ_{rr0} in Eq. (3) can be rewritten as

$$\sigma_{rr0} = T_{\max}F(r). \quad (10)$$

The off-plane deflection w satisfies

$$w = w_{,r} = 0 \quad \text{as } r \to a \text{ and } w_{,r} = 0 \quad \text{as } r \to 0,$$

so w must be in the form

$$w(r) = (r^2 - a^2)^2 \sum_{n=0}^{\infty} C_n r^{2n} = \sum_{n=0}^{\infty} R_n C_n. \quad (11)$$

To find the critical buckling temperature T_{cr}, a minimum state of the potential energy has to be reached. Substituting Eqs. (10) and (11) into Eq. (2), then, it leads to a infinite Eigen system equation:

$$\frac{\partial V}{\partial C_i} = (A + T_{cr}B)C = 0, \text{ or } (A_{ij} + T_{cr}B_{ij})C_j = 0, \quad (12)$$

in which only the undetermined coefficients C_i and T_{cr} are involved in the system equations. For those coefficients to have a nontrivial solution, the determinant of the matrix has to be zero. That is, T_{cr} is represented as a value in the matrix. Some numerical results are listed by choosing different numbers of undermined coefficients C_i. The component of matrices A and B are summarized as:

$$A_{ij} = D\int_0^a\left[\nabla^2 R_i \nabla^2 R_j - (1-\nu)\frac{(R_i'' R_j' + R_j'' R_i')}{r} rdr\right], \quad (13)$$

$$B_{ij} = t\int_0^a\left[R_i' R_j' F(r)\right] rdr. \quad (14)$$

It is interesting to see that the second term of the integrals in Eq. (13) vanishes.

IV. PARAMETRIC STUDIES

We assumed the diamond disk is the first component in the x-ray path. The total power deposited on the diamond disk is calculated by the code "UA"[5] (see [2] for more details.). Substituting Eqs. (5), (6), and (7) into Eq. (3), σ_{rr0} becomes

$$\sigma_{rro} = \frac{q_0\sigma_0^2\alpha E}{4kta^2r^2}\left\{\begin{array}{l}2a^2\sigma_0^2 + 2r^2\sigma_0^2 E_2\left(\frac{a^2}{2\sigma_0^2}\right) - \\ 2r^2\sigma_0^2 - 2a^2\sigma_0^2 E_2\left(\frac{r^2}{2\sigma_0^2}\right) - \\ a^2r^2\ln\left(\frac{a^2}{2\sigma_0^2}\right) + a^2r^2\ln\left(\frac{r^2}{2\sigma_0^2}\right)\end{array}\right\}.$$

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For simplicity, if only three terms are chosen, the off-plane deflection w in Eq. (11) can be simplified as:

$$w = (r^2 - a^2)(C_o + C_1 r^2 + C_2 r^4),$$

and matrix A in Eq. (13) is solved as:

$$\frac{Et^3}{12(1-\nu^2)}\begin{bmatrix} \frac{32a^6}{3} & \frac{8a^8}{3} & \frac{16a^{10}}{15} \\ \frac{8a^8}{3} & \frac{64a^{10}}{15} & \frac{8a^{12}}{3} \\ \frac{16a^{10}}{15} & \frac{8a^{12}}{3} & \frac{256a^{14}}{105} \end{bmatrix}.$$

Figure 2 shows the T_{max} (in Eq. (8)) vs. t (μm; thickness) and $2a$ (mm; diameter) of the diamond disk (k = 1000 W/(m K)). Figure 3 shows the maximum σ_{rr0} (MPa) during operation vs. t and $2a$ (E = 11.8 × 10^5 MPa, $\alpha = 2.6 \times 10^{-6}$/K, $\nu = 0.15$). The yield strength of diamond is about 900 MPa on the nucleation side and about 350 MPa on the growth side (the off-plane direction). The temperature is uniform along the off-plane direction; the thermal stress is almost zero in that direction. Therefore, σ_{rr0} in Fig. 3 is compared with the yield strength of 900 MPa. Figure 4 shows the T_{cr} vs. t (μm) and $2a$ (mm) of the disk. For a given a, t, q_0, σ_0, E, α, and ν, T_{cr} is determined. If the calculated T_{max} (in Fig. 2) during operation is smaller than T_{cr} (in Fig. 4), the diamond disk is free from thermal buckling.

V. PARAMETRIC STUDIES WITH ANSYS CODE

This section will discuss the parametric studies (using ANSYS) for various thicknesses of the diamond disk as the first component in the x-ray path. The thickness of the diamond disk (contamination barrier) is varied from 127 to 200 μm in the following analyses.

The diamond disk is clamped on a 7 × 7 mm^2 opening in the fixed mask and has a 4.5 × 4.5 mm^2 aperture upstream[1] as shown in Fig. 5. The power absorption (generated by Code "HIT"[6]) in Fig. 5 is for a 175-μm-thick diamond disk. The thermal contact resistances between the diamond disk and fixed mask are modeled as a 0.5-mm-thick element with various k (from 0.5 W/(m K) to 5 W/(m K)). At normal operation conditions (no missteering), the fixed mask is subjected to an 11 W/mm^2 heat flux due to its smaller aperture size than the x-ray beam size (see [7, 8] for more details); Fig. 6 shows the temperature contour of such a case when k (contact conductance) = 5 W/(m K). The temperature rises in the fixed mask become important for the diamond disk when the T_{max} (during operation) of the disk is close to its T_{cr}.

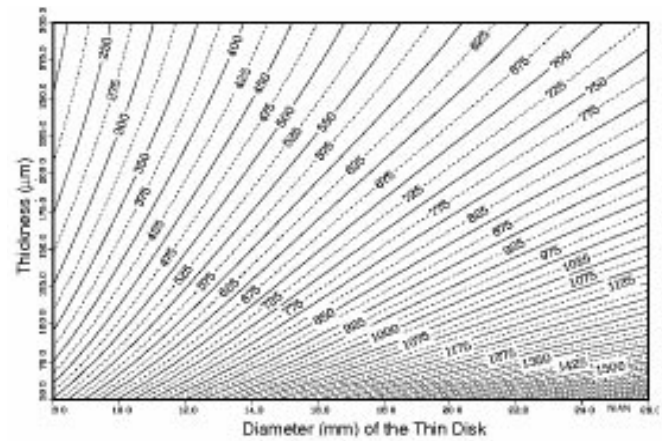

FIG. 2. The maximum temperature (°C) vs. diameter (mm) and thickness (μm) of a thin disk (k = 1000 W/(m K)).

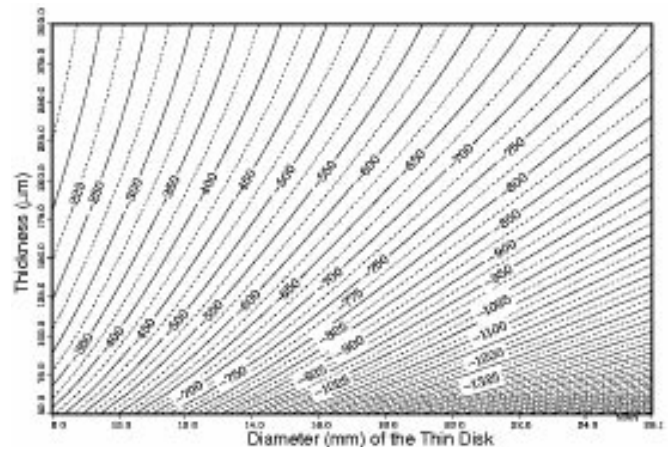

FIG. 3. The maximum radial stress (MPa) vs. diameter (mm) and thickness (μm) of a thin disk (k = 1000 W/(m K)).

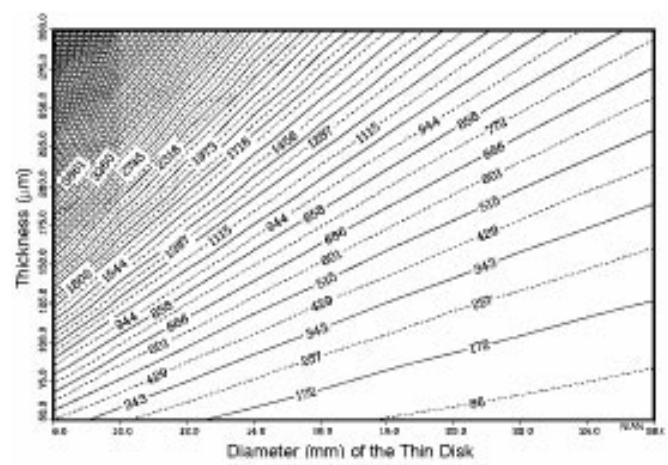

FIG. 4. The critical temperature (T_{cr}) (°C) vs. diameter (mm) and thickness (μm) of a thin disk (k = 1000 W/(m K)).

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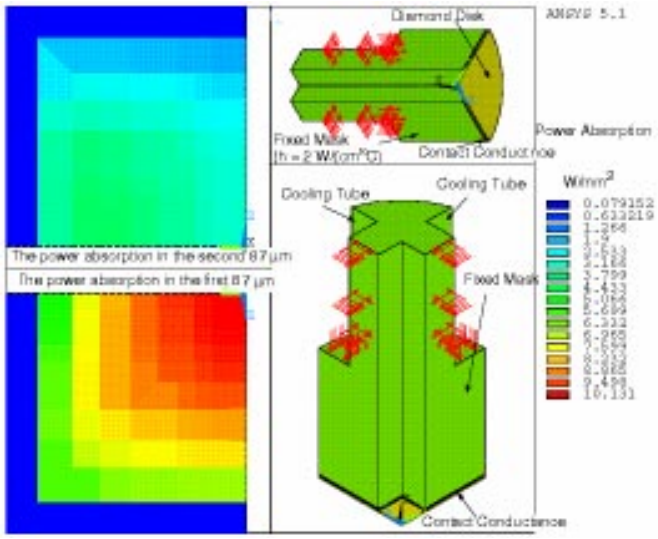

FIG. 5. The computer model for the diamond disk.

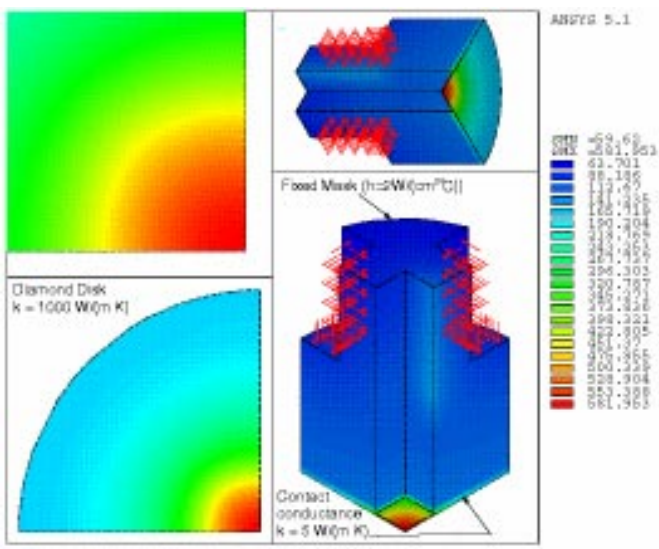

FIG. 6. The temperature contour of the assembly.

Figure 7 shows the calculated T_{max} and T_{cr} for both the various thicknesses of the diamond disk and the contact conductance. For the radiatively cooled case, the curves of T_{cr} and T_{max} intersect at about 170 µm, which implies that at thicknesses larger than 170 µm, the diamond disk is free from buckling with any cooling scheme. However, the better the cooling scheme, the less thick the disk can be and still avoid thermal buckling. Figure 8 shows the maximum compressive stress σ_y (MPa) during operation and the maximum compressive stress at buckle. The maximum compressive stress σ_z is about 3-5% smaller than σ_y. The T_{max} and T_{cr} for a 250-µm-thick beryllium window (sitting on a 8 × 5.5 mm^2 opening in the fixed mask) after a 175-µm-diamond disk are 95°C and > 200°C, respectively[8] (σ_{eff} = 120 MPa).

The thermal buckling analyses for a diamond disk after various thicknesses of graphite filter are presented in [8]. Also, that analysis[8] shows that a 127-µm-thick diamond disk after a 100-µm-thick graphite filter is safe (T_{max} =340°C,

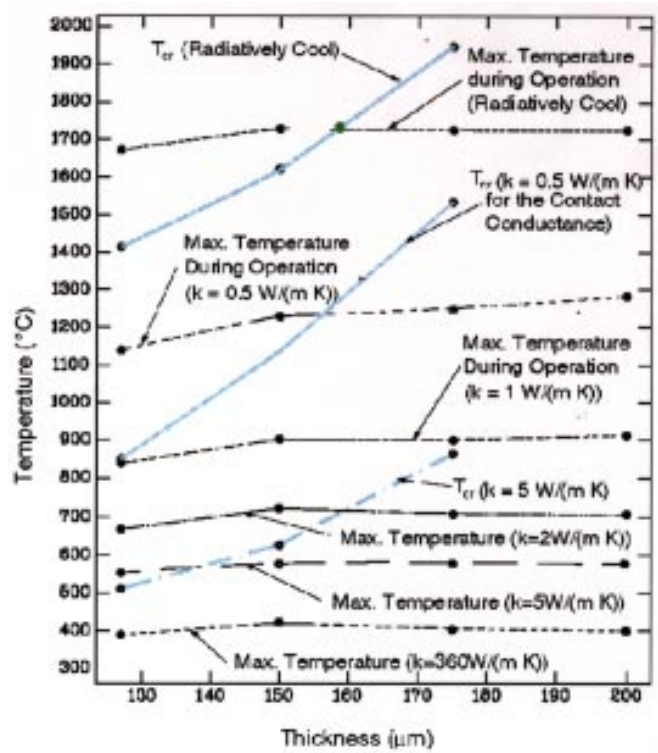

FIG. 7. The calculated maximum temperature during operation and the critical temperature (T_{cr}) for various thicknesses of diamond disk with various conduct conductances (k).

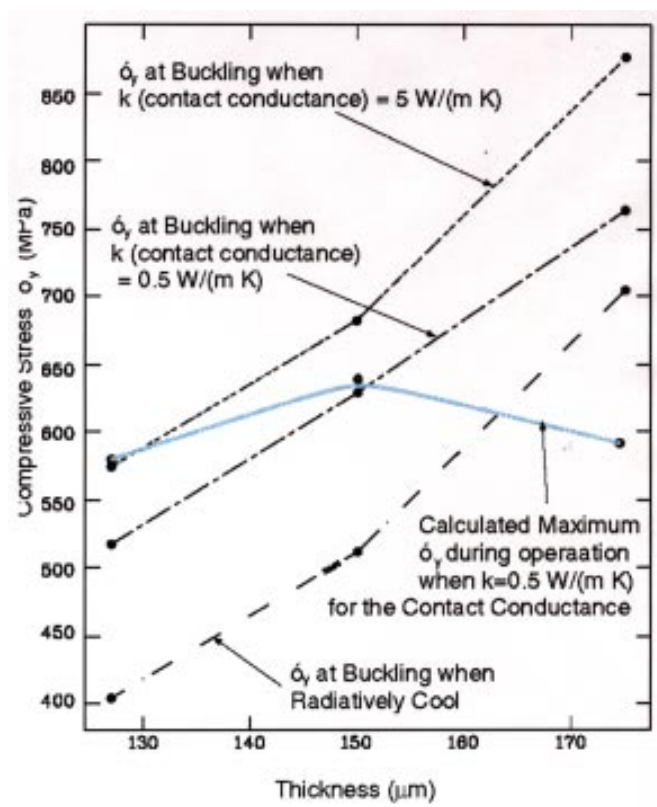

FIG. 8. The calculated maximum compressive stress σ_y (MPa) during operation and the maximum compressive stress σ_y (MPa) at buckle.

T_{cr} > 400°C, and σ_{eff} = 305 Mpa when k the contact conductance equals 5 W/(m K)). A 125-μm-thick graphite filter (T_{max} =2200°C, T_{cr} > 2300°C[8]) or thicker is free from thermal buckle.

ACKNOWLEDGMENTS

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